$$\boxed{2\sin x - 3\cos x \geq 3}$$

3° modo: passando a $t = \operatorname{tg}\frac{x}{2}$.

$$\boxed{\begin{aligned} \sin x &= \frac{2t}{1+t^2} \\ \cos x &= \frac{1-t^2}{1+t^2} \end{aligned}}$$

$\forall x$ t.c. $\frac{x}{2} \neq (2k+1)\frac{\pi}{2}$ $\quad t = \operatorname{tg}\frac{x}{2}$

$x \neq (2k+1)\pi$.

Dimostriamole:

$$\sin(2x) = 2\sin x\cos x = 2\underbrace{\frac{\sin x}{\cos x}}_{\operatorname{tg}(x)} \cdot \underbrace{\cos^2 x}_{\frac{1}{1+\operatorname{tg}^2 x}} = \frac{2\operatorname{tg}x}{1+\operatorname{tg}^2 x}$$

$$\frac{1}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + \operatorname{tg}^2 x$$

$$\Rightarrow \cos^2 x = \frac{1}{1+\operatorname{tg}^2 x}$$

metto x al posto di 2x

$$\sin x = \frac{2\operatorname{tg}\frac{x}{2}}{1+\operatorname{tg}^2\frac{x}{2}} = \frac{2t}{1+t^2}$$

$$\cos(2x) = 2\underbrace{\cos^2 x}_{\frac{1}{1+\operatorname{tg}^2 x}} - 1 = \frac{2}{1+\operatorname{tg}^2 x} - 1 = \frac{2-1-\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x} = \frac{1-\operatorname{tg}^2 x}{1+\operatorname{tg}^2 x}$$

mettendo x al posto di 2x

$$\cos x = \frac{1-\operatorname{tg}^2\left(\frac{x}{2}\right)}{1+\operatorname{tg}^2\left(\frac{x}{2}\right)} = \frac{1-t^2}{1+t^2}$$

$$2\sin x - 3\cos x \geq 3$$

Passo a $\boxed{t = tg\,\frac{x}{2}}$

$$\boxed{x \neq \pi + 2k\pi}$$

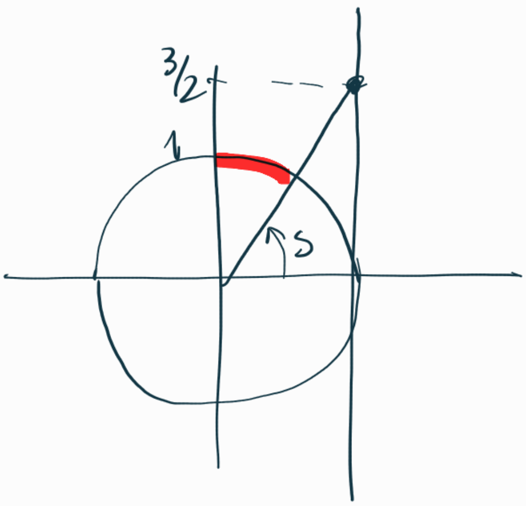
$$\boxed{2\cdot\frac{2t}{1+t^2} - 3\,\frac{(1-t^2)}{1+t^2} \geq 3}$$

molt. per $1+t^2>0$

$$4t - 3 + \cancel{3t^2} \geq 3(1+t^2) = 3 + \cancel{3t^2}$$

$$4t \geq 6$$

$$\boxed{t \geq \frac{3}{2}}$$



$$tg\left(\frac{x}{2}\right) \geq \frac{3}{2}$$

$$\frac{x}{2} = s$$

$$tg(s) \geq \frac{3}{2}$$

$$\Updownarrow$$

$$\operatorname{arctg}\frac{3}{2} \leq s < \frac{\pi}{2}$$

$$s = \frac{x}{2}$$

$$2\operatorname{arctg}\frac{3}{2} + 2k\pi \leq x \leq \pi + 2k\pi$$

Verifico la diseq^{ne} in $x = \pi + 2k\pi$ è sol^{ne}.

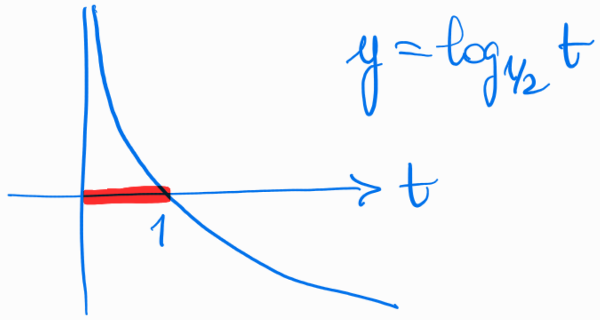
$$\underbrace{2\sin\pi}_{0} - 3\underbrace{\cos\pi}_{-1} \overset{?}{\geq} 3 \qquad \text{OK.}$$

Trovare il dominio di

$$f(x) = \sqrt{\log_{1/2}\left(\log_2\left(2\sin^2 x - \cos x\right)\right)}$$

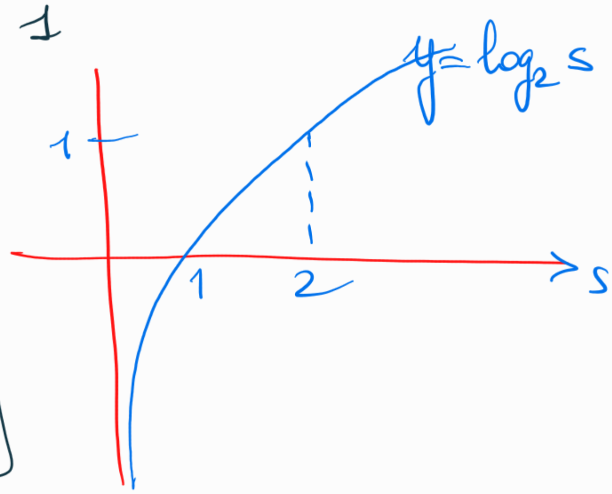
$$\log_{1/2}\big(\underbrace{\log_2\left(2\sin^2 x - \cos x\right)}_{t}\big) \geq 0$$

$$\log_{1/2} t \geq 0 \iff 0 < t \leq 1$$



$$0 < \log_2 (\underbrace{2\sin^2 x - \cos x}_{s}) \leq 1$$

$$1 < s \leq 2.$$



$$\boxed{1 \underset{A}{<} 2\sin^2 x - \cos x \underset{B}{\leq} 2}$$

A $\quad 2\sin^2 x - \cos x - 1 > 0 \qquad \sin^2 x = 1 - \cos^2 x$

$$2 - 2\cos^2 x - \cos x - 1 > 0$$

$$2\cos^2 x + \cos x - 1 < 0$$

$\boxed{t = \cos x}$ $\qquad 2t^2 + t - 1 < 0$

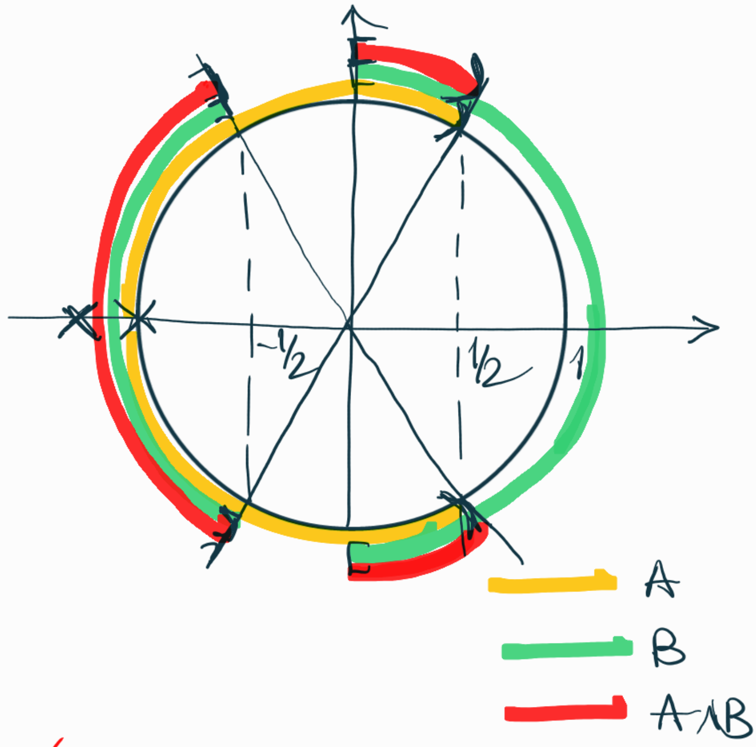
$$t_{1,2} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

$$-1 < t < \frac{1}{2}.$$

$$-1 < \cos x < \frac{1}{2}$$

solⁿ di A $\qquad \frac{\pi}{3} + 2k\pi \leq x \leq \frac{5}{3}\pi + 2k\pi$

$$x \neq \pi + 2k\pi$$



(B) $2\sin^2 x - \cos x \le 2$.

$\cancel{2} - 2\cos^2 x - \cos x \le \cancel{2}$.

$2\cos^2 x + \cos x \ge 0$.

$\cos x\,(2\cos x + 1) \ge 0$

$\left(\cos x \le -\frac{1}{2}\right) \vee \left(\cos x \ge 0\right)$

$$\boxed{\begin{aligned}
\operatorname{dom} f = \Big\{x :\ & \left(\tfrac{\pi}{3}+2k\pi < x \le \tfrac{\pi}{2}+2k\pi\right) \vee \\
\vee\ & \left(\tfrac{2}{3}\pi+2k\pi \le x \le \tfrac{4}{3}\pi+2k\pi,\ x \ne \pi+2k\pi\right) \vee \\
\vee\ & \left(\tfrac{3}{2}\pi+2k\pi \le x < \tfrac{5}{3}\pi+2k\pi\right)\Big\}
\end{aligned}}$$

Risolvere la diseq^{ne}

$$\boxed{2\sin x - 1 < 2\cos x - tg\, x.}$$

$$\boxed{x \neq \frac{\pi}{2}(2k+1)}$$

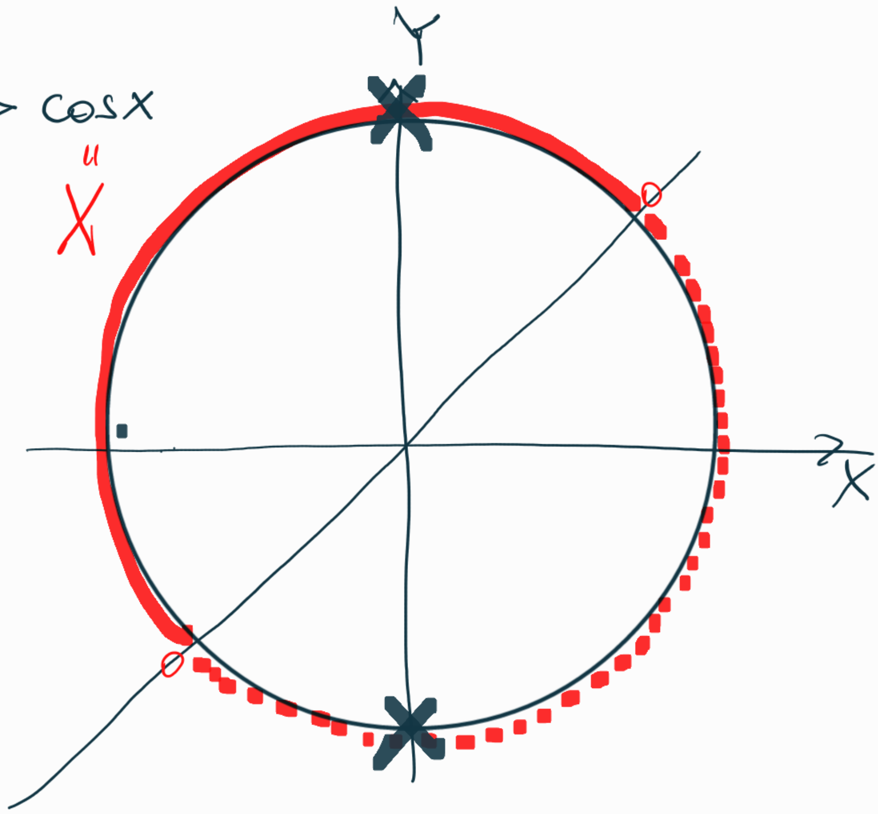
$$2\sin x - 1 < 2\cos x - \frac{\sin x}{\cos x}$$

$$\frac{2\sin x\cos x - \cos x - 2\cos^2 x + \sin x}{\cos x} < 0$$

$$\frac{\sin x\,(2\cos x + 1) - \cos x\,(2\cos x + 1)}{\cos x} < 0$$

$$\frac{\overbrace{(\sin x - \cos x)}^{A}\overbrace{(2\cos x + 1)}^{B}}{\underbrace{\cos x}_{C}} < 0$$

$$A > 0 \iff \underset{\text{"}Y\text{"}}{\sin x} > \underset{\text{"}X\text{"}}{\cos x}$$



... da completare ...