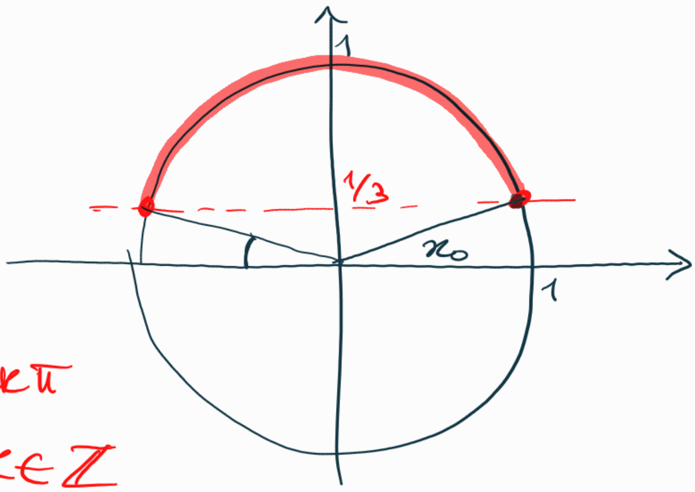
Uso delle funzioni trigonometriche inverse per le disequazioni

$$\boxed{\begin{array}{l}\text{Risolvere la diseq}^{ne} \\ \qquad 3\sin x \geq 1.\end{array}}$$



$$\sin x \geq \frac{1}{3}$$

Sol^{ne}

$$x_0 + 2k\pi \leq x \leq \pi - x_0 + 2k\pi$$

$k \in \mathbb{Z}$

dove $x_0 = \arcsin \frac{1}{3}$

$$\arcsin \frac{1}{3} + 2k\pi \leq x \leq \pi - \arcsin \frac{1}{3} + 2k\pi \qquad (k \in \mathbb{Z})$$

Avevamo risolto (in tre modi) la diseq

$$\sin x - \sqrt{3}\cos x \geq \underbrace{\sqrt{3}}_{\approx 1,7}$$

mettiamo $\frac{3}{2}$ al posto di $\sqrt{3}$

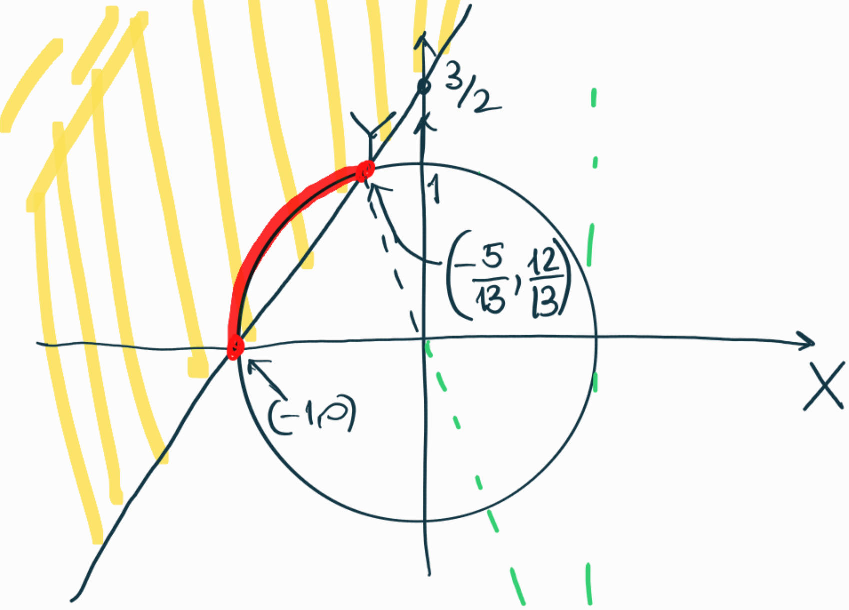
$$\sin x - \frac{3}{2}\cos x \geq \frac{3}{2}$$

$$\boxed{2\sin x - 3\cos x \geq 3}$$

1° modo (Intersezione con la circ. goniometrica)

$\sin x = Y$
$\cos x = X$

$$\begin{cases} X^2 + Y^2 = 1 \\ 2Y - 3X \geq 3 \iff Y \geq \frac{3}{2}(1+X) \end{cases}$$



Intersezioni circonferenza – retta

$$\begin{cases} X^2 + Y^2 = 1 \\ Y = \frac{3}{2}(X+1) \end{cases} \Rightarrow X^2 + \frac{9}{4}(X+1)^2 = 1$$

$$4X^2 + 9X^2 + 18X + 9 = 4$$

$$13X^2 + 18X + 5 = 0$$

$$X = \frac{-9 \pm \sqrt{81-65}}{13} = \frac{-9 \pm 4}{13} = \begin{cases} -1 \\ -\frac{5}{13} \end{cases}$$

$$X = -1 \Rightarrow Y = \frac{3}{2}(X+1) = 0$$

$$X = -\frac{5}{13} \Rightarrow Y = \frac{3}{2}(X+1) = \frac{3}{2} \cdot \frac{8}{13} = \frac{12}{13}$$

due pti: $(-1,0)$, $\left(-\frac{5}{13}, \frac{12}{13}\right)$

Sol^{ni} $\quad x_0 + 2k\pi \leq x \leq \pi + 2k\pi$

Chi è x_0?

$$\cos x_0 = -\frac{5}{13}$$

$$\sin x_0 = \frac{12}{13}$$

$$\operatorname{tg} x_0 = \frac{\sin x_0}{\cos x_0} = -\frac{12}{5}$$

$$x_0 = \arccos\left(-\frac{5}{13}\right)$$ OK!

$$x_0 = \cancel{\arcsin\left(\frac{12}{13}\right)} = \pi - \arcsin\left(\frac{12}{13}\right)$$

$$x_0 = \cancel{\operatorname{arctg}\left(-\frac{12}{5}\right)} = \pi + \operatorname{arctg}\left(-\frac{12}{5}\right)$$

$$= \pi - \operatorname{arctg}\left(\frac{12}{5}\right)$$

<u>2° modo</u> (metodo dell'angolo aggiunto)

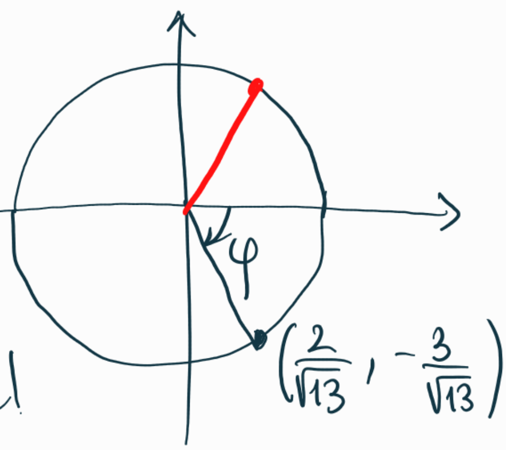
$$A \sin x + B \cos x = \sqrt{A^2+B^2}\,\sin(x+\varphi) \qquad (A,B) \neq (0,0)$$

dove
$$\begin{cases} \cos\varphi = \dfrac{A}{\sqrt{A^2+B^2}} \\ \sin\varphi = \dfrac{B}{\sqrt{A^2+B^2}} \end{cases}$$

Nel nostro caso $A = 2$ $B = -3$, $\sqrt{A^2+B^2} = \sqrt{13}$

$$2\sin x - 3\cos x = \sqrt{13}\,\sin(x+\varphi)$$

dove
$$\begin{cases} \cos\varphi = \dfrac{2}{\sqrt{13}} \\ \sin\varphi = -\dfrac{3}{\sqrt{13}} \end{cases}$$



$$\varphi = \cancel{\arccos\frac{2}{\sqrt{13}}} = -\arccos\frac{2}{\sqrt{13}}$$ OK!

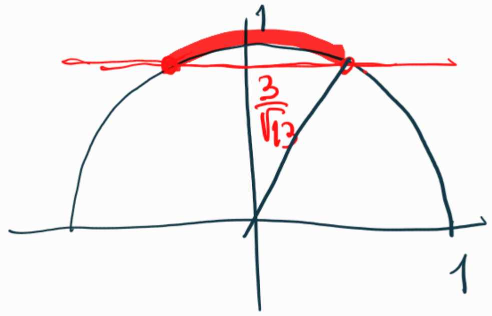
$$\varphi = \arcsin\left(-\frac{3}{\sqrt{13}}\right) = -\arcsin\left(\frac{3}{\sqrt{13}}\right)$$ OK!

$$\varphi = \operatorname{arctg}\left(-\frac{3}{2}\right) = -\operatorname{arctg}\left(\frac{3}{2}\right) \qquad \text{ok!}$$

La diseq^{ne} diventa

$$\sqrt{13}\ \operatorname{sen}(\underbrace{x+\varphi}_{t}) \geq 3$$

$$\sin t \geq \frac{3}{\sqrt{13}} \iff \arcsin\frac{3}{\sqrt{13}} + 2k\pi \leq \underset{\underset{x+\varphi}{\shortparallel}}{t} \leq \pi - \arcsin\frac{3}{\sqrt{13}} + 2k\pi$$



$$\arcsin\frac{3}{\sqrt{13}} + 2k\pi \leq x+\varphi \leq \pi \underbrace{\boxed{-\arcsin\frac{3}{\sqrt{13}}}}_{\varphi} + 2k\pi.$$

$$-\varphi \ + 2k\pi \leq x+\varphi \leq \pi + \varphi \ + 2k\pi$$

$$\underbrace{-2\varphi}_{\shortparallel} + 2k\pi. \leq x \leq \pi \ + 2k\pi$$

$$2\operatorname{arctg}\frac{3}{2} = 2\arcsin\frac{3}{\sqrt{13}}$$

E' la stessa sol^{ne} di prima?

Prima avevamo trovato

$$\arccos\left(\frac{-5}{13}\right) + 2k\pi. \leq x \leq \pi + 2k\pi$$

$$2\arcsin\left(\frac{3}{\sqrt{13}}\right) \overset{?}{=} \arccos\left(-\frac{5}{13}\right)$$

Entrambi gli angoli stanno tra [0 e π].

Sono uguali se e solo se i loro coseni sono uguali

$$\cos\left(2\arcsin\frac{3}{\sqrt{13}}\right) \stackrel{?}{=} \cos\left(\arccos\left(-\frac{5}{13}\right)\right) = -\frac{5}{13}$$

$$\cos(2t) = 1 - 2\sin^2 t$$

$$\cos\left(2\arcsin\frac{3}{\sqrt{13}}\right) = 1 - 2\left(\sin\left(\arcsin\frac{3}{\sqrt{13}}\right)\right)^2 =$$

$$= 1 - 2\cdot\left(\frac{3}{\sqrt{13}}\right)^2 = 1 - \frac{18}{13} = -\frac{5}{13} \quad \text{ok}$$