Funzioni trigonometriche inverse:

$f(x) = \sin x : \mathbb{R} \longrightarrow \mathbb{R}$

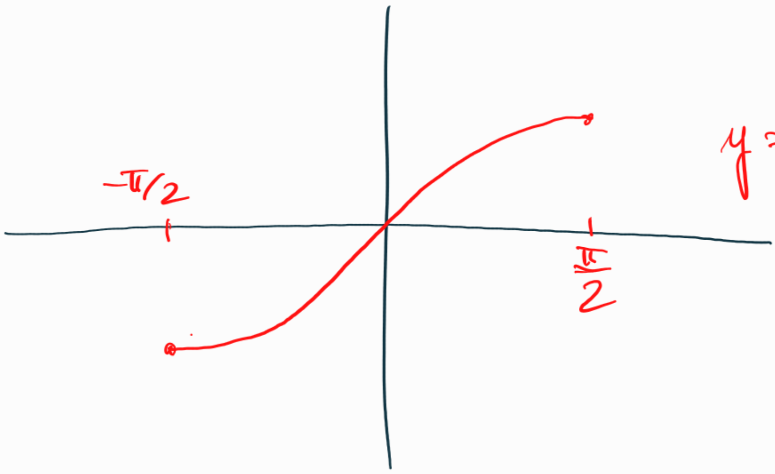
non è iniettiva né suriettiva.

La suriettività si recupera restringendo l'insieme di arrivo a $[-1,1]$

Per l'iniettività si deve scegliere un intervallo in cui $\sin x$ è strett. crescente.

Per convenzione, si sceglie $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

$$\sin x \Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1,1].$$
$$x \longmapsto \sin x$$



$y = \sin x \Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$.

La funzione così definita è biiettiva da $\left[\frac{\pi}{2}, \frac{\pi}{2}\right]$ in $[-1,1]$.

- iniettiva perché strett. crescente
- suriettiva perché f continua in un intervallo assume tutti i valori compresi tra $\inf f = -1$ e $\sup f = 1$.

$$\Rightarrow \exists\, f^{-1}(y) = \arcsin y.$$

Quindi l'arcsin è l'inversa di $\sin \Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]}$

$\arcsin y : [-1,1] \to \left[-\frac{\pi}{2},\frac{\pi}{2}\right]$

$\qquad\qquad\quad y \mapsto \boxed{f^{-1}(y) = \arcsin y = \text{l'unico } x \in \left[\frac{\pi}{2},\frac{\pi}{2}\right] \text{ t.c. } \sin x = y.}$

$\arcsin\left(\frac{1}{2}\right) = \frac{\pi}{6} \quad \cancel{= \frac{5\pi}{6}} \quad \cancel{\frac{13\pi}{6}} \quad \cancel{\frac{\pi}{6} + 2k\pi}$

$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$

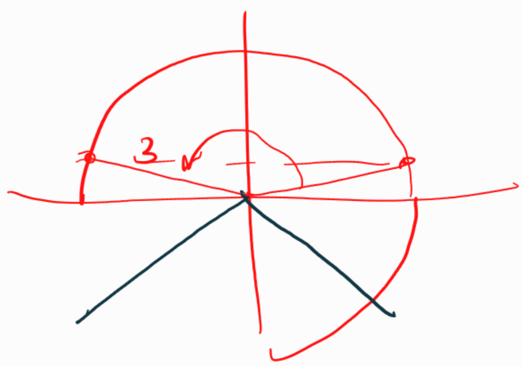
$\arcsin(-1) = -\frac{\pi}{2}$

$x \in \left[-\frac{\pi}{2},\frac{\pi}{2}\right] \qquad y \in [-1,1].$

$\sin x = y \iff x = \arcsin y$

$\arcsin\left(\sin\frac{1}{4}\right) = \frac{1}{4}$

$\arcsin(\sin 3) = \cancel{3} \quad \pi - 3$



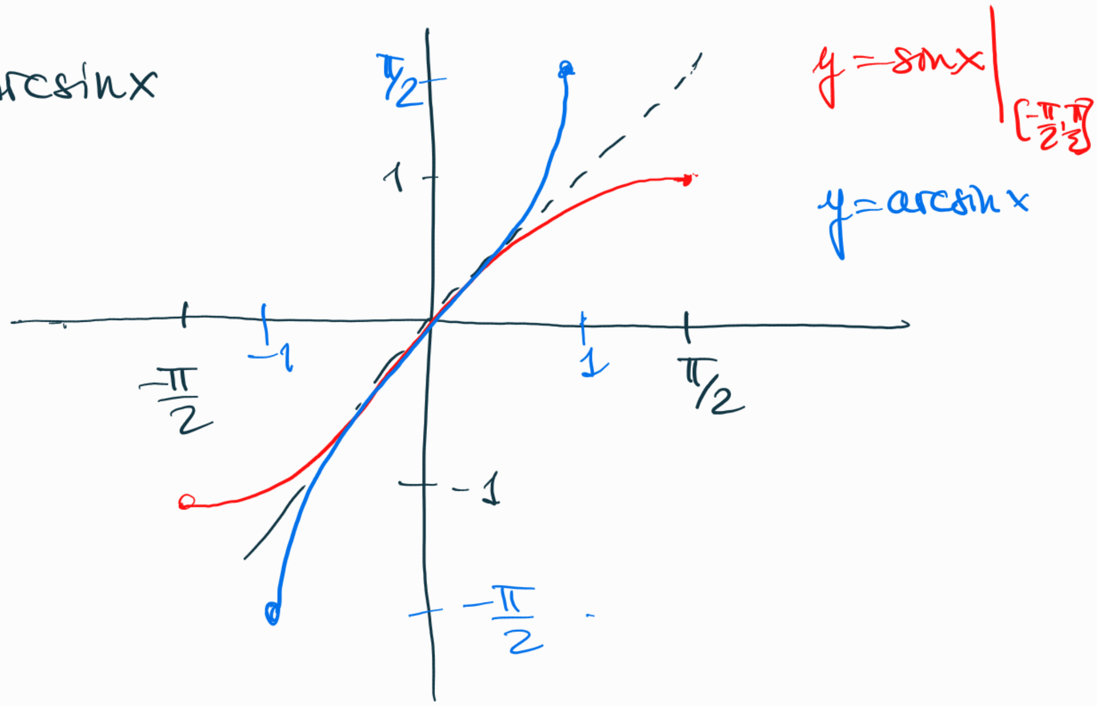
$\arcsin\left(\sin\frac{5\pi}{4}\right) = \cancel{\frac{5\pi}{4}} \quad -\frac{\pi}{4}$

$\arcsin(\sin x) = x \qquad \forall x \in \left[\frac{\pi}{2},\frac{\pi}{2}\right].$

ma non fuori da questo interv

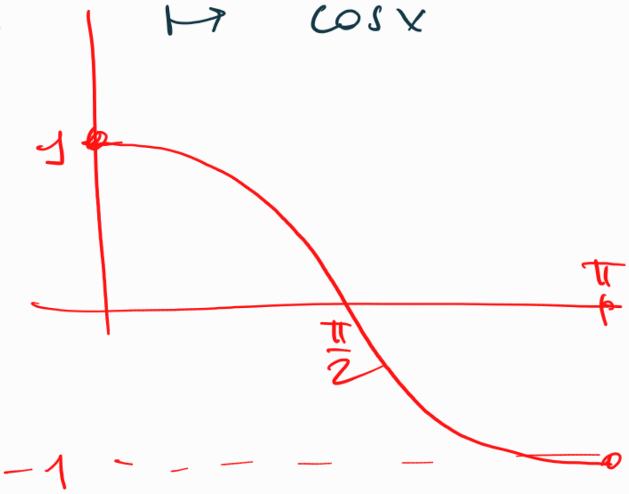
$$\sin(\arcsin x) = x \qquad \forall x \in [-1,1].$$

Grafico di $\arcsin x$



$\cos x : \mathbb{R} \to \mathbb{R}$ non è iniettiva né suriettiva.

$$\cos x\Big|_{[0,\pi]} : [0,\pi] \longrightarrow [-1,1]. \quad \text{biiettiva.}$$
$$x \longmapsto \cos x$$



$$f^{-1}(y) = \arccos y : [-1,1] \longrightarrow [0,\pi].$$
$$\begin{array}{c}\psi\\ y\end{array} \longmapsto \boxed{\begin{array}{l}\arccos y = \text{l'unico } x \in [0,\pi] \\ \text{t.c. } \cos x = y.\end{array}}$$

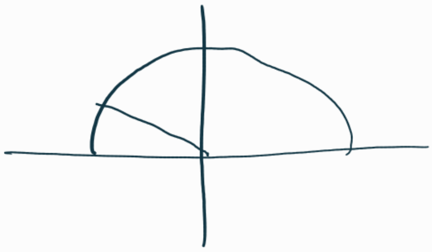
$\arccos 1 = 0$

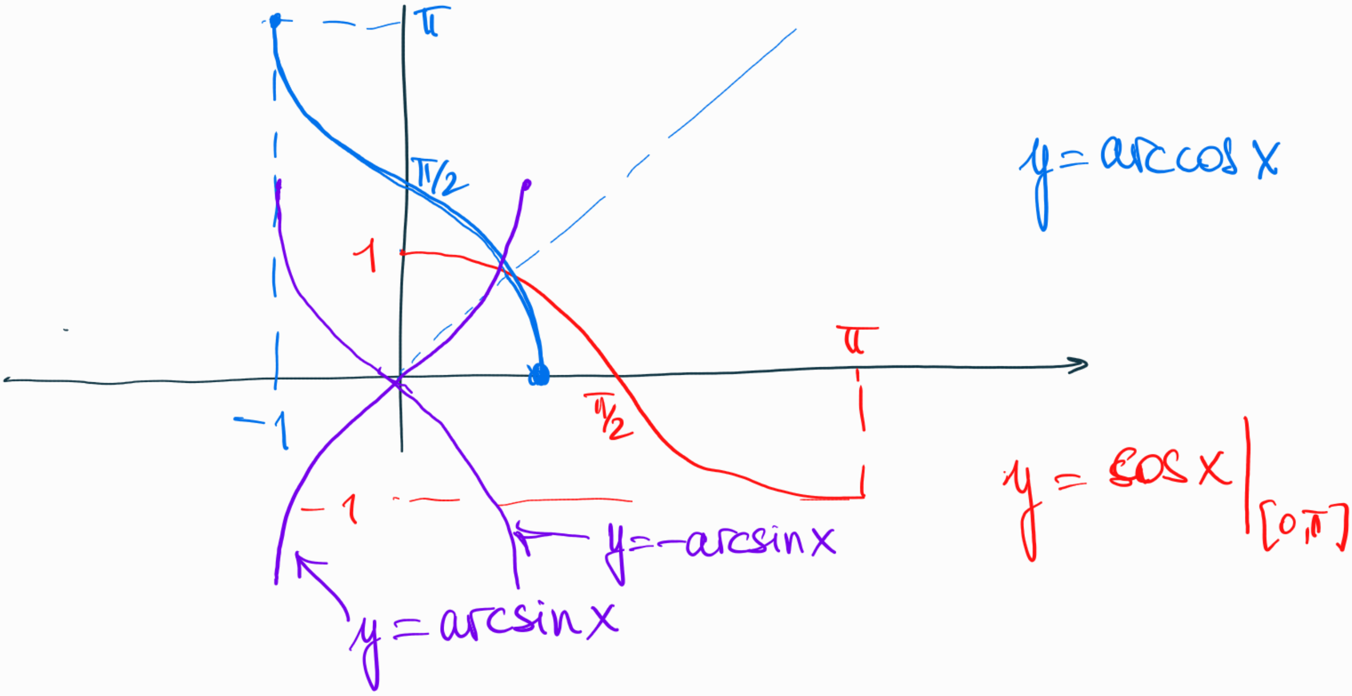
$\arccos \frac{1}{2} = \frac{\pi}{3}$

arccos è l'inversa di $\cos\Big|_{[0,\pi]}$

$\arccos 0 = \frac{\pi}{2}$

$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \frac{5}{6}\pi$





Congettura: $\arccos x = \underbrace{\frac{\pi}{2} - \arcsin x}_{t} \quad \forall x \in [-1,1]$.

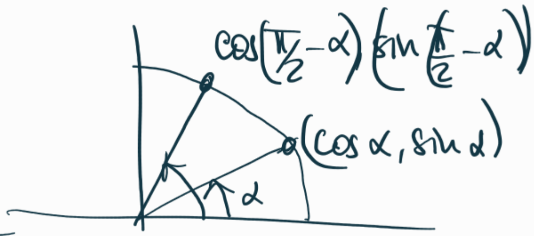
Devo provare

1) $t \in [0,\pi]$? sì, perché $-\arcsin x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \Rightarrow \frac{\pi}{2} - \arcsin x \in [0,\pi]$

2) $\cos t = x$? sì.

$$\cos\left(\frac{\pi}{2} - \arcsin x\right) = \sin(\arcsin x) = x$$

$$\cos\left(\frac{\pi}{2} - \alpha\right) = \sin \alpha \quad \forall \alpha \in \mathbb{R}$$

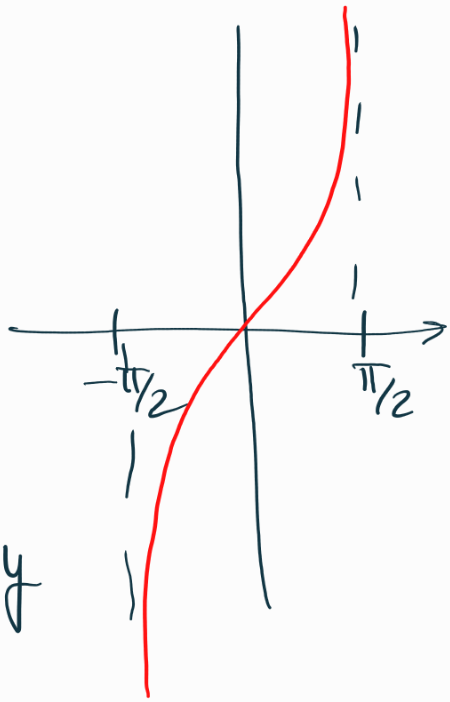


$\arccos\left(\cos \frac{7}{4}\pi\right) = \frac{\pi}{4}$

$\arccos\left(\cos \frac{5}{4}\pi\right) = \frac{3}{4}\pi$

$$\operatorname{tg} x \big|_{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)} : \left(-\frac{\pi}{2},\frac{\pi}{2}\right) \longrightarrow \mathbb{R}$$

biiettiva



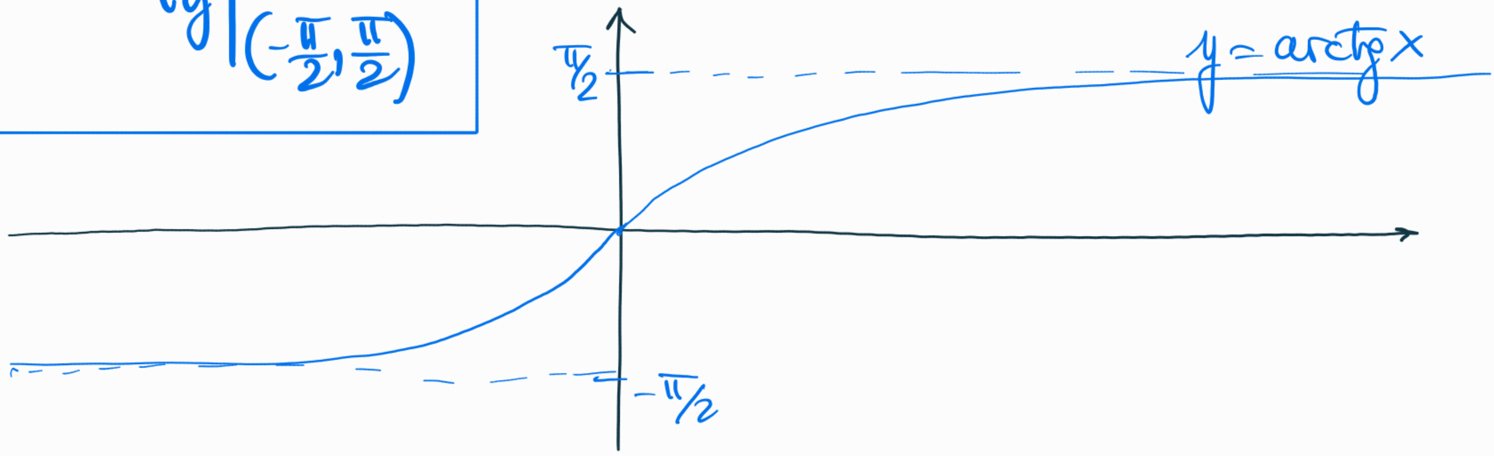
La sua inversa si chiama

$$f^{-1}(y) = \operatorname{arctg} y = \arctan y$$

$$\operatorname{arctg} : \mathbb{R} \longrightarrow \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

$$y \longmapsto \boxed{\operatorname{arctg} y \doteq \text{l'unico } x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right) \text{ t.c. } \operatorname{tg} x = y.}$$

$\boxed{\operatorname{arctg} \text{ è l'inversa di } \operatorname{tg}\big|_{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}}$



$$\operatorname{arctg}(\sqrt{3}) = \frac{\pi}{3}$$

$$\operatorname{arctg}(1) = \frac{\pi}{4}$$

$$\operatorname{arctg}\left(-\frac{\sqrt{3}}{3}\right) = -\frac{\pi}{6} \qquad \cancel{\frac{5}{6}\pi}$$

OSS $\quad \arcsin(-x) = \underbrace{-\arcsin x}_{t} \quad \forall x \in [-1,1]$.

verificare

$\arcsin(-x) = t$ significa provare:

1) $t \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$? $\quad -\arcsin x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$? $\quad$ ok!

2) $\sin t = -x$ $\quad \sin(-\arcsin x) \stackrel{?}{=} -x$

$$= -\sin(\arcsin x) = -x$$

Idem. arctg x.