



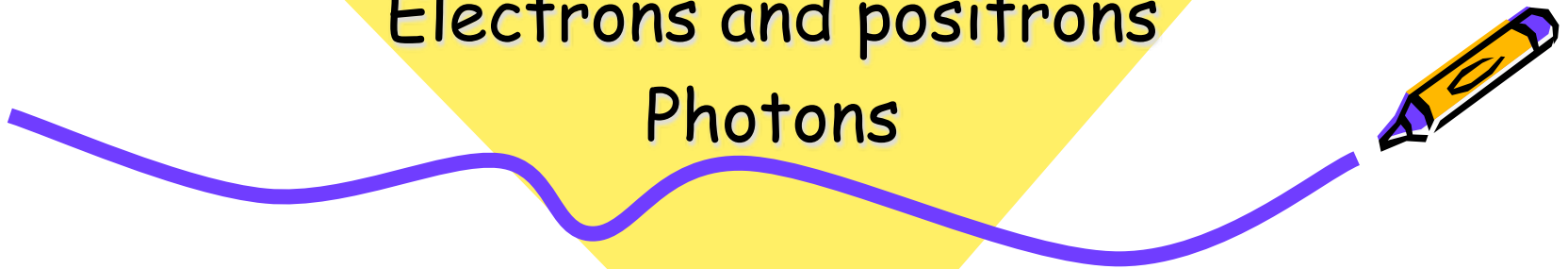
Radiation/Matter

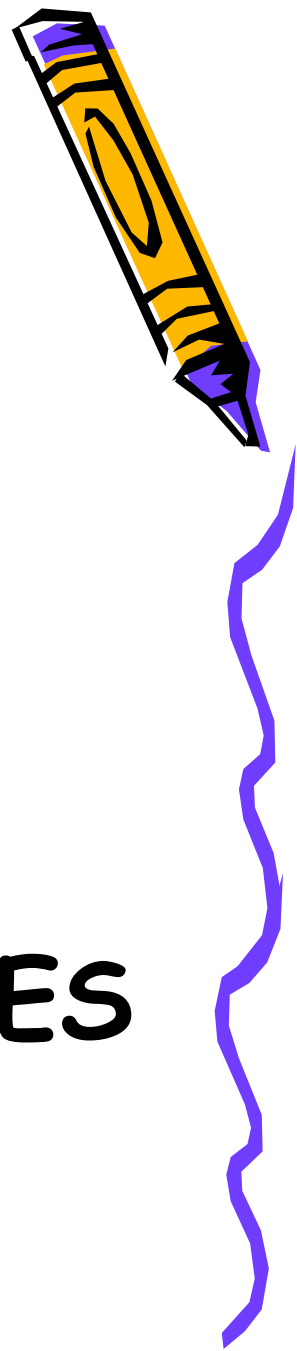
e.m. interactions

Heavy Charged Particles

Electrons and positrons

Photons

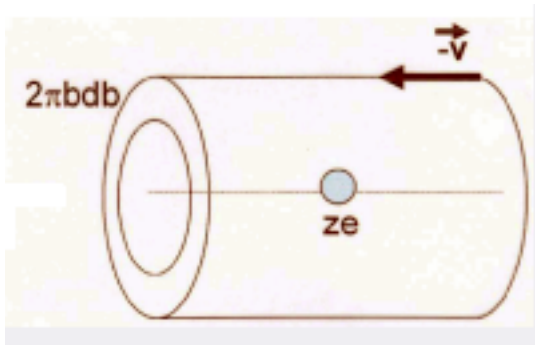




HEAVY CHARGED PARTICLES



Rutherford scattering



Hypothesis: $v_H \ll 1$; $\Delta E_H \ll E_H$

LAB

NUCLEON M, v_H

INITIAL STATE

FINAL STATE

\vec{P}_e

$\vec{P}_H^L = \vec{P}_H^L + \vec{P}_e^L$

NUCLEON

INITIAL STATE

M, ze

e^-

$-\vec{P}_H$

\vec{P}_e^L

\vec{P}_H^L

INITIAL STATE

$\vec{P}_e^N = \vec{P}_H^{N'} + \vec{P}_e^{N'}$

RELATIONSHIP AMONG RS

Momentum transferred to e^-

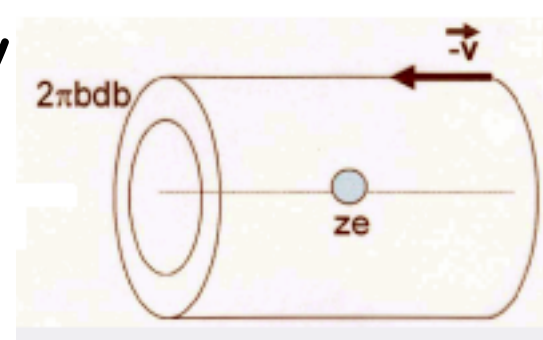
$\vec{P}_e^L = \vec{P}_H^L - \vec{P}_H^L = \Delta \vec{P}_H^L = \Delta \vec{P}_H^N = \vec{P}_H^{N'}$



Rutherford scattering



- Projectile with velocity v and $Q=ze$ on atomic electron
- Calculate in the projectile rest frame considering as negligible:
 - The Speed of electrons
 - The Loss of energy from projectile



$$p_e = \int e \cdot \mathcal{E}_\perp dt = \frac{e}{v} \int \mathcal{E}_\perp dx.$$

$$\Phi(\vec{\mathcal{E}}) = \int \vec{\mathcal{E}} \cdot \vec{n} dS = 2\pi b \int \mathcal{E}_\perp dx = \frac{ze}{\epsilon_0}.$$

$$T_e = \frac{p_e^2}{2m_e} = \left(\frac{ze^2}{4\pi\epsilon_0 b} \right)^2 \frac{2}{m_e v^2}.$$



Transferred energy

Underlying hypothesis



- $\Delta p \ll p$

$$\frac{\Delta p}{p} = \frac{ze^2}{2\pi\epsilon_0} \frac{\hbar c}{m_e c^2 \beta^2 b} = \frac{2z}{137} \frac{200 \text{ MeV fm}}{0.5 \beta^2 b} \approx \frac{5.8z}{\beta^2 b (\text{fm})}$$

- For a 100 MeV proton $\beta=0.41$, $z=1$

$$\frac{\Delta p}{p} \ll 1 \Rightarrow b \gg 30 \text{ fm}$$



Numerical example I

- K=100 MeV protons

$$p = \sqrt{2MK} \sim 430 \text{ MeV} \Rightarrow \beta = \frac{p}{E} = 0.41; \gamma = 1.1$$

$$p_t = \frac{e^2}{4\pi\epsilon_0} \frac{1}{bv} = \frac{1}{137} \frac{1}{0.41b} [\text{NU}]$$

- Assuming $b \sim 1 \text{ \AA} = 10^5 \text{ fm}$

$$p_t = \frac{\hbar c}{137 * 0.41 * 10^5 \text{ fm}} = \frac{200 \text{ MeV fm}}{137 * 0.41 * 10^5 \text{ fm}} \sim 30 \text{ eV}$$

$$E_t = \frac{p_t^2}{2m} \sim 10^{-3} \text{ eV}$$

The transferred momentum is really small because MeV \leftrightarrow fm

Too low to ionize \rightarrow for $I \sim 10 \text{ eV}$ $E_t > I \rightarrow b < 0.01 \text{ \AA}$



Numerical example II

- $K=300 \text{ MeV/u}$ carbons

$$p = \sqrt{2MK} \sim 9011 \text{ MeV} \Rightarrow \beta = \frac{p}{E} = 0.61; \gamma = 1.3$$

$$p_t = \frac{Ze^2}{4\pi\epsilon_0} \frac{1}{bv} = \frac{6}{137} \frac{1}{0.61b} [\text{NU}]$$

- Assuming $b \sim 1 \text{ \AA} = 10^5 \text{ fm}$

$$p_t = \frac{6\hbar c}{137 * 0.61 * 10^5 \text{ fm}} = \frac{1200 \text{ MeVfm}}{137 * 0.61 * 10^5 \text{ fm}} \sim 121 \text{ eV}$$

$$E_t = \frac{p_t^2}{2m} \sim 10^{-2} \text{ eV}$$

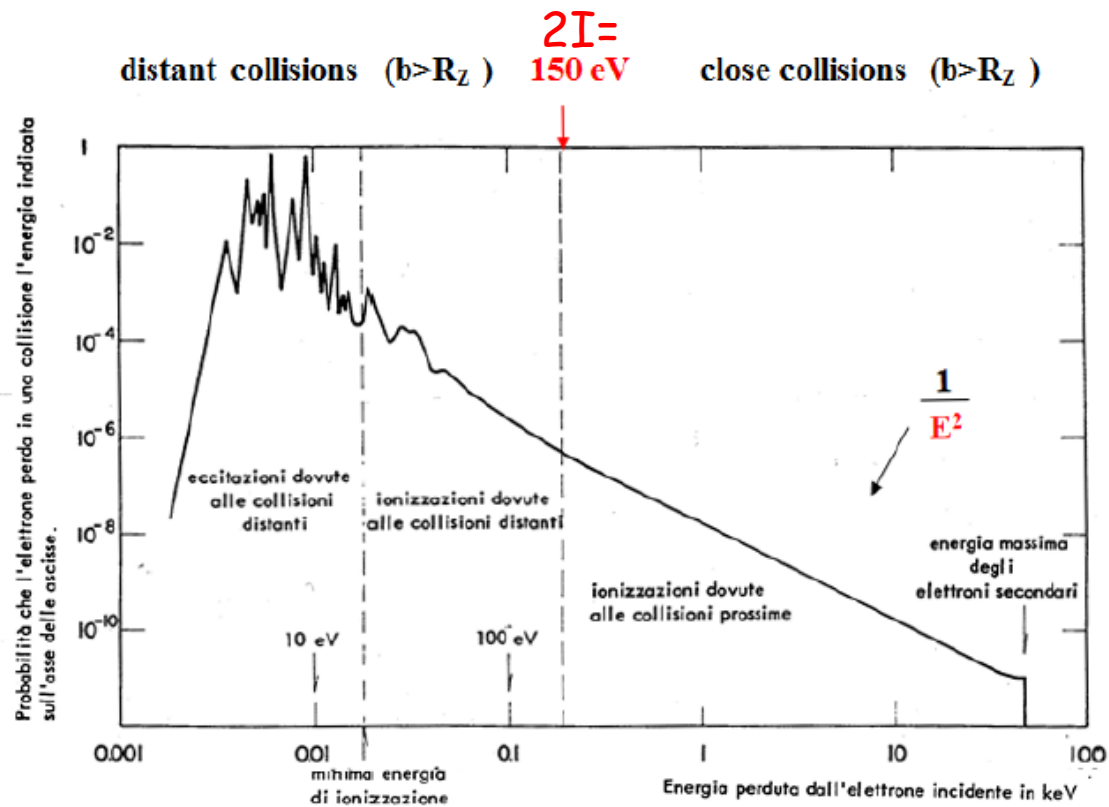
The transferred momentum is really small because $\text{MeV} \leftrightarrow \text{fm}$

Too low to ionize \rightarrow for $I \sim 10 \text{ eV}$ $E_t > I \rightarrow b < 0.04 \text{ \AA}$



Distant and close collisions

→ Need to consider in LET only the collisions that are close enough to ionize



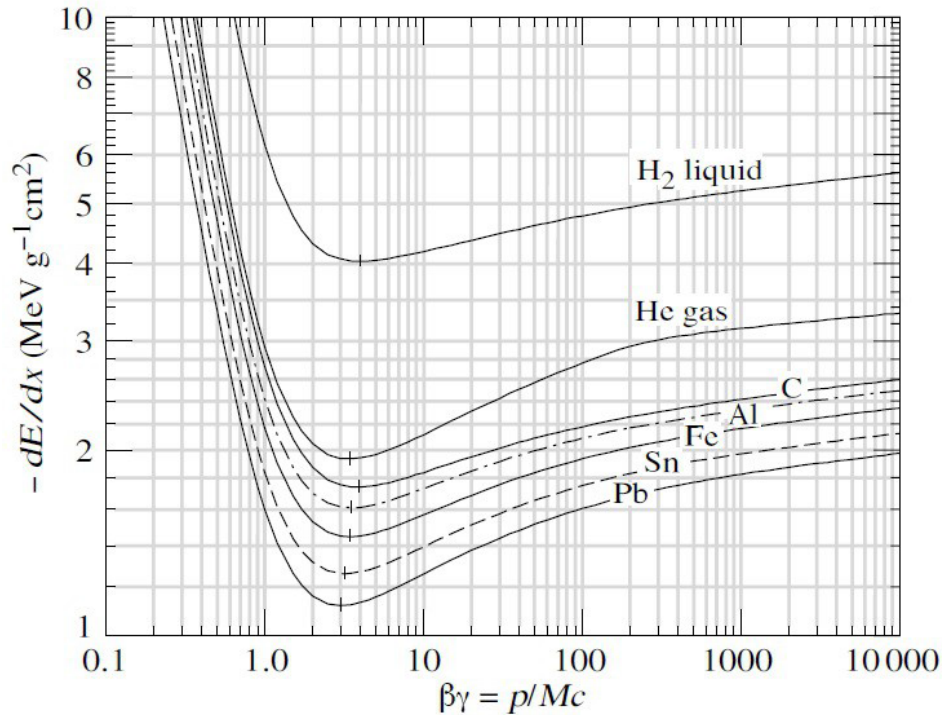
LET



- Linear Energy Transfer: mean energy released by ionization to the matter → note: dE/dX is the energy lost by the radiation > energy transferred locally
- Integrate between
 - E_{min} corresponding to b_M needed to allow for transition time $< T \rightarrow b_M/v = 1/v = h/I$
$$E_{min} = I^2 / 2m\gamma^2\beta^2$$
 - E_{max} corresponding to max transferrable energy in elastic scattering
$$Vf(e) = 2v_i$$



Bethe Bloch

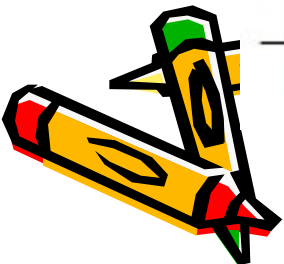


Examples of
how to read
this plot

$$-\frac{dE}{dx} = 4\pi N_A \frac{Z\rho}{A} r_e^2 m_e c^2 \frac{z^2}{\beta^2} \left(\ln \frac{2m_e c^2 \beta^2 \gamma^2}{I} - \beta^2 - \frac{\delta(\gamma)}{2} \right)$$

$$r_e = \alpha \hbar / mc$$

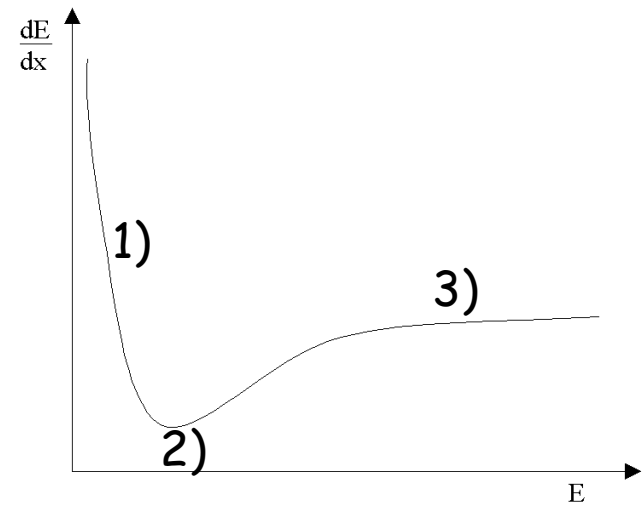
$I = 10Z$ (eV) \rightarrow empirical mean Ionization Energy



Rappresentiamo la perdita di energia in funzione dell'energia della particella:

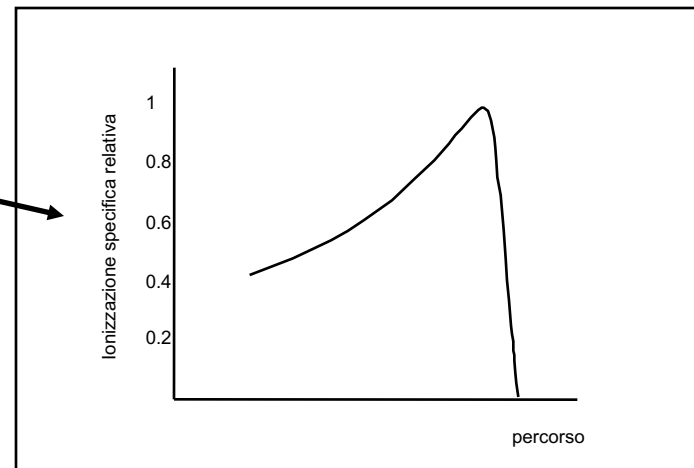
presenta un minimo, detto minimo di ionizzazione, e poi risale lentamente fino a raggiungere una saturazione

- 1) A basse energie domina il termine $1/\beta^2$
- 2) Minimo di Ionizzazione: $2-3 m_0c^2$
- 3) Risalita relativistica



Il numero di coppie create per unità di lunghezza di percorso è proporzionale alla frazione dE/dx d'energia persa dalla particella. Quest'ultima aumenta man mano che l'energia della particella diminuisce passando per un massimo alla fine del percorso.

Curva di Bragg

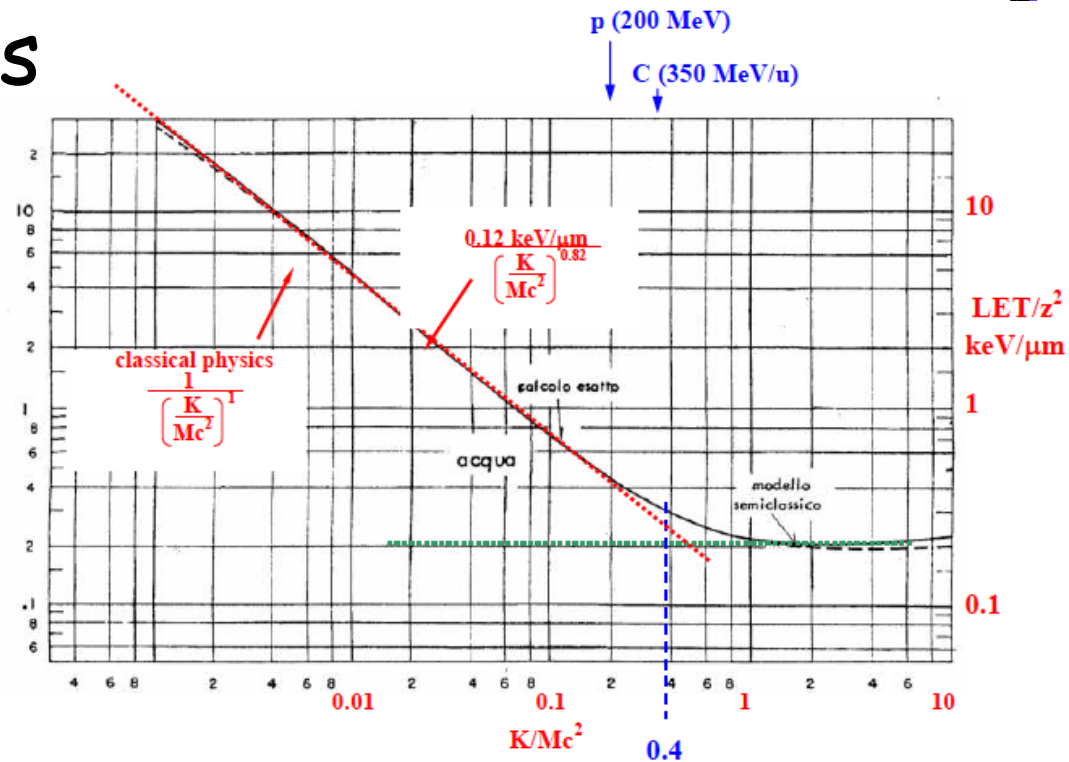


$1/\beta^2$ regime



Achieved for $K < 0.4M$

- 200 keV electrons
- 370 MeV/u hadrons



$$LET_{\text{acqua}} : \frac{0.12z^2 \text{keV}/\mu\text{m}}{\left(K/Mc^2\right)^{0.82}}$$



Range



- Range in the path of a particle before stopping

$$R = \int_K^0 \frac{dK}{-LET(K)}$$

$$R_{acqua} = \frac{1}{z^2 0.21 \frac{MeV}{mm}} \int \left(\frac{K}{Mc^2} \right)^{0.82} dK = \overset{R^*}{(425cm)} \frac{A}{z^2} \left(\frac{K}{Mc^2} \right)^{1.82} .$$



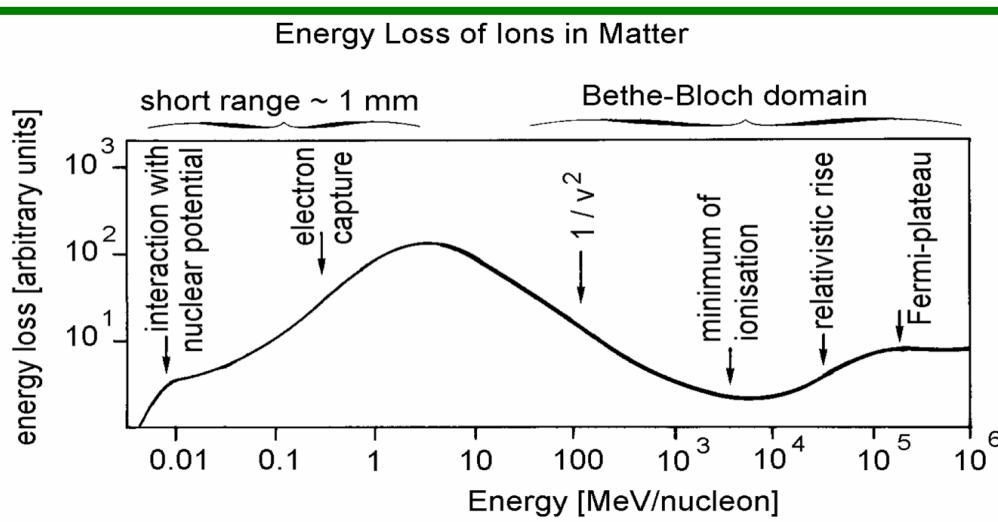
LET vs Range

- From the previous relationship

$$LET = \underbrace{0.12}_{L^*} z^2 \frac{\text{keV}}{\mu\text{m}} \frac{1}{\left(\frac{z^2 R_{acq}}{A R^*}\right)^{\frac{0.82}{1.82}}} = L^* z^{1.1} A^{0.45} \left(\frac{R^*}{R_{acq}}\right)^{0.45}$$

$$R^* = 425 \text{ cm (water)}$$

- $1/\beta^2$ does not hold indefinitely

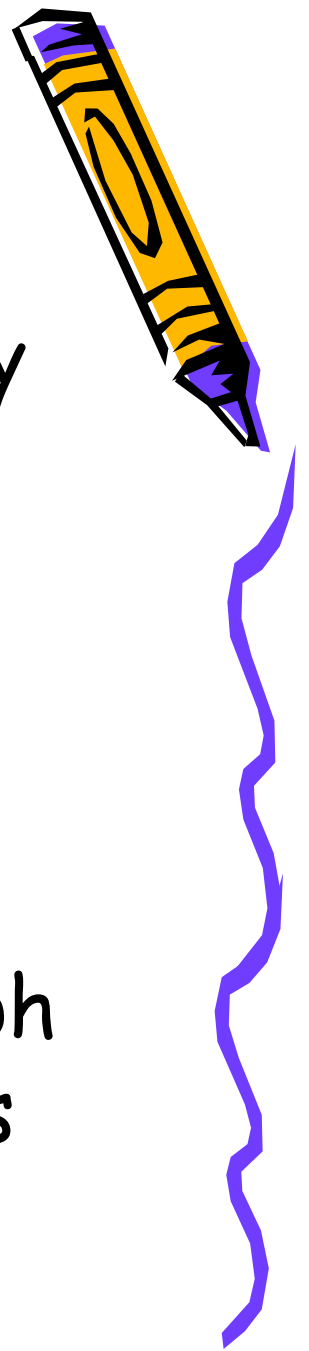


$$K_{\min} = 5 \text{ MeV/u}$$



$$LET_{\max} (\text{keV}/\mu\text{m}) = 0.12 z^2 / 0.005^{0.82} = 9.3 z^2$$

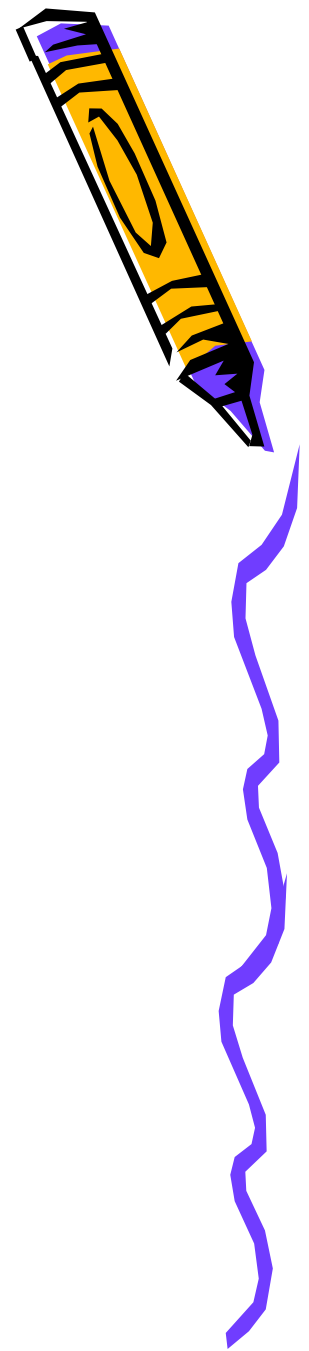
Exercises (Set 1)



- Graph the range in water vs energy for
 - Protons
 - Alpha (He-4 nuclei)
 - Carbon nuclei
- Fixing the entrance energy to the one required to have $R=20\text{cm}$, graph the LET in water vs x in 1mm steps



Numerical examples



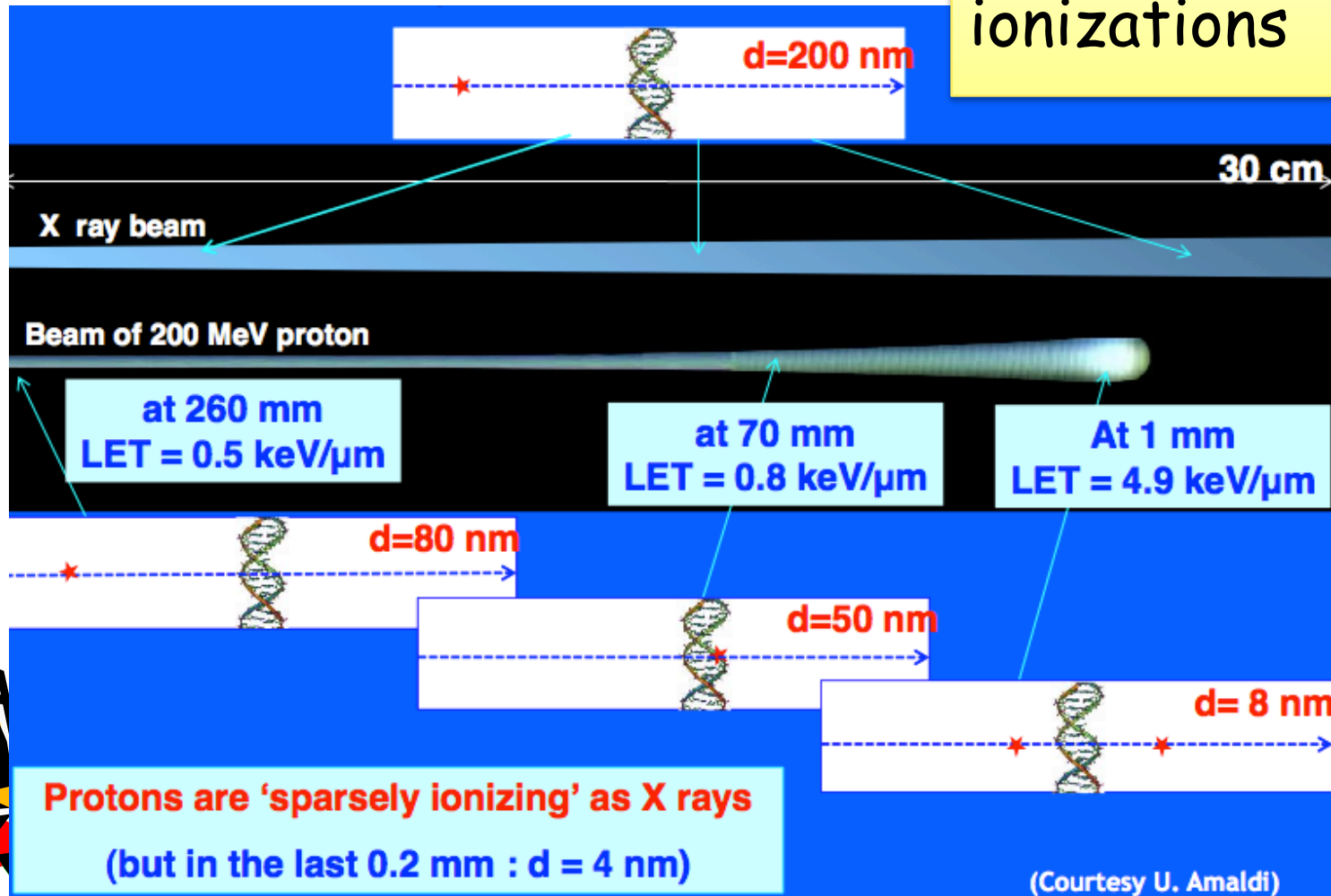
Charged particle	E/A [MeV/A]	p [MeV/c]	p at inject. [MeV/c]	LET [eV/nm] at various residual ranges in water [cm]				
				26.2	15	7	3	0.1
M_NZ	<i>range</i> 26.2 cm	<i>range</i> 26.2 cm	$E = 7$ MeV/u	26.2	15	7	3	0.1
e^{-*}	56.0	56.0	-	0.2	0.2	0.2	0.2	0.2
${}^1\text{H}^{+1}$	200.0	645	115	0.5	0.6	0.8	1.1	4.9
${}^2\text{D}^{+1}$	136.0	1045	160	0.6	0.7	1.0	1.5	6.9
${}^3\text{T}^{+1}$	108.0	1385	345	0.7	0.9	1.2	1.8	8.3
${}^3\text{He}^{+2}$	238.5	2125	345	1.6	2.0	2.7	3.9	17.8
${}^4\text{He}^{+2}$	202.0	2580	457	1.8	2.2	3.1	4.4	19.6
${}^7\text{Li}^{+3}$	234.1	4905	800	3.7	4.6	6.2	8.9	41.0
${}^9\text{Be}^{+4}$	283.7	7050	1035	5.9	7.2	9.6	13.7	62.4
${}^{11}\text{B}^{+5}$	329.5	9350	1260	8.5	10.2	13.6	19.3	87.7
${}^{12}\text{C}^{+6}$	391.0	11260	1370	11.1	13.4	17.7	24.7	111.8
${}^{14}\text{N}^{+7}$	430.5	13910	1600	14.5	17.3	22.6	31.6	142.2
${}^{16}\text{O}^{+8}$	468.6	16710	1830	18.2	21.6	28.1	39.2	175.1
${}^{20}\text{Ne}^{+10}$	540.8	22847	2297	26.8	31.5	40.6	55.9	248.7



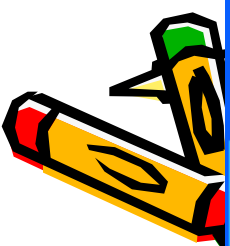
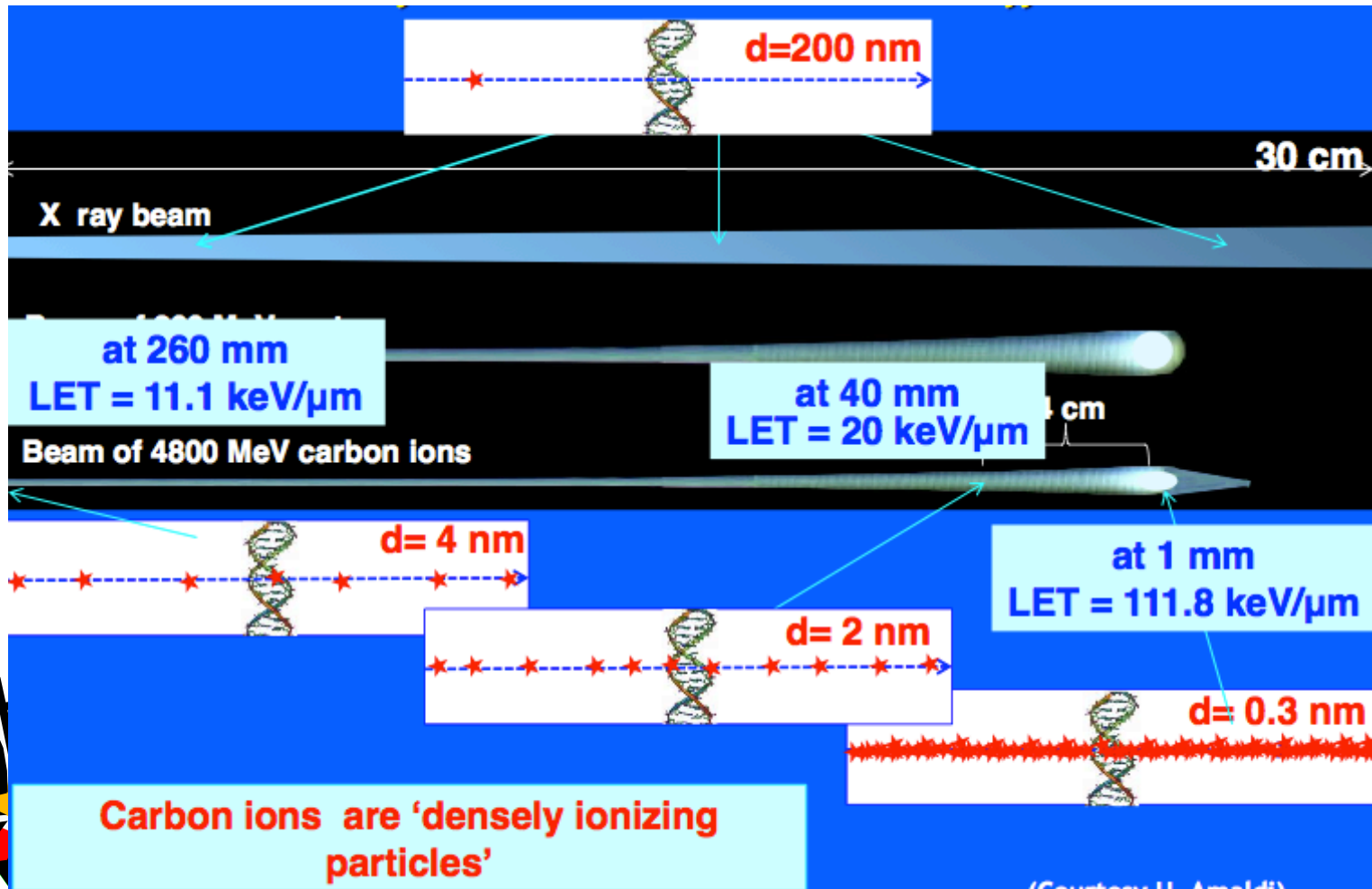
DNA size ~2nm, Ionization ~50 eV → 25 eV/nm required to break DNA twice

Proton therapy: effectiveness

$d \sim 40 \text{ eV/LET}$
distance
between
ionizations



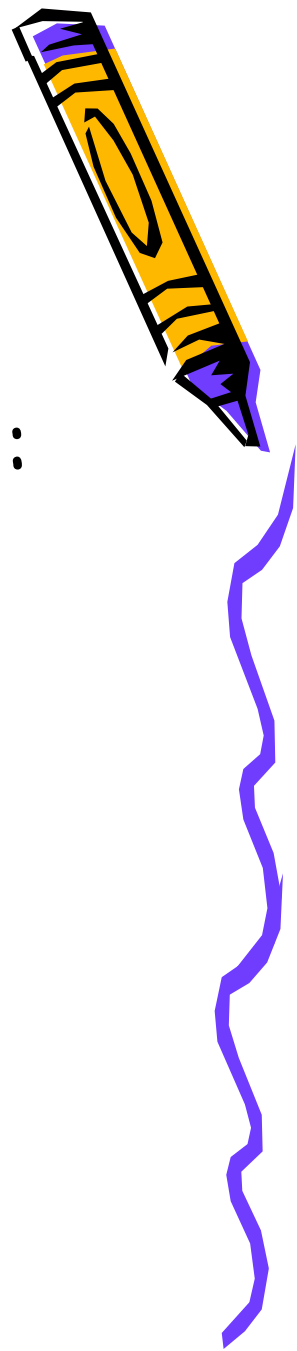
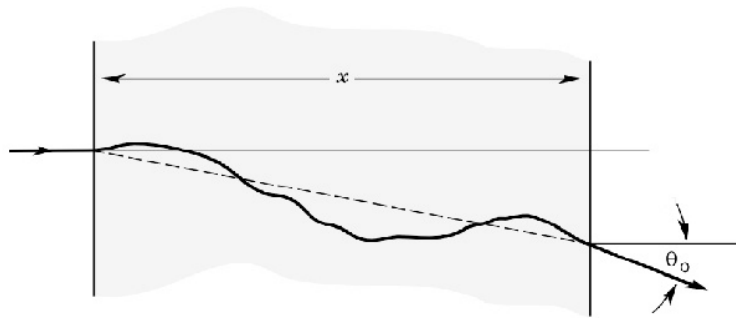
Carbon therapy



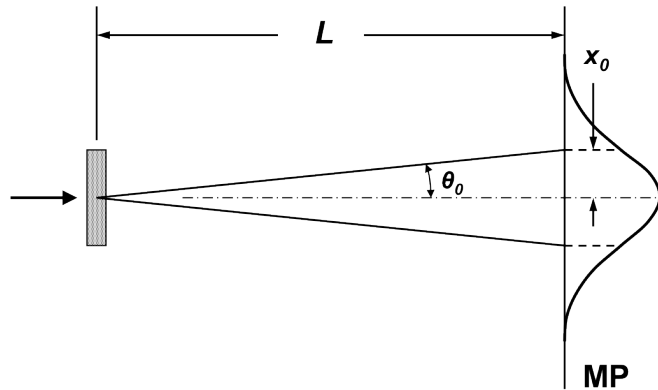
(Courtesy U. Amaldi)

Multiple scattering

- Particles can also scatter on nuclei:
 - No energy loss
 - Angular deviation
- When traversing a thick material multiple scatterings occur



Multiple scattering dispersion



Incident radiation gets scattered by interaction with nuclei

$$\sqrt{\langle \vartheta^2 \rangle} = 21 \text{MeV} \frac{z}{\beta p} \sqrt{\frac{x}{X_0}}$$

x: spessore materiale attraversato

$$X_0 \cong \frac{1}{4\alpha_{em} r_e^2 \log\left(\frac{184}{Z^{1/3}}\right)} \frac{A}{N_A Z^2 \rho}$$

$$r_e = \frac{\alpha_{em}}{m_e}$$

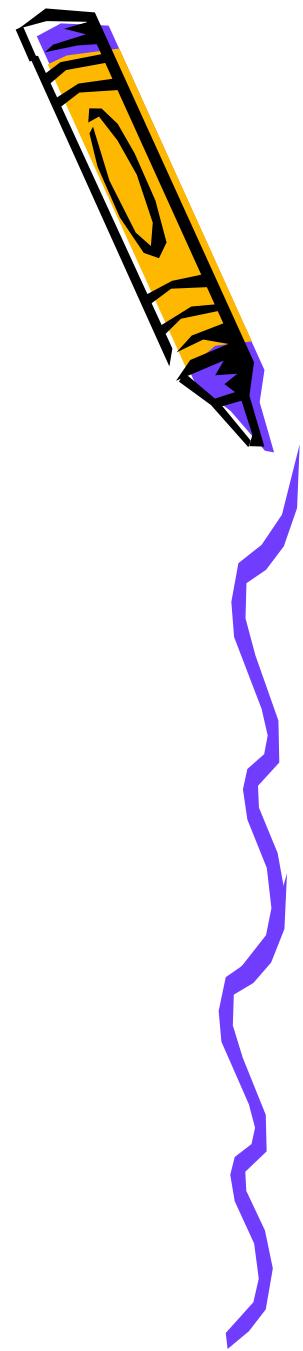
$$N_A = \text{avogadro}$$



Table 6.1 Abridged from pdg.lbl.gov/AtomicNuclearProperties by D. E. Grob (2007). See web pages for more detail about entries in this table including chemical formulae, and for several hundred other entries. Quantities in parentheses are for NTP (20°C and 1 atm), and square brackets indicate quantities evaluated at STP. Boiling points are at 1 atm. Refractive indices n are evaluated at the sodium D line blend (589.2 nm); values ≥ 1 in brackets are for $(n - 1) \times 10^6$ (gases).

Material	Z	A	(Z/A)	Nucl.coll. length λ_T {g cm ⁻² }	Nucl.inter. length λ_I {g cm ⁻² }	Rad.len. X_0 {g cm ⁻² }	dE/dx _{min} {MeV g ⁻¹ cm ⁻² }	Density {g cm ⁻³ {(gℓ ⁻¹)}	Melting point (K)	Boiling point (K)	Refract. index (@ Na D)
H ₂	1	1.00794(7)	0.99212	42.8	52.0	63.04	(4.103)	0.071(0.084)	13.81	20.28	1.11[132.]
D ₂	1	2.01410177803(8)	0.49650	51.3	71.8	125.97	(2.053)	0.169(0.168)	18.7	23.65	1.11[138.]
He	2	4.002602(2)	0.49967	51.8	71.0	94.32	(1.937)	0.125(0.166)		4.220	1.02[35.0]
Li	3	6.941(2)	0.43221	52.2	71.3	82.78	1.639	0.534	453.6	1615.	
Be	4	9.012182(3)	0.44384	55.3	77.8	65.19	1.595	1.848	1560.	2744.	
C diamond	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.725	3.520			2.42
C graphite	6	12.0107(8)	0.49955	59.2	85.8	42.70	1.742	2.210			
N ₂	7	14.0067(2)	0.49976	61.1	89.7	37.99	(1.825)	0.807(1.165)	63.15	77.29	1.20[298.]
O ₂	8	15.9994(3)	0.50002	61.3	90.2	34.24	(1.801)	1.141(1.332)	54.36	90.20	1.22[271.]
F ₂	9	18.9984032(5)	0.47372	65.0	97.4	32.93	(1.676)	1.507(1.580)	53.53	85.03	[195.]
Ne	10	20.1797(6)	0.49555	65.7	99.0	28.93	(1.724)	1.204(0.839)	24.56	27.07	1.09[67.1]
Al	13	26.9815386(8)	0.48181	69.7	107.2	24.01	1.615	2.699	933.5	2792.	
Si	14	28.0855(3)	0.49848	70.2	108.4	21.82	1.664	2.329	1687.	3538.	3.95
Cl ₂	17	35.453(2)	0.47951	73.8	115.7	19.28	(1.630)	1.574(2.980)	171.6	239.1	[773.]
Ar	18	39.948(1)	0.45059	75.7	119.7	19.55	(1.519)	1.396(1.662)	83.81	87.26	1.23[281.]
Ti	22	47.867(1)	0.45961	78.8	126.2	16.16	1.477	4.540	1941.	3560.	
Fe	26	55.845(2)	0.46557	81.7	132.1	13.84	1.451	7.874	1811.	3134.	
Cu	29	63.546(3)	0.45636	84.2	137.3	12.86	1.403	8.960	1358.	2835.	
Ge	32	72.64(1)	0.44053	86.9	143.0	12.25	1.370	5.323	1211.	3106.	
Sn	50	118.710(7)	0.42119	98.2	166.7	8.82	1.263	7.310	505.1	2875.	
Xe	54	131.293(6)	0.41129	100.8	172.1	8.48	(1.255)	2.953(5.483)	161.4	165.1	1.39[701.]
W	74	183.84(1)	0.40252	110.4	191.9	6.76	1.145	19.300	3695.	5828.	
Pt	78	195.084(9)	0.39983	112.2	195.7	6.54	1.128	21.450	2042.	4098.	
Au	79	196.966569(4)	0.40108	112.5	196.3	6.46	1.134	19.320	1337.	3129.	
Pb	82	207.2(1)	0.39575	114.1	199.6	6.37	1.122	11.350	600.6	2022.	
U	92	[238.02891(3)]	0.38651	118.6	209.0	6.00	1.081	18.950	1408.	4404.	
Air (dry, 1 atm)			0.49919	61.3	90.1	36.62	(1.815)	(1.205)		78.80	
Shielding concrete			0.50274	65.1	97.5	26.57	1.711	2.300			
Borosilicate glass (Pyrex)			0.49707	64.6	96.5	28.17	1.696	2.230			
Lead glass			0.42101	95.9	158.0	7.87	1.255	6.220			
Standard rock			0.50000	66.8	101.3	26.54	1.688	2.650			
Methane (CH ₄)			0.62334	54.0	73.8	46.47	(2.417)	(0.667)	90.68	111.7	[444.]
Ethane (C ₂ H ₆)			0.59861	55.0	75.9	45.66	(2.304)	(1.263)	90.36	184.5	
Propane (C ₃ H ₈)			0.58962	55.3	76.7	45.37	(2.262)	0.493(1.868)	85.52	231.0	
Butane (C ₄ H ₁₀)			0.59497	55.5	77.1	45.23	(2.278)	(2.489)	134.9	272.6	
Octane (C ₈ H ₁₈)			0.57778	55.8	77.8	45.00	2.123	0.703	214.4	398.8	
Paraffin (CH ₃ (CH ₂) _{n≈23} CH ₃)			0.57275	56.0	78.3	44.85	2.088	0.930			
Nylon (type 6, 6/6)			0.54790	57.5	81.6	41.92	1.973	1.18			
Polycarbonate (Lexan)			0.52697	58.3	83.6	41.50	1.886	1.20			
Polyethylene ((CH ₂ CH ₂) _n)			0.57034	56.1	78.5	44.77	2.079	0.89			
Polyethylene terephthalate (Mylar)			0.52037	58.9	84.9	39.95	1.848	1.40			
Polyimide film (Kapton)			0.51264	59.2	85.5	40.58	1.820	1.42			
Polymethylmethacrylate (acrylic)			0.53937	58.1	82.8	40.55	1.929	1.19			1.49
Polypropylene			0.55998	56.1	78.5	44.77	2.041	0.90			
Polystyrene ((C ₆ H ₅ CHCH ₂) _n)			0.53768	57.5	81.7	43.79	1.936	1.06			1.59
Polytetrafluoroethylene (Teflon)			0.47992	63.5	94.4	34.84	1.671	2.20			
Polyvinyltoluene			0.54141	57.3	81.3	43.90	1.956	1.03			1.58
Aluminum oxide (sapphire)			0.49038	65.5	98.4	27.94	1.647	3.970	2327.	3273.	1.77
Barium fluoride (BaF ₂)			0.42207	90.8	149.0	9.91	1.303	4.893	1641.	2533.	1.47
Bismuth germanate (BGO)			0.42065	96.2	159.1	7.97	1.251	7.130	1317.		2.15
Carbon dioxide gas (CO ₂)			0.49989	60.7	88.9	36.20	1.819	(1.842)			[449.]
Solid carbon dioxide (dry ice)			0.49989	60.7	88.9	36.20	1.787	1.563		Sublimes at 194.7 K	
Cesium iodide (CsI)			0.41569	100.6	171.5	8.39	1.243	4.510	894.2	1553.	1.79
Lithium fluoride (LiF)			0.46262	61.0	88.7	39.26	1.614	2.635	1121.	1946.	1.39
Lithium hydride (LiH)			0.50321	50.8	68.1	79.62	1.897	0.820	965.		
Lead tungstate (PbWO ₄)			0.41315	100.6	168.3	7.39	1.229	8.300	1403.		2.20
Silicon dioxide (SiO ₂ , fused quartz)			0.49930	65.2	97.8	27.05	1.699	2.200	1986.	3223.	1.46
Sodium chloride (NaCl)			0.55509	71.2	110.1	21.91	1.847	2.170	1075.	1738.	1.54
Sodium iodide (NaI)			0.42697	93.1	154.6	9.49	1.305	3.667	933.2	1577.	1.77
Water (H ₂ O)			0.55509	58.5	83.3	36.08	1.992	1.000(0.756)	273.1	373.1	1.33
Silica aerogel			0.50093	65.0	97.3	27.25	1.740	0.200	(0.03 H ₂ O, 0.97 SiO ₂)		

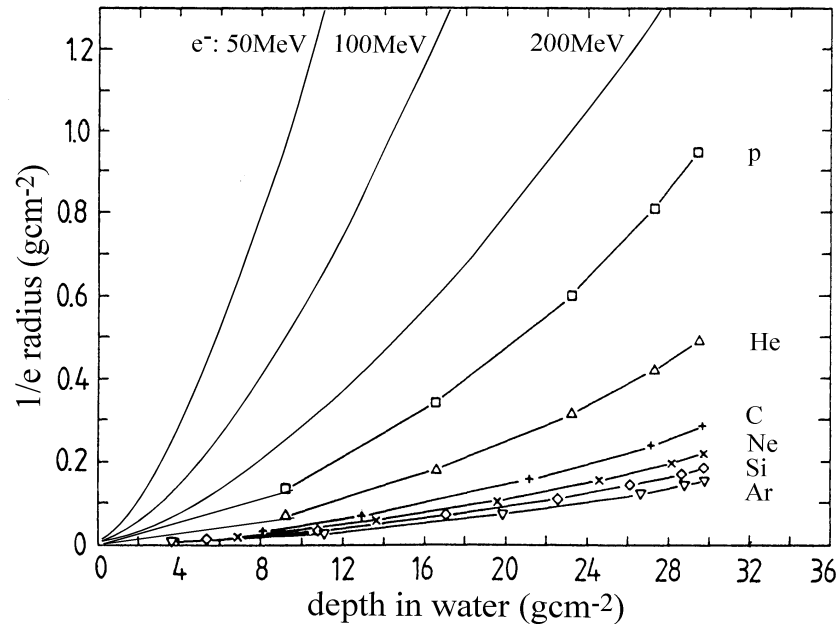
X₀ for
selected
materials



Hadrontherapy: straggling



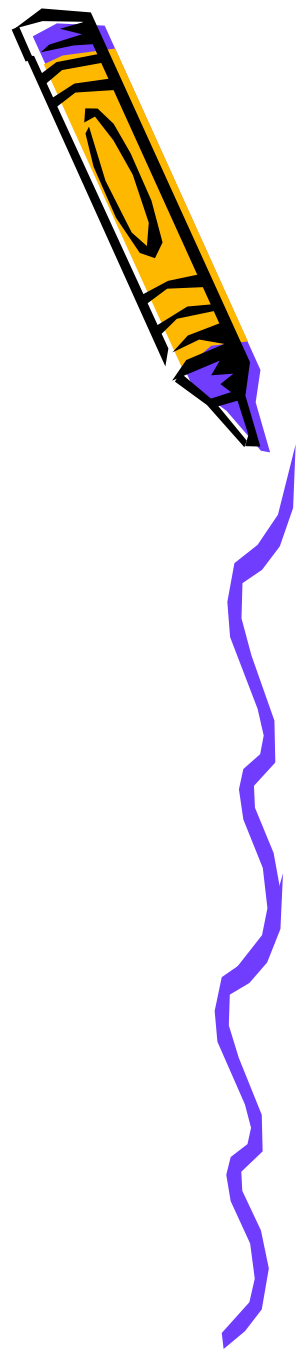
- Limit to narrowness of bragg peak




Kahoot!



ELECTRONS



Interazione delle particelle cariche leggere



Col termine particelle cariche leggere intendiamo gli elettroni (e^- ed β^-) ed i positroni (e^+ ed β^+).

Per le particelle cariche leggere, gli **effetti relativistici** non possono essere trascurati, avendo queste una massa a riposo molto più piccola delle rispettive particelle cariche pesanti.

Le particelle cariche leggere sono soggette non solo alla collisione con gli elettroni atomici del mezzo in cui interagiscono, ma subiscono anche un secondo tipo di meccanismo di perdita di energia dovuto alla interazione coi nuclei atomici. Questo secondo tipo di interazione, importante per energie elevate dell'elettrone incidente, è detta **perdita di energia per irraggiamento**.



Perdita di energia per ionizzazione

La perdita di energia per unità di percorso è **più fluttuante** che nel caso delle particelle pesanti; la lunghezza della traiettoria subisce quindi una dispersione statistica più importante.

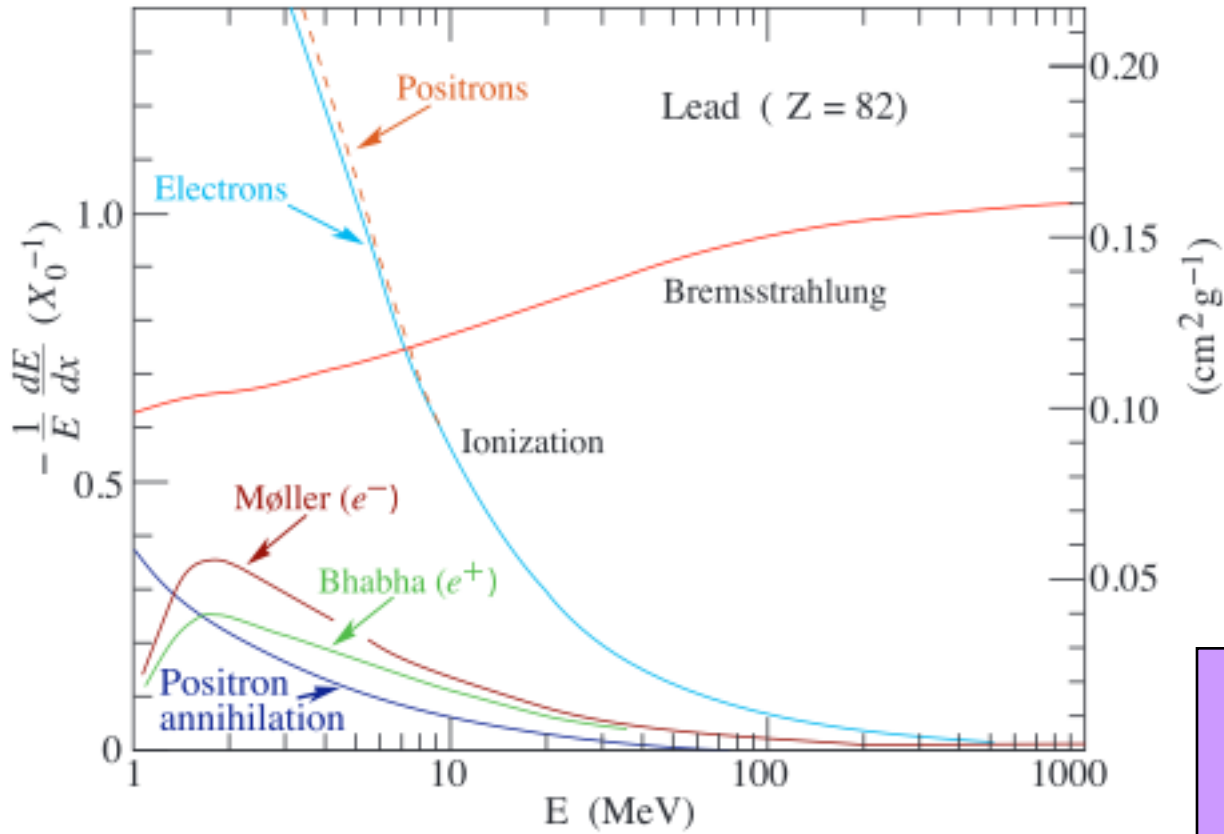
La formula Bethe e Bloch è data per due domini di energia dell'elettrone incidente:

$$-\left(\frac{d\bar{E}}{dx}\right) = 0.306 \frac{N_A Z \rho}{A} \frac{1}{\beta^2} \ln\left(\frac{1.16 m_e c^2 \beta^2}{2 I}\right) \left(\frac{\text{MeV}}{\text{cm}}\right) \quad (\text{per } \beta < 0.5)$$

$$-\left(\frac{d\bar{E}}{dx}\right) \cong 0.153 \frac{N_A Z \rho}{A} \frac{1}{\beta^2} \ln\left(\frac{E(E + m_e c^2)^2 \beta^2}{2 I^2 m_e c^2}\right) \left(\frac{\text{MeV}}{\text{cm}}\right) \quad (\text{per } \beta \approx 1)$$

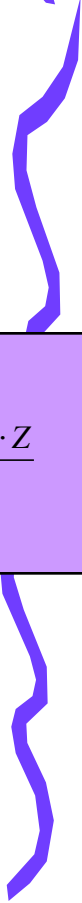
Nel caso **non relativistico** il potere di rallentamento decresce in funzione dell'energia E dell'elettrone come avveniva per le particelle cariche pesanti, mentre nel caso **relativistico** il potere di rallentamento cresce lentamente con $\ln E$.



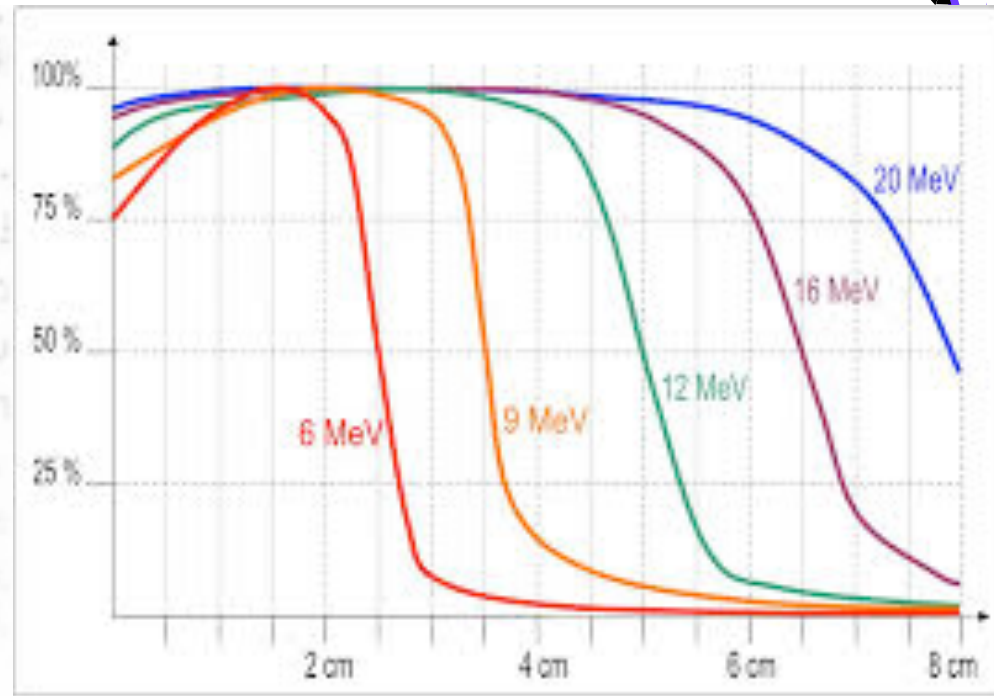
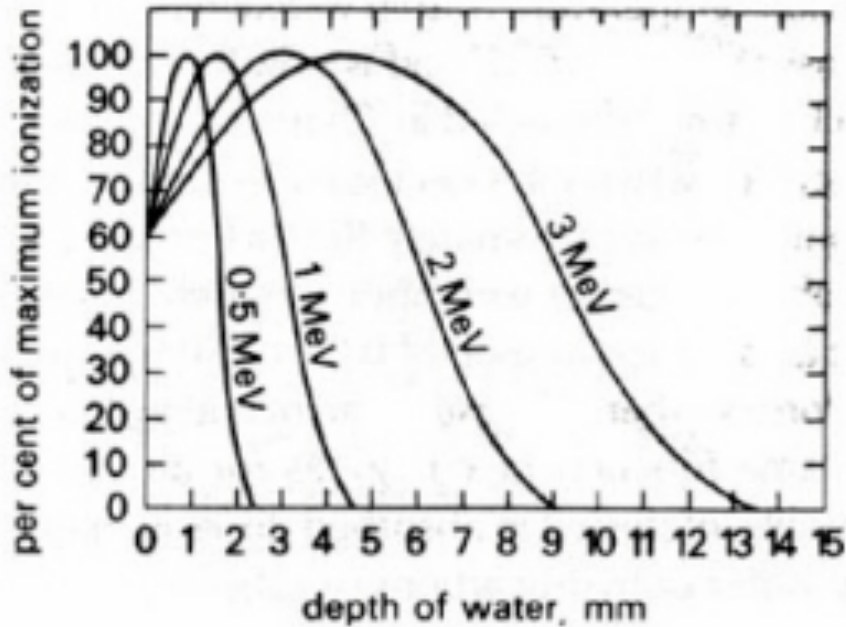


$$\frac{\left(\frac{d\bar{E}}{dx}\right)_{Brems}}{\left(\frac{d\bar{E}}{dx}\right)_{Ion}} \approx \frac{E(\text{MeV}) \cdot Z}{800}$$

This plot is valid for electrons
 In the case of protons the
 Bremsstrahlung curve would be six orders
 of magnitude below



Energy release electrons



Bremsstrahlung



Per grandi energie, gli elettroni possono subire grandi perdite di energia passando nelle vicinanze di un nucleo pesante; qui gli elettroni deviano dalla loro traiettoria incidente; tale cambiamento di direzione equivale ad una accelerazione, rilasciando una certa quantità della loro energia sotto forma di radiazione elettromagnetica. L'emissione di fotoni attraverso questo processo è chiamato **irraggiamento da frenamento** o *bremsstrahlung*.

- Il fenomeno è descritto dalla formula di Larmor che si ricava a partire dal potenziale ritardato

$$\vec{A} = \frac{1}{4\pi\epsilon_0 c^2} \frac{q\vec{v}(t - \frac{r}{c})}{r}$$

Con v diretto lungo l'asse z

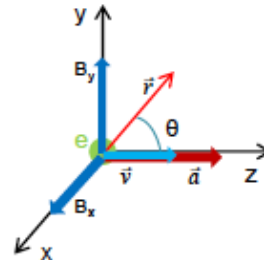
$$\vec{B} = \vec{\nabla} \times \vec{A}$$

$$B_x = \frac{q}{4\pi\epsilon_0 c^2} \left(-\frac{ay}{cr^2} - \frac{vy}{r^3} \right)$$

$$B_y = \frac{q}{4\pi\epsilon_0 c^2} \left(-\frac{ax}{cr^2} - \frac{vx}{r^3} \right)$$

Termine che
descrive
irraggiamento

Espressione
campo prodotto
da particella con
velocità v



Bremsstrahlung cont.

Il modulo quadro del campo (dipendente dall'accelerazione) in coordinate polari vale

$$|B|^2 = \frac{q^2}{16\pi^2 \varepsilon_0^2 c^4} \left(\frac{a^2 \sin^2 \theta}{c^2 r^2} \right)$$

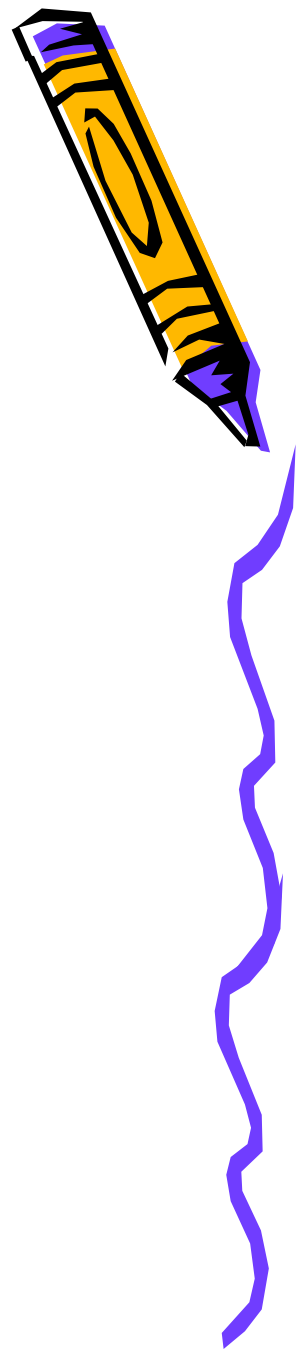
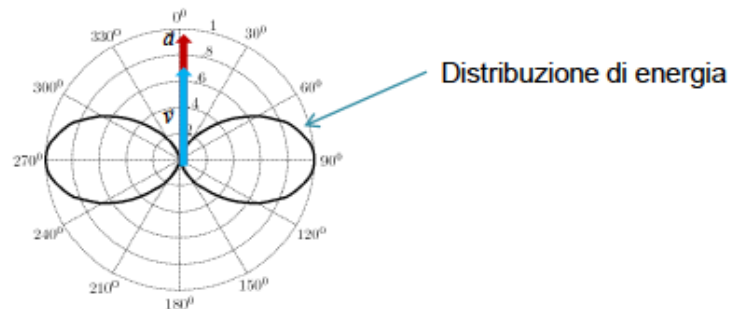
Si trova così la densità di energia trasportata dal campo

$$U = \frac{|B|^2 \varepsilon_0 c^2}{2}$$

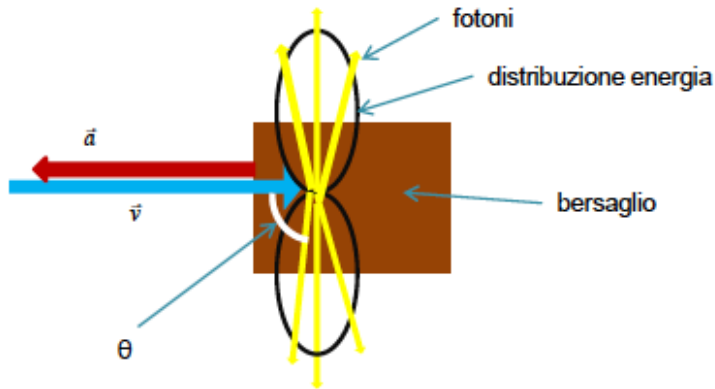
Da cui si ricava la potenza irradiata

$$W = \frac{dE}{dt} = \frac{q^2 a^2 \sin^2 \theta}{4\pi \varepsilon_0 c^3}$$

La distribuzione di energia irradiata sarà del tipo $\sin^2 \theta$



Bremsstrahlung III



Distribuzione di energia per un fascio **elettroni** che urta contro un **bersaglio**

La perdita di energia per unità di percorso da parte di un elettrone è data da:

$$-\frac{dE}{dx} = \frac{E}{X_0}$$

Dove X_0 è la lunghezza di radiazione, proporzionale a

$$X_0 \propto \frac{A}{Z^2}$$

Dato che $A \approx 2Z$ per tutti gli elementi eccetto l'idrogeno si ha

$$X_0 \propto \frac{1}{Z}$$

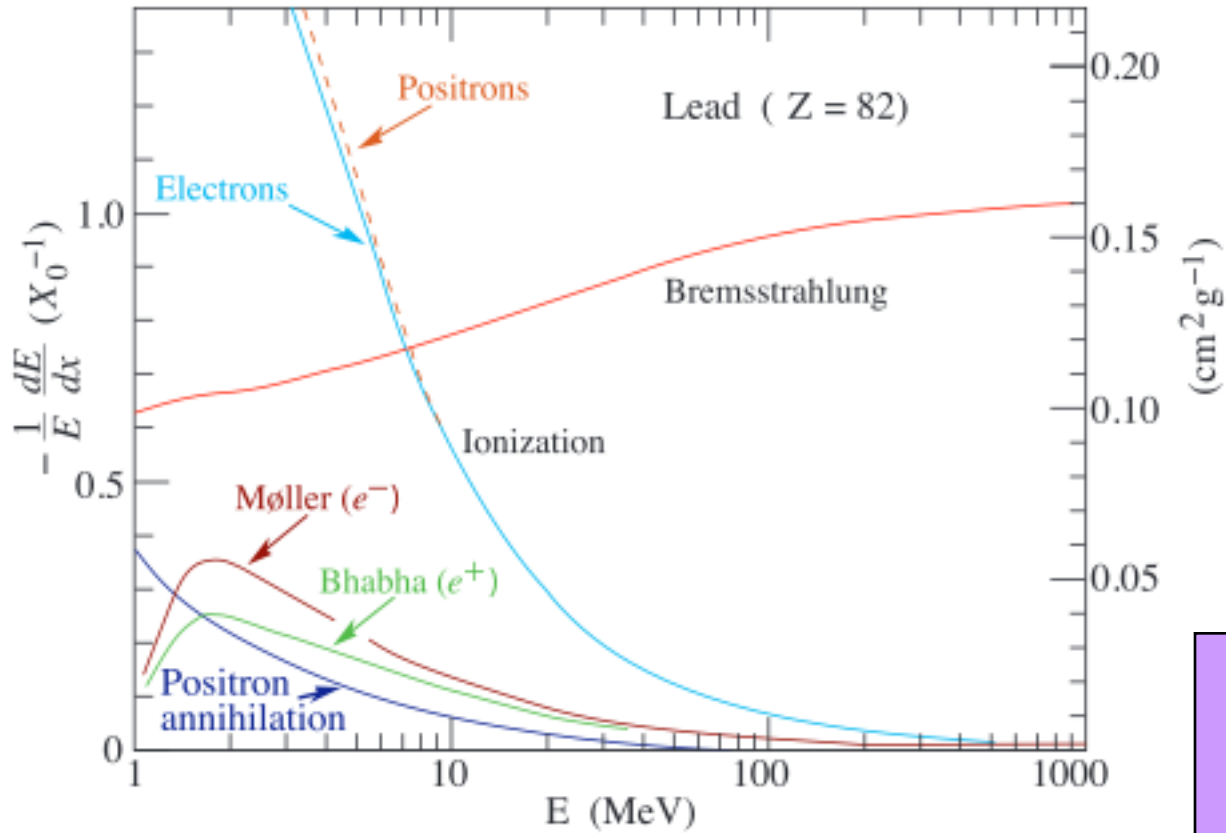
Per Z elevati l'irraggiamento è più intenso.

$$X_0 \cong \frac{1}{4\alpha_{em} r_e^2 \log\left(\frac{184}{Z^{1/3}}\right)} \frac{A}{N_A Z^2 \rho}$$

$$r_e = \frac{\alpha_{em}}{m_e}$$

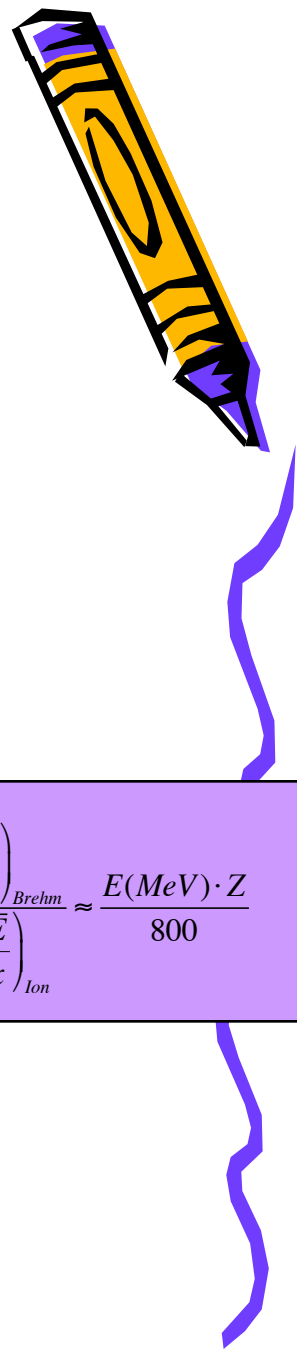
$$N_A = \text{avogadro}$$

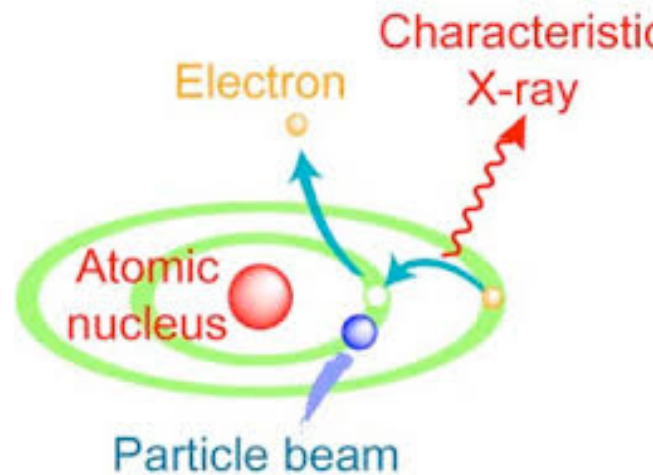
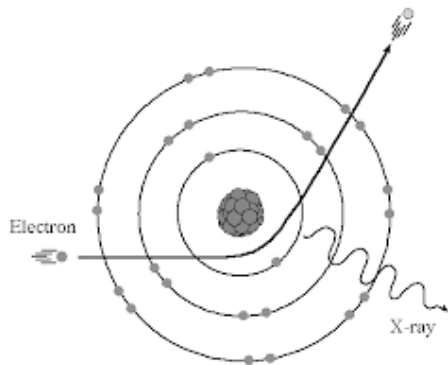




$$\frac{\left(\frac{d\bar{E}}{dx}\right)_{\text{Brems}}}{\left(\frac{d\bar{E}}{dx}\right)_{\text{Ion}}} \approx \frac{E(\text{MeV}) \cdot Z}{800}$$

This plot is valid for electrons
 In the case of protons the
 Bremsstrahlung curve would be six orders
 of magnitude below



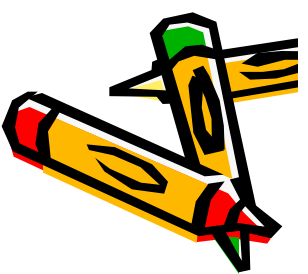
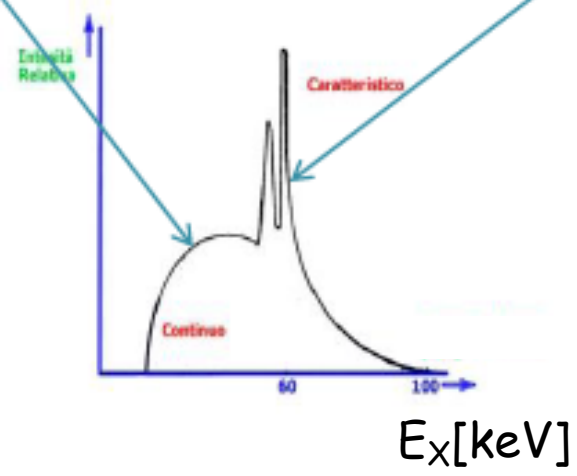


Bremsstrahlung

Diseccitazione

Spettro continuo

Spettro caratteristico



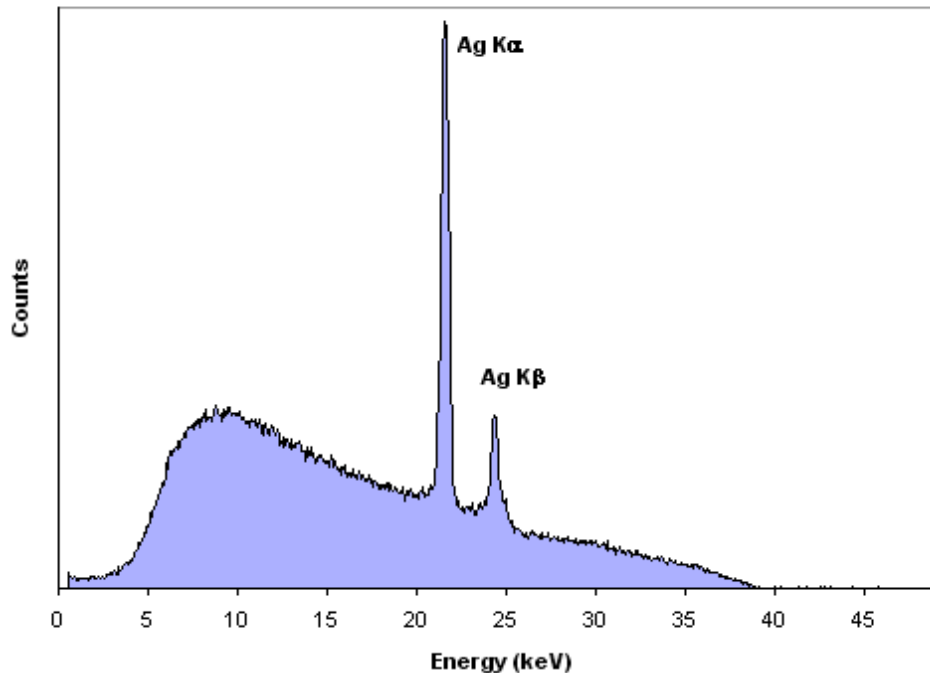
Application: radiography

Typical electron energy $E_e = 50\text{-}100\text{ keV}$

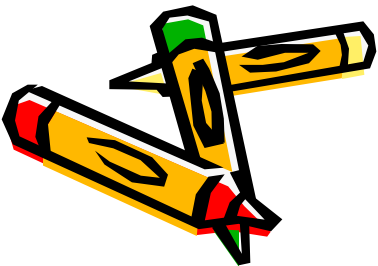
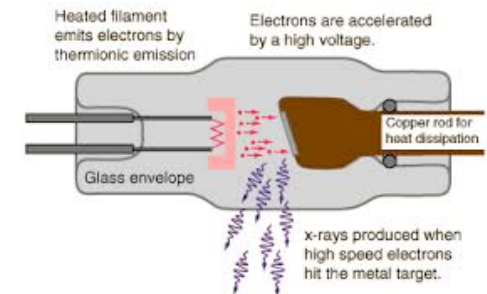
→ maximum X-ray energy $\sim E_e$



Mini-X Output X-Ray Spectrum: Ag Target @ 40 kV

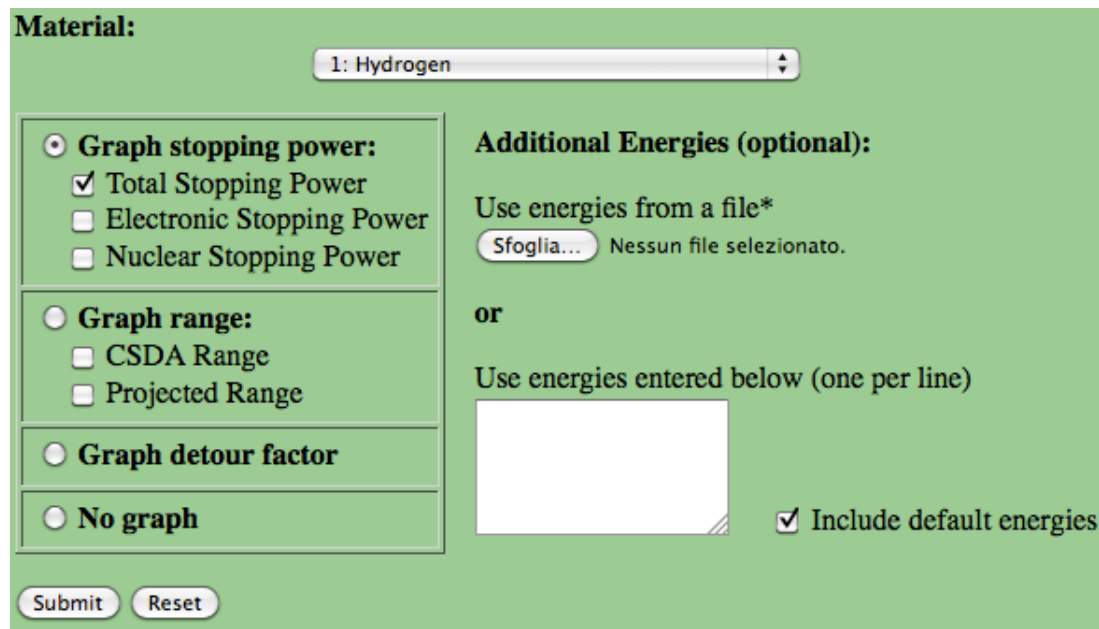


How does an X-ray tube work



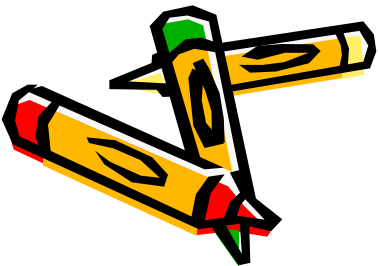
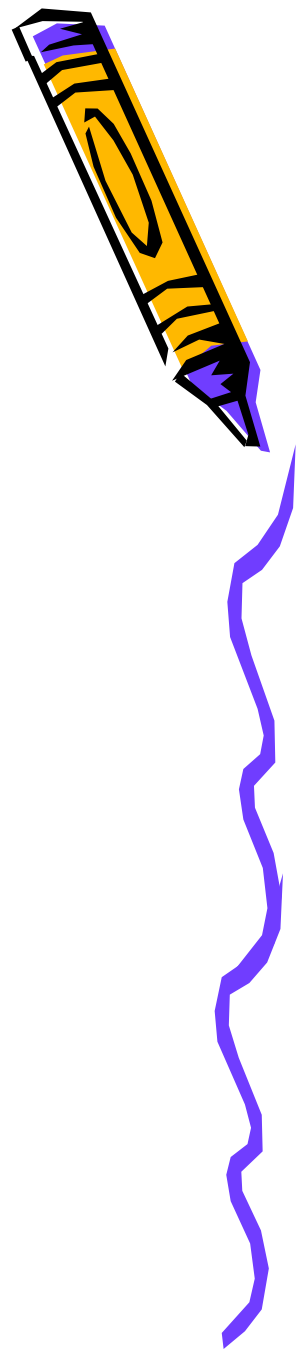
Diffusion and range

- Projected range is shorter than CSDA (linear) range



The screenshot shows the 'Material:' section of the NIST STAR software interface. It features a dropdown menu set to '1: Hydrogen'. Below this are four radio button options for graphing: 'Graph stopping power:' (with sub-options for Total, Electronic, and Nuclear Stopping Power), 'Graph range:' (with sub-options for CSDA Range and Projected Range), 'Graph detour factor', and 'No graph'. To the right, there are sections for 'Additional Energies (optional):', including a file selection button ('Sfoggia...') and a checkbox for 'Include default energies'. At the bottom are 'Submit' and 'Reset' buttons.

<http://www.nist.gov/pml/data/star/index.cfm>



Stopping power and range tables



<http://www.nist.gov/pml/data/star/index.cfm>

electrons

Select a common material:

1: Hydrogen

or enter a [unique material](#)

Graph stopping power:

- Total Stopping Power
- Collision Stopping Power
- Radiative Stopping Power

Graph density effect parameter

Graph CSDA range

Graph radiation yield

No graph

Additional Energies (optional):

Use energies from a file*

Sfogliala... Nessun file selezionato.

or

Use energies entered below (one per line)

Include default energies

Note: Only stopping powers and the density effect parameter will be calculated if additional energies are used.

Submit Reset



Exercises (Set 2)



- Determine the energy that a proton and an alpha must have to have a negligible multiple scattering
 $\langle \theta \rangle \leq 0.01 \text{ rad}$

In case of

- 20 cm of water
- 20cm of air

- Estimated CSDA and projected range in water for:
 - alpha particles of $T=1, 10$ and 100 MeV
 - Protons of $T=1, 10, 100 \text{ MeV}$
 - Electrons of $1, 9, 20 \text{ MeV}$
- Estimate the thickness of
 - Carbon
 - Leadneeded to stop $1, 10, 100 \text{ MeV}$ alpha and protons

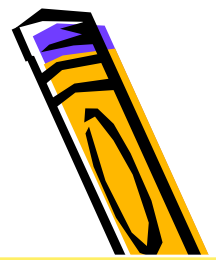
In case of composite materials

$$1/X_0 = \sum w_j / X_j ,$$

w_j = fraction by weight



Shieldings

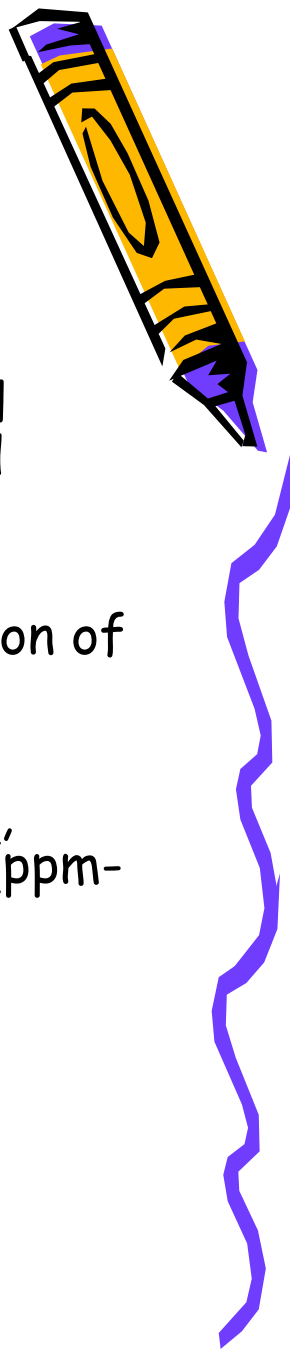


Range(mm)	WATER(proj/ CSDA)	CARBON (Proj.)	IRON (Proj.)	LEAD (Proj.)
P(1MeV)	24/24 10^{-3}	12 10^{-3}	6.6 10^{-3}	9.1 10^{-3}
P(10MeV)	1.2/1.2	0.61	0.26	0.30
P(100MeV)	77/77	38	14.5	14
α (1 MeV)	5.9/5.7 10^{-3}	2.6 10^{-3}	1.7 10^{-3}	2.2 10^{-3}
α (10 MeV)	112/112 10^{-3}	58 10^{-3}	26 10^{-3}	37 10^{-3}
α (100 MeV)	6.4/6.4	3.2	1.3	1.4
e(1 MeV)	~4.0/4.4			
e (9 MeV)	~40/45			
e(20 MeV)	~91/93			



→ Utilizzo di particelle alpha per terapia radiometabolica

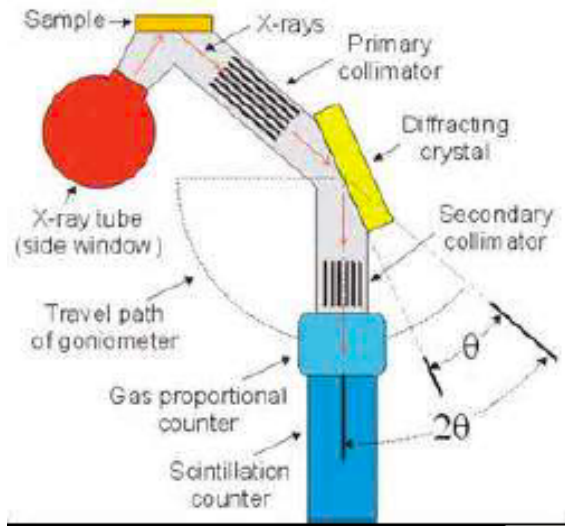
Application: XRF



- X-ray fluorescence (XRF) spectrometry is an elemental analysis technique with broad application in science and industry.
- XRF is routinely used for the simultaneous determination of elemental composition and film thickness.
- Modern XRF instruments are capable of analyzing solid, liquid, and thin-film samples for both major and trace (ppm-level) components.
- The analysis is rapid and usually sample preparation is minimal or not required at all.



XRF Setup



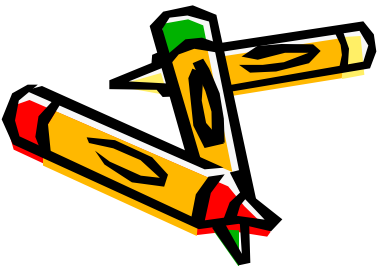
1) X-ray irradiates specimen

2) Specimen emits characteristic X-rays

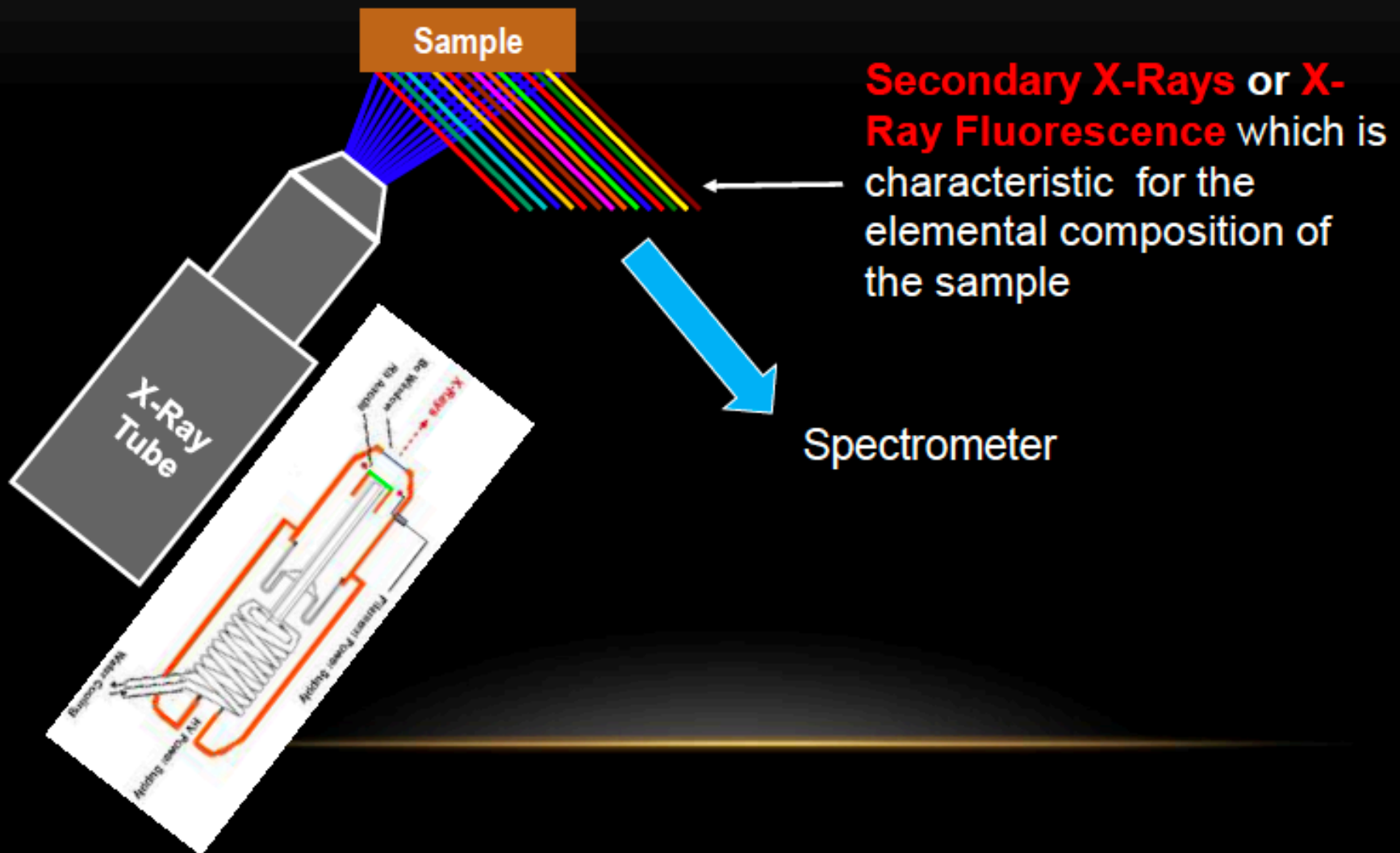
3) Analyzing crystal rotates to accurately reflect each wavelength and satisfy Bragg's Law $n\lambda = 2d\sin\theta$

4) Detector measures position and intensity of XRF peaks

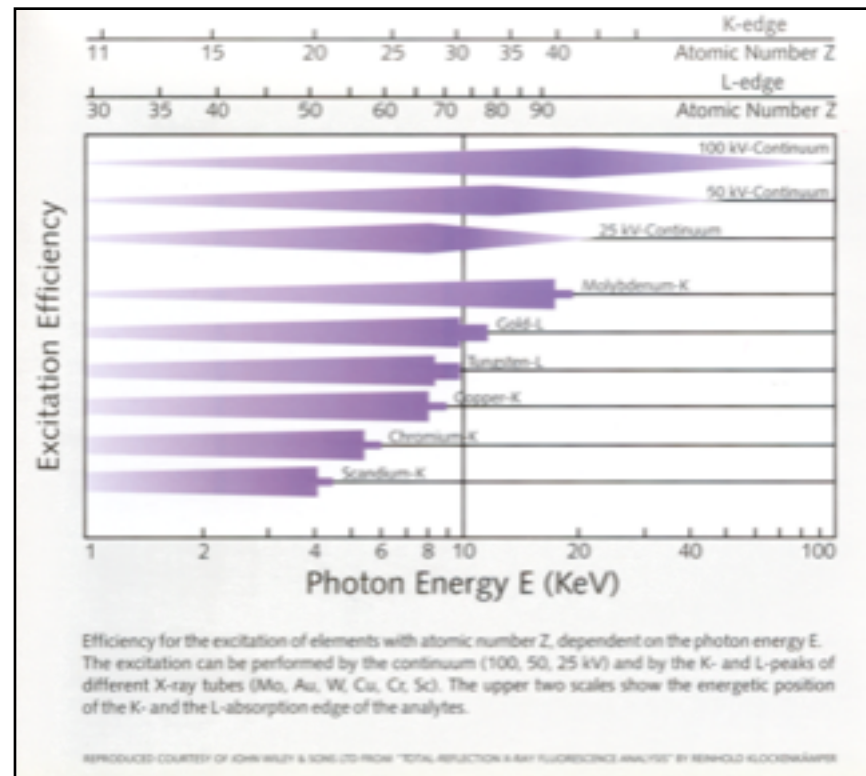
5) XRF is diffracted by a crystal at different . To separate X-ray I and to identify elements



Excitation of the sample

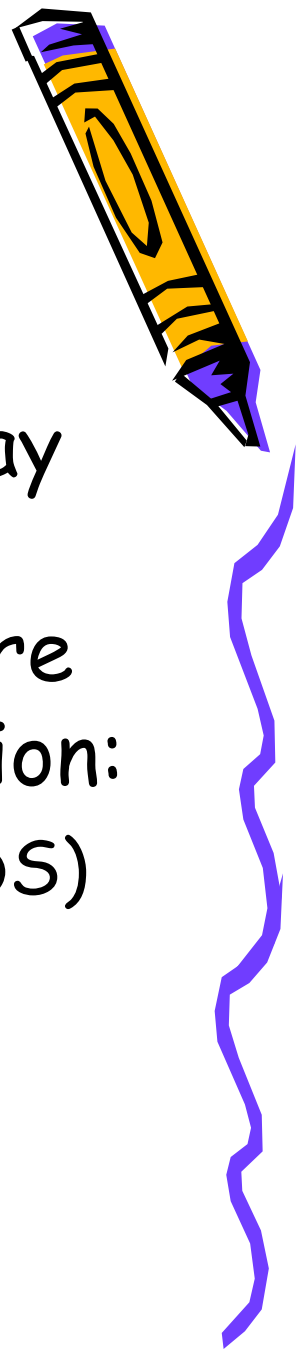


Choice of the anode



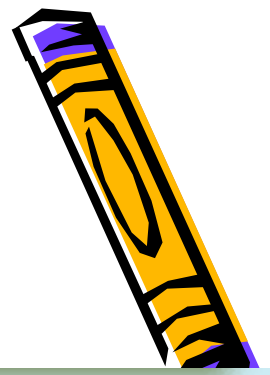
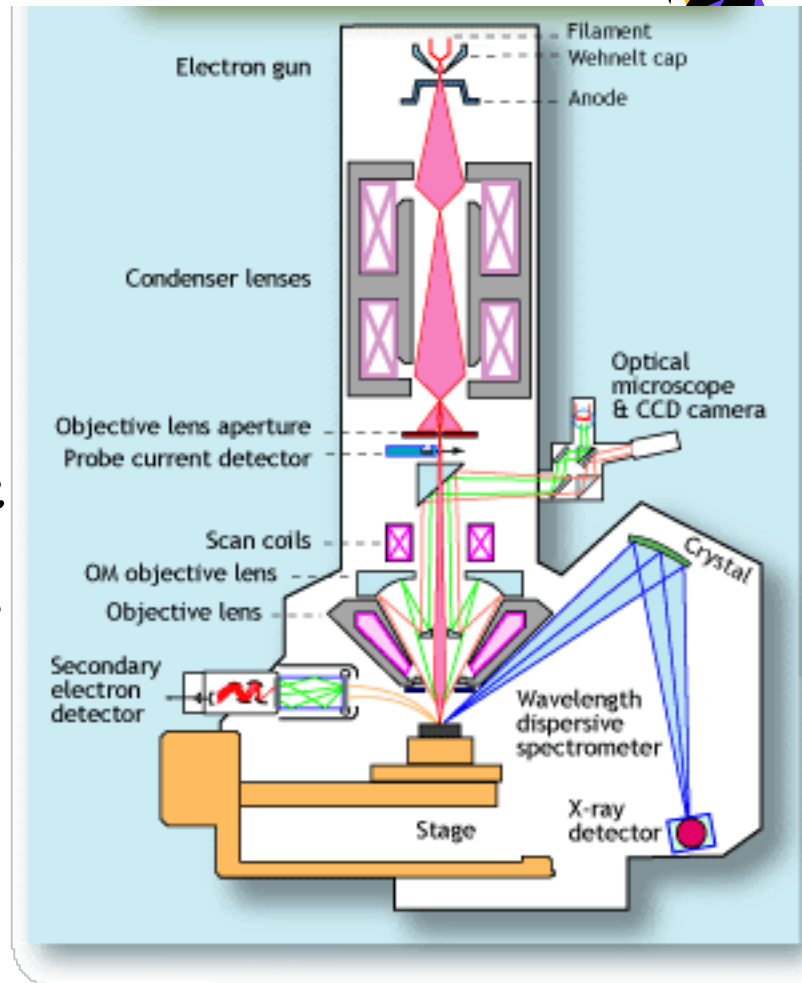
Detection system

- No matter how the secondary X-ray radiation (X-Ray fluorescence) is produced in XRF machines there are **TWO WAYS** to detect this radiation:
 1. Wavelength Dispersive System (WDS)
 2. Energy Dispersive System (EDS).



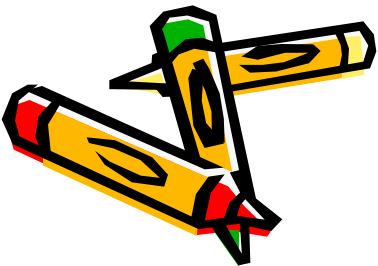
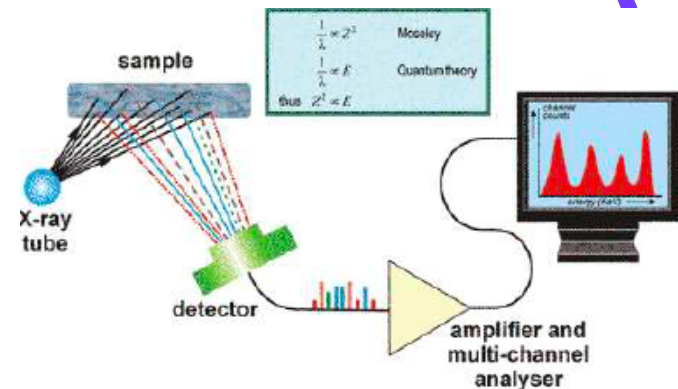
WDS

- A wavelength dispersive detection system physically separates the X-Rays according to their wavelengths.
- The x-rays are directed to a crystal, which diffracts (according to Bragg's Law) the X-Rays in different directions according to their wavelengths (energies).



EDS

- EDS is an analytical technique used for the elemental analysis or chemical characterization of a sample.
- The secondary x-rays (XRF) are directed to a detector.
- A detector is used to convert X-ray energy into voltage signals; this information is sent to a pulse processor, which measures the energy of the signals and passes them onto an analyzer.
- The analyzer converts the analog into a digital signal which is proportional to the energy of the incoming pulse.
- Received pulses are actually amplified and converted into digital signals.
- They are sorted by energy with help of multi-channel analyzer and sent to data display and analysis.
- The most common detector now is Si(Li) detector cooled to cryogenic temperatures with liquid nitrogen.



What can be analysed by X-Ray Fluorescence?



Oils



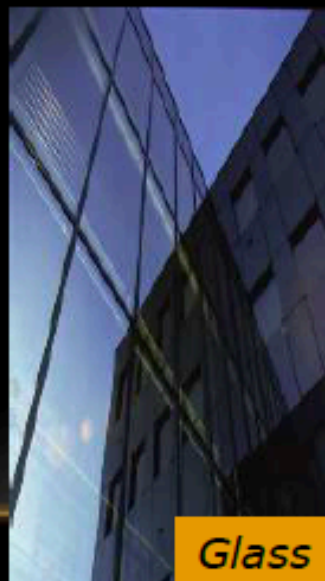
Polymers



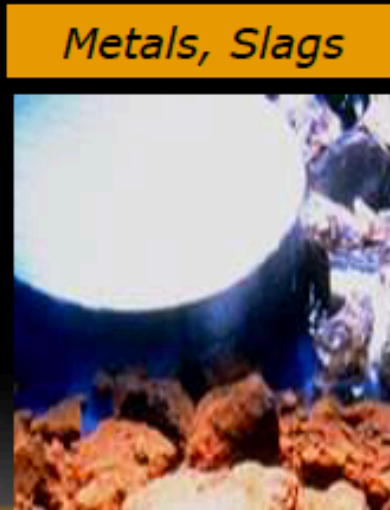
Ores and Raw Materials



Chemicals



Glass



Metals, Slags

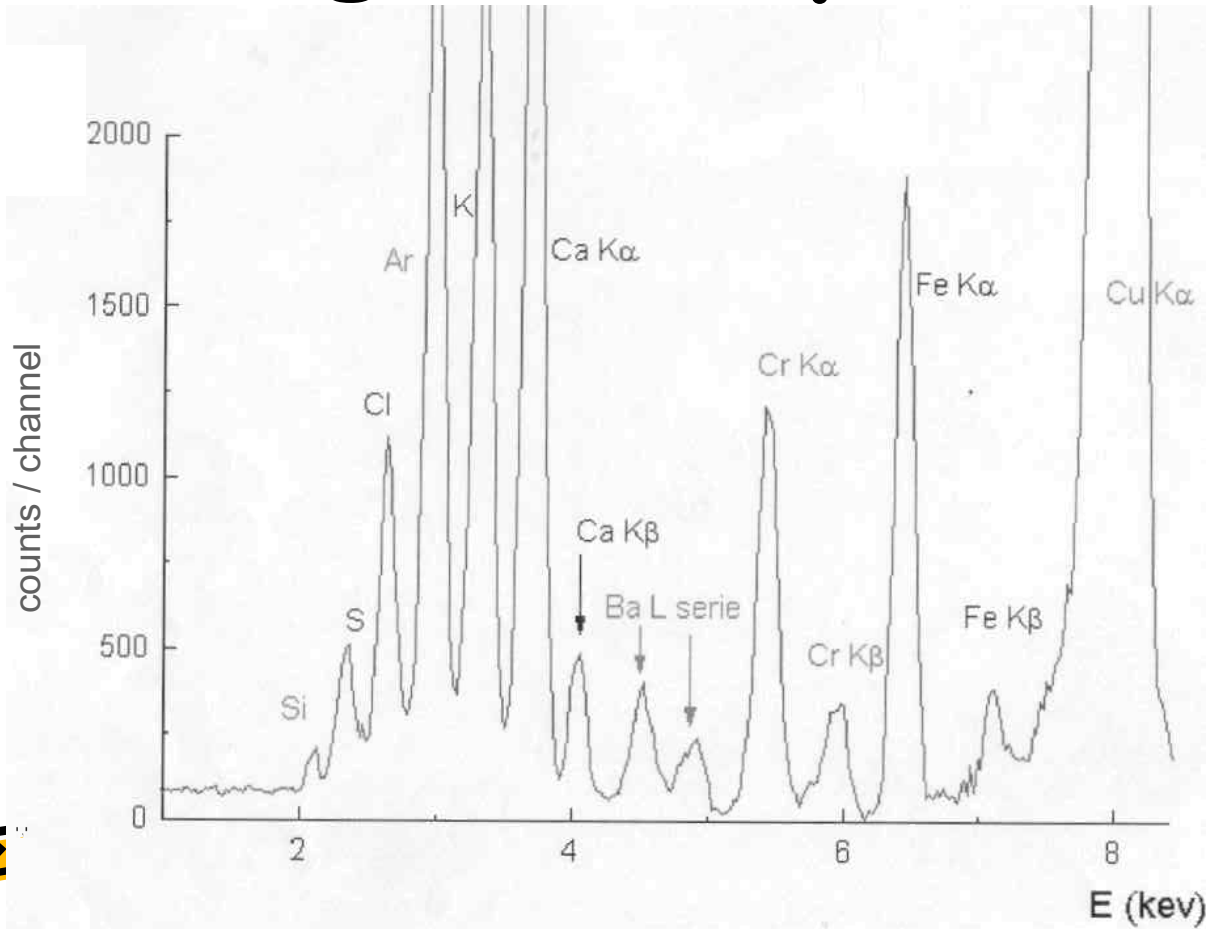


Ceramic

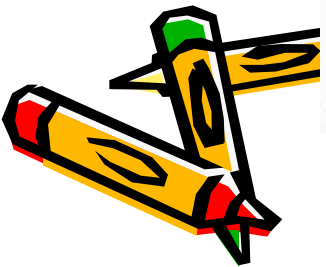
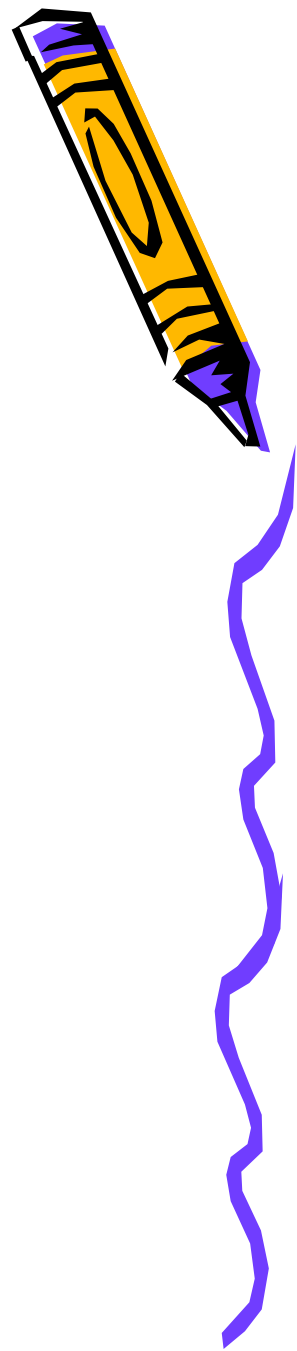


Food Products

Forensic: gunshot powder



Adhesive (chromium) and gunpowder (Barium)



XRF advantages

- XRF is a versatile, rapid technique .
- It is non destructive method of chemical analysis. Important as in case of samples in limited amounts, or valuable or irreplaceable.
- It is precise and with skilled operations it is accurate.
- .
- Applicable to a wide variety of samples from powders to liquids.
- It is convenient and economical to use.
- .
- With the major input cost being the hardware itself, which averages around \$75,000 for a modern industrial-use spectrometer or \$125,000 for a research-quality instrument.
- The instruments have few moving parts, tend to be low-maintenance, and on a regular basis consume only liquid nitrogen and electricity





Table 2. The extrapolated range of electrons in some commonly used solvents

Energy (MeV)	Extrapolated range (g/cm ²)				
	Diethylether	Benzene	Ethanol	Glycerol	Water
0.01	2.32 E-04	2.46 E-04	2.29 E-04	2.31 E-04	2.21 E-04
0.02	6.98 E-04	7.43 E-04	6.97 E-04	7.22 E-04	6.98 E-04
0.05	3.52 E-03	3.75 E-03	3.52 E-03	3.63 E-03	3.51 E-03
0.1	1.18 E-02	1.25 E-02	1.18 E-02	1.21 E-02	1.17 E-02
0.2	3.74 E-02	3.96 E-02	3.73 E-02	3.83 E-02	3.70 E-02
0.5	1.50 E-01	1.59 E-01	1.49 E-01	1.53 E-01	1.48 E-01
1	3.77 E-01	3.99 E-01	3.76 E-01	3.87 E-01	3.73 E-01
2	8.63 E-01	9.13 E-01	8.62 E-01	8.86 E-01	8.57 E-01
5	2.35 E+00	2.48 E+00	2.35 E+00	2.42 E+00	2.35 E+00
10	4.84 E+00	5.11 E+00	4.85 E+00	5.01 E+00	4.87 E+00
20	9.82 E+00	1.04 E+01	9.84 E+00	1.02 E+01	9.89 E+00
50	2.40 E+01	2.52 E+01	2.40 E+01	2.46 E+01	2.39 E+01
100	4.41 E+01	4.63 E+01	4.39 E+01	4.46 E+01	4.30 E+01

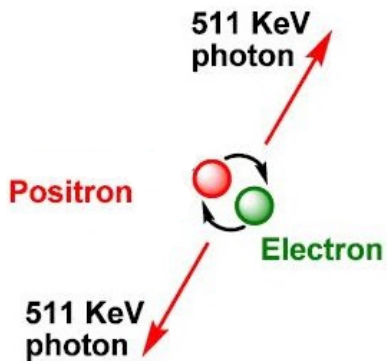
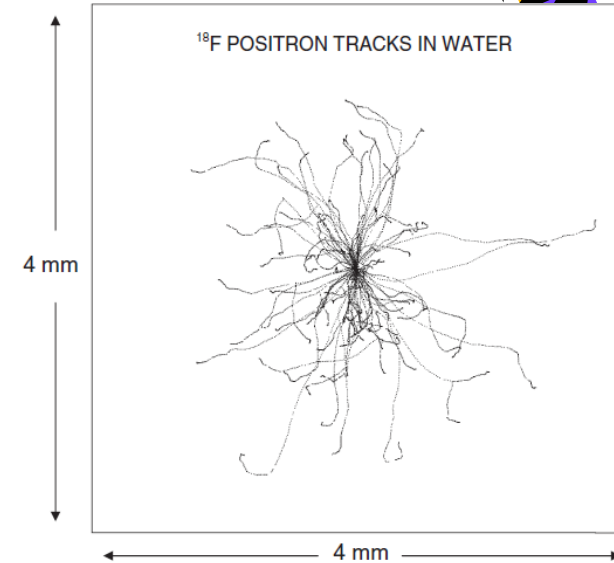


Positron annihilation

A high energy positron will lose energy by collision and radiation → only 20% probability of annihilation in flight

$e^+e^- \rightarrow \gamma\gamma$ (free electron)

$e^+e^- N \rightarrow \gamma N$ (bound electron ~20%)



After it stops, positron binds with electrons → positronium

Para $S=0$ $\tau=125$ ps $CP=+$ → $\gamma\gamma$

Orto $S=1$ $\tau=140$ ns $CP=-$ → $\gamma\gamma\gamma$

Decays of Positronium

Positronium is an e^+e^- atom with parity states:

$$P(e^+e^-) = P_{e^-}P_{e^+}(-1)^L = (-1)^{L+1}$$

Note this is the opposite parity to the Hydrogen atom states because of the intrinsic parities of the e^+ and e^-

Charge conjugation symmetry depends on the spin orientations:

$$C(\uparrow\downarrow) = (-1)^L \quad C(\uparrow\uparrow) = (-1)^{L+1}$$

$S=0$ is ortho-positronium $\uparrow\downarrow$, $S=1$ is para-positronium $\uparrow\uparrow$

Photon has $C = -1$ from symmetry of electromagnetic fields

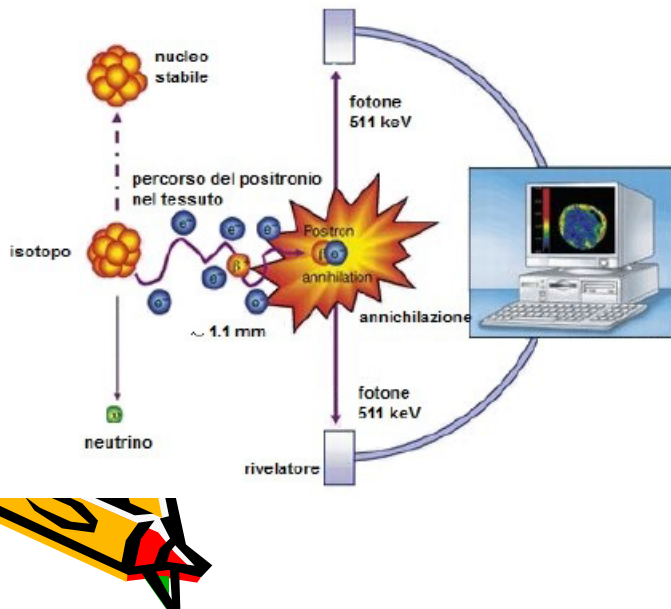
Ortho-positronium states with even(odd) L decay to 2(3) photons

Para-positronium states with odd(even) L decay to 2(3) photons

C-parity is conserved in electromagnetic interactions

PET: spatial resolution

- The range of positrons is the main limiting factor to the resolution in PET



ISOTOPO	VITA MEDIA	PERCORSO RESIDUO	E_{max}
F_{18}	110 min	0.64 mm	0.64 MeV
C_{11}	20.4 min	1.03 mm	0.96 MeV
N_{13}	9.97 min	1.32 mm	1.19 MeV
O_{15}	122 s	2.01 mm	1.72 MeV

Escapes

If positron is create one or both the electrons can escape.

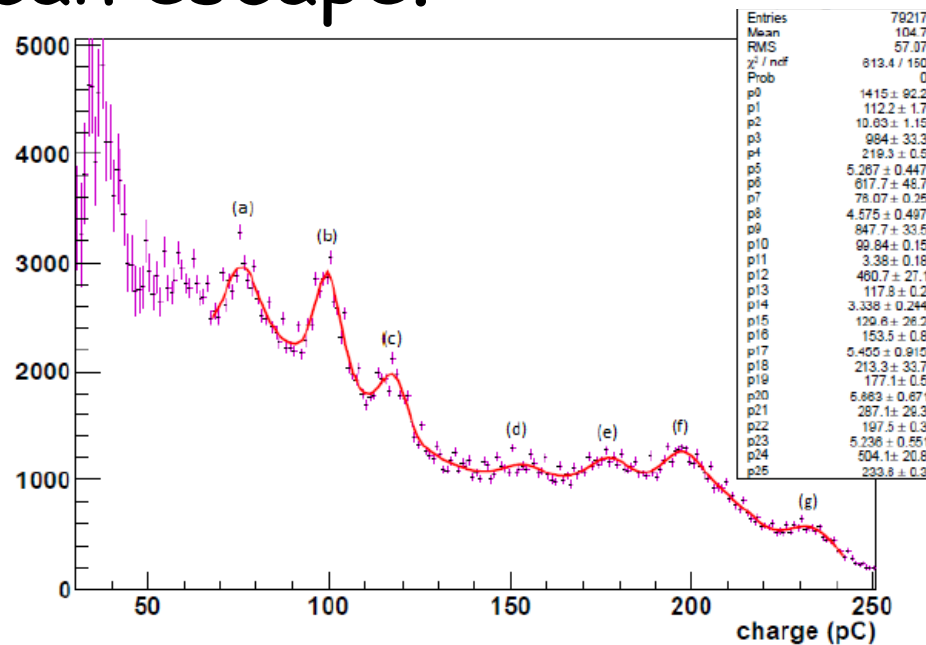


Figure 5.2. Americium Beryllium on paraffin spectrum acquired with the *LYSOI* detector crystal in *GSI* configuration. From the left (a) is the single escape of (b), the 2.22 MeV line, (c) is identified as the peak from the gammas of the interaction of neutrons with the iron of the steel capsule, (d) and (e) are respectively the double and single escape peaks of the 4.44 line (f), and (g) likely comes from the interaction of neutrons with the iron of the steel capsule, as well.

Interazione della radiazione elettromagnetica

I fotoni nell'attraversare un mezzo assorbente possono interagire sia con gli elettroni degli atomi sia con il nucleo atomico:

Interazione con gli elettroni:

Diffusione Compton

Diffusione Rayleigh

Effetto fotoelettrico

Interazione col nucleo:

Reazioni fotonucleari

Produzione di coppie





A differenza delle particelle cariche i fotoni interagiscono con la materia in modo discontinuo e la loro intensità non viene mai ridotta a zero.

$$I(x) = I_0 e^{-\mu x}$$

dove μ è detto **coefficiente di attenuazione** (o di assorbimento) e dipende sia dall'energia del fotone sia dalle caratteristiche del mezzo attraversato.

$$\left[\frac{1}{cm} \right]$$

La **lunghezza di attenuazione** λ è definita come l'inverso del coefficiente di attenuazione μ : $\lambda = \frac{1}{\mu} \text{ [cm]}$

Indichiamo con σ la **sezione d'urto** che esprimeremo in cm^2 :

$$\sigma = \frac{\mu}{n_A} = \frac{\mu}{\rho / m_A} \cong \frac{\mu A}{\rho [g/cm^3] N_A}$$

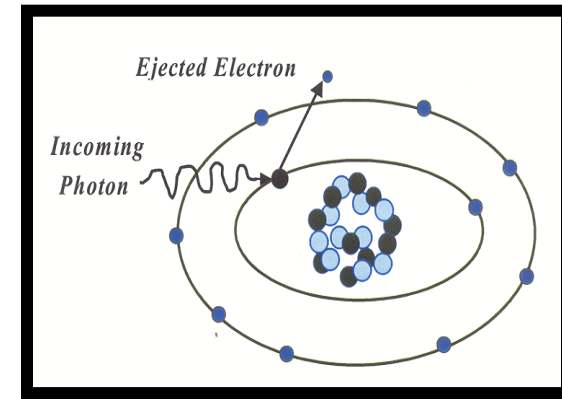
*rappresenta la **probabilità** che una data collisione tra due particelle avvenga. Essa ha le dimensioni di una superficie e spesso viene misurata in barn ($1 \text{ barn} = 10^{-24} \text{ cm}^2$).*



Effetto fotoelettrico

Interazione di un fotone con un elettrone atomico.

Durante l'interazione il fotone cede tutta la sua energia all'elettrone.



$$E_{e^-} = h\nu - E_b$$

L'effetto fotoelettrico è un effetto a soglia, potendosi verificare solo quando l'energia del fotone incidente è superiore all'energia di legame dell'elettrone.

La sezione d'urto per effetto fotoelettrico, che indicheremo con σ_{foto} risulta:

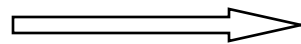
$$\sigma_{foto} \cong Z^5 \alpha \left(\frac{m_e c^2}{E_\gamma} \right)^n \quad \begin{array}{l} n = 3.5 \text{ per } E_\gamma < m_e c^2 \\ n = 1 \text{ per } E_\gamma \gg m_e c^2 \end{array}$$

costante di struttura fine

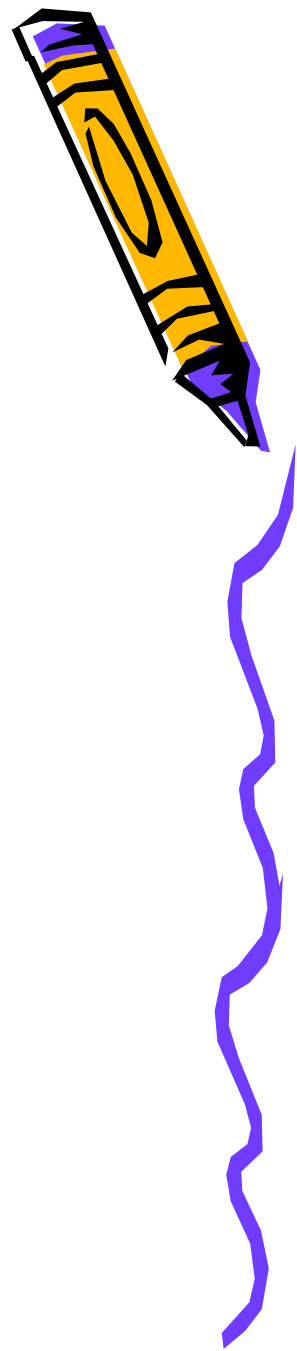
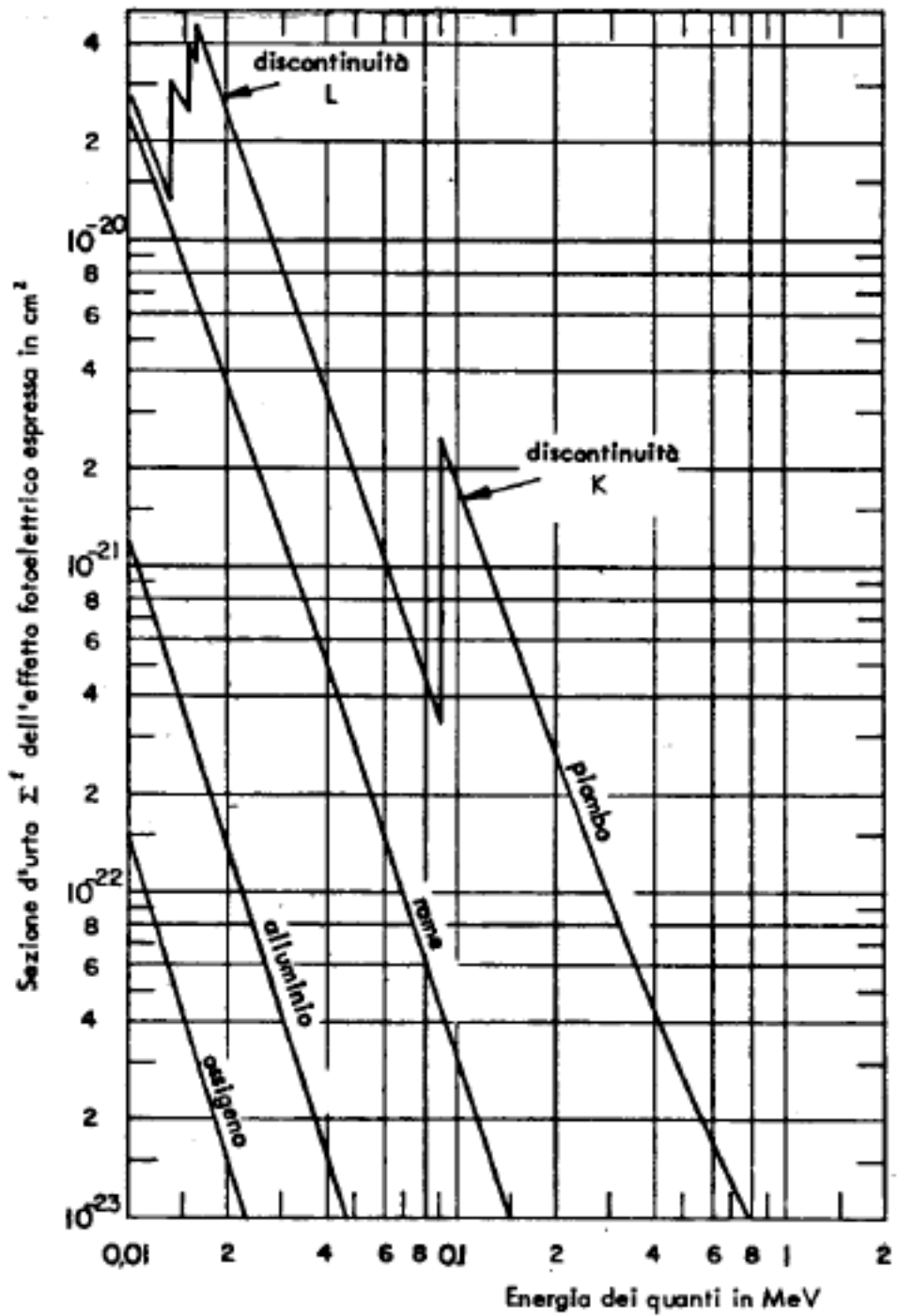
$$\alpha = \frac{e^2}{hc} \approx \frac{1}{137}$$



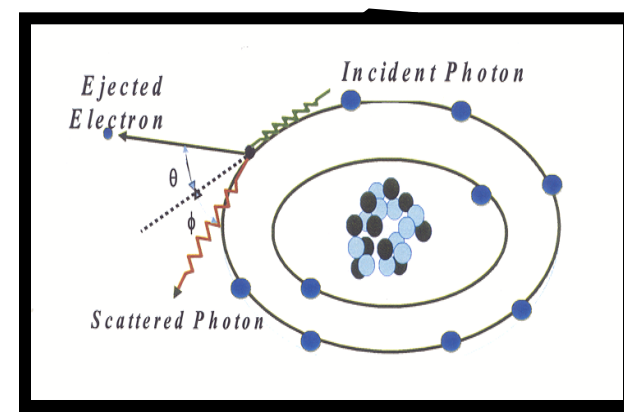
ricordando che $\sigma = \mu \frac{A}{\rho N_A}$



$$\mu_{foto} \approx \rho \frac{Z^5}{A}$$

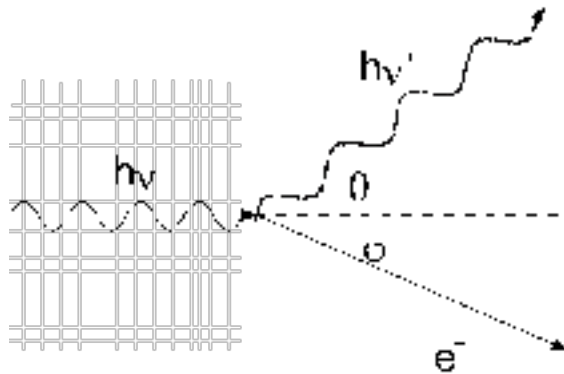


Effetto Compton



Interazione di un fotone con un elettrone "libero"

La differenza di energia tra fotone incidente e fotone diffuso sarà impartita all'e



A differenza dell'effetto fotoelettrico il fotone non cede tutta la sua energia in una sola interazione, ma rilascia solo una frazione della propria energia deviando rispetto alla direzione incidente.

La sezione d'urto per l'effetto Compton (nella trattazione non relativistica) risu

$$\sigma_{Compton} \cong \frac{8\pi}{3} r_e^2$$

Raggio classico dell' e^-

$$r_e = \frac{e^2}{m_e c^2} = 2.8 \cdot 10^{-15} m$$

$$\mu_{Compton} \approx \rho \frac{Z}{A}$$

Per la conservazione dell'energia abbiamo:

$$E_\gamma + E_e = E_{\gamma'} + E_{e'}$$

ora $E_\gamma = h\nu$

$$E_e = \sqrt{m_e^2 c^4 + p_e^2 c^2} = m_e c^2$$

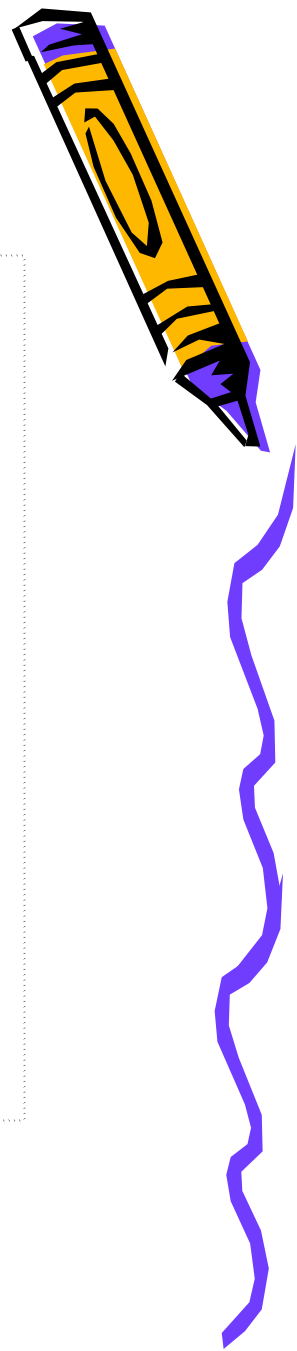
$$E_{e'} = \sqrt{m_e^2 c^4 + p_{e'}^2 c^2}$$

$$E_{\gamma'} = h\nu'$$

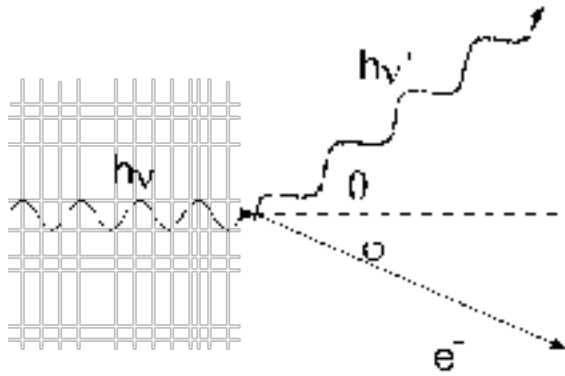
Quindi $h\nu + m_e c^2 = h\nu' + \sqrt{m_e^2 c^4 + p_{e'}^2 c^2}$

esplicitando rispetto a $p_{e'}^2$

$$p_{e'}^2 = \frac{(h\nu + m_e c^2 - h\nu')^2 - m_e^2 c^4}{c^2}$$



Calcoliamo la perdita di energia del fotone incidente in funzione dell'angolo di diffusione.



Sia γ il fotone (di frequenza ν) incidente su un elettrone a riposo
Sia γ' il fotone diffuso (di frequenza ν')
Sia e' l'elettrone dopo l'urto
Indichiamo θ l'angolo di diffusione

Per la conservazione della quantità di moto abbiamo:

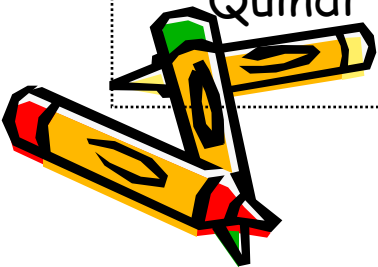
ma
$$p_\gamma = \frac{E_\gamma}{c} = \frac{h\nu}{c} \quad p_{\gamma'} = \frac{E_{\gamma'}}{c} = \frac{h\nu'}{c}$$

$$p_\gamma = p_{\gamma'} + p_{e'}$$

$$p_{e'}^2 = (p_\gamma - p_{\gamma'})^2 = p_\gamma^2 + p_{\gamma'}^2 - 2p_\gamma p_{\gamma'} \cos\theta$$

Quindi

$$p_{e'}^2 = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2\frac{h\nu}{c}\frac{h\nu'}{c}\cos\theta$$



Combinando le 2 equazioni si ottiene: $\frac{(h\nu + m_e c^2 - h\nu')^2 - m_e^2 c^4}{c^2} = \left(\frac{h\nu}{c}\right)^2 + \left(\frac{h\nu'}{c}\right)^2 - 2 \frac{h\nu}{c} \frac{h\nu'}{c} \cos\theta$

Sviluppando e ricordando che $\nu = \frac{c}{\lambda}$

si ottiene $h \frac{c}{\lambda} \frac{c}{\lambda'} (1 - \cos\theta) = m_e c^2 \left(\frac{c}{\lambda} - \frac{c}{\lambda'} \right)$

e quindi $\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos\theta)$

$\lambda_C = \frac{h}{m_e c}$ è detta **lunghezza d'onda Compton dell'elettrone** e vale $2.426 \cdot 10^{-12}$ m.

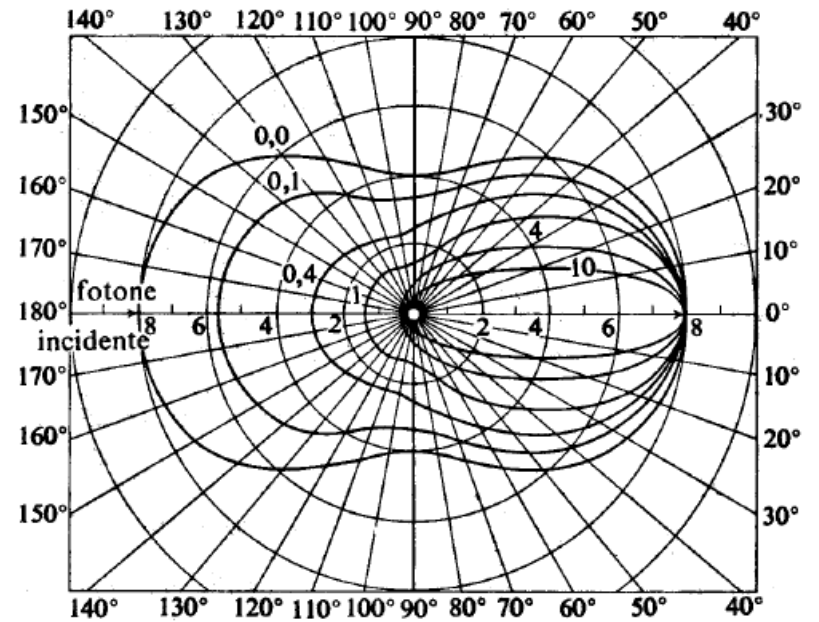
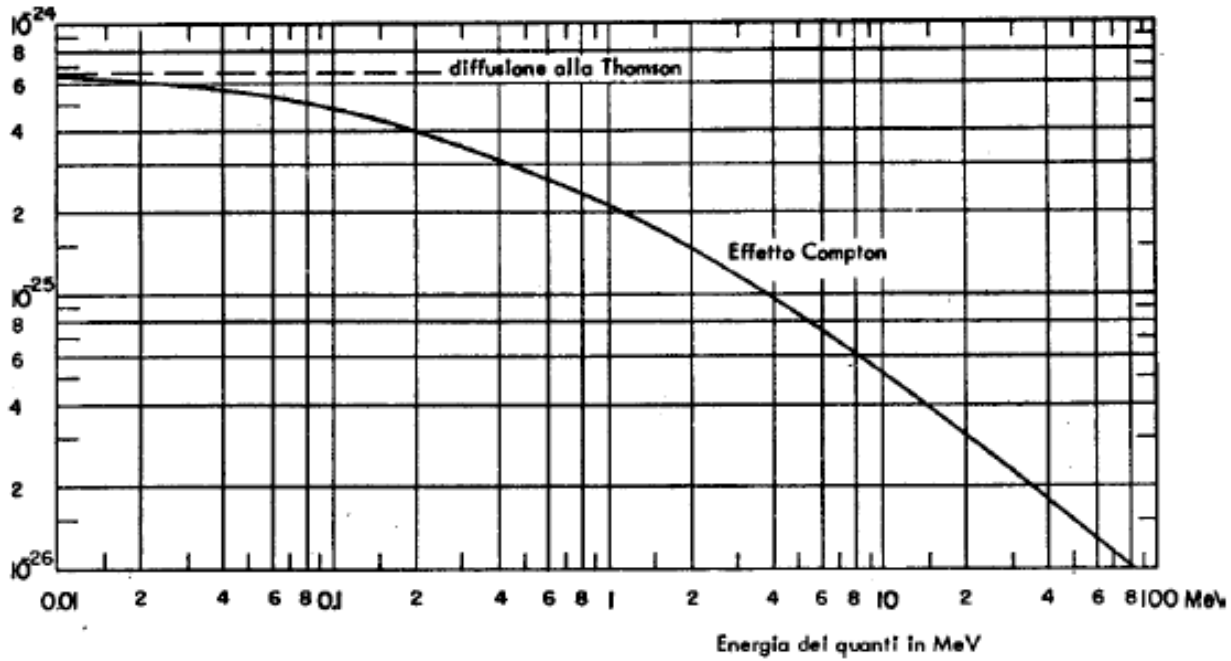
La conoscenza della lunghezza d'onda λ del fotone incidente (e quindi la sua energia $E=h\nu$) e l'angolo di diffusione θ permettono di calcolare il valore dell'energia cinetica impressa all'elettrone:

$$E_{\text{cinetica}} = h\nu - h\nu' = \frac{(h\nu)^2 (1 - \cos\theta)}{m_e c^2 + h\nu(1 - \cos\theta)}$$

• Per $\theta=0^\circ$ l'energia trasferita è nulla, e quindi l'energia del fotone è conservata.

• Per $\theta=180^\circ$ il fotone è rimbalzato all'indietro ed l'energia trasferita è massima e vale $\frac{2(h\nu)^2}{m_e c^2 + 2(h\nu)}$

Sezione d'urto σ_{Σ}^C dell'effetto Compton su un elettrone espressa in cm^2



COMPTON

$$\lambda + \lambda' - \lambda' \cos \theta = \lambda' - \lambda' \cos \theta$$

$$e = \lambda' - \lambda'$$

$$m_0 c^2 + m_0 c^2 + 2 \lambda' E - 2 \lambda' E \cos \theta = 2 \lambda' E$$

$$\Rightarrow \Delta \frac{1}{E_2} = \frac{1 - \cos \theta}{m_0 c} \quad E_1 \left[0, \frac{2}{m_0 c} \right]$$



Puncte $E_1 = 0, 1, 2$ Ke

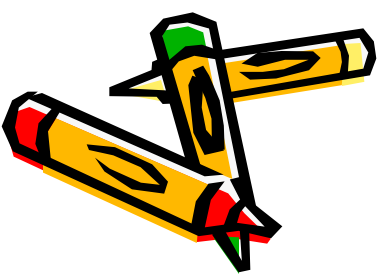
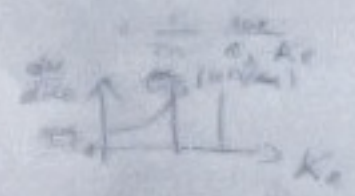
$$K_0^{max} = \sum_{i=1}^n E_2 (E_1 + K_0^{max}) \Rightarrow$$

$$\Rightarrow K_0^{max} = \frac{2 E_1^2}{m_0 c^2 + 2 E_1}$$

E_1 (KeV)	K_0^{max} (KeV)
663	176
211	541

distributie:

$$\frac{dN}{dE_0} = \frac{dN}{dE_2} = \frac{dN}{dE_1} \frac{dE_1}{dE_2} = \frac{1}{20} \frac{m_0 c^2}{(E_1)^2}$$



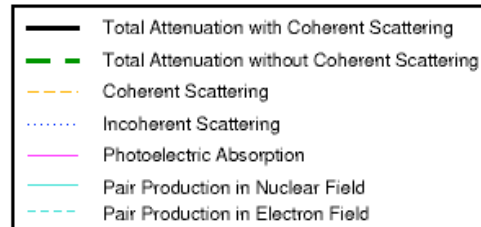
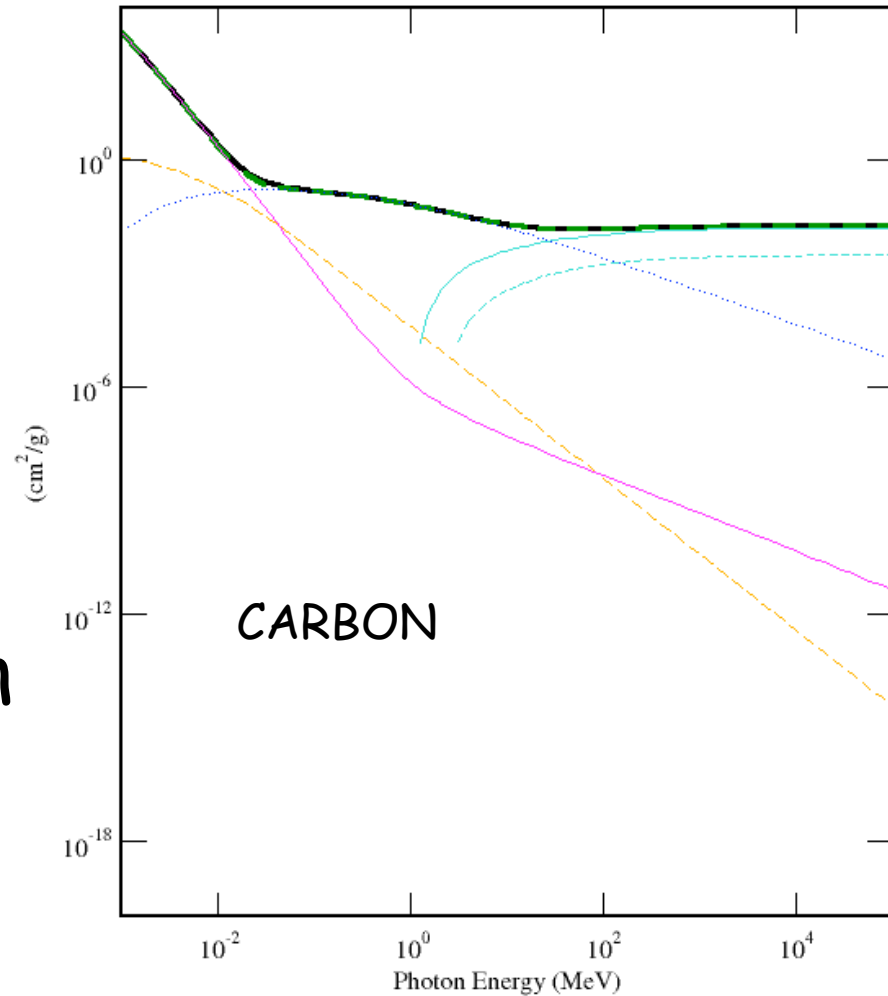
DATABASE:

<http://physics.nist.gov/PhysRefData/Xcom/html/xcom1.html>

Photon attenuation

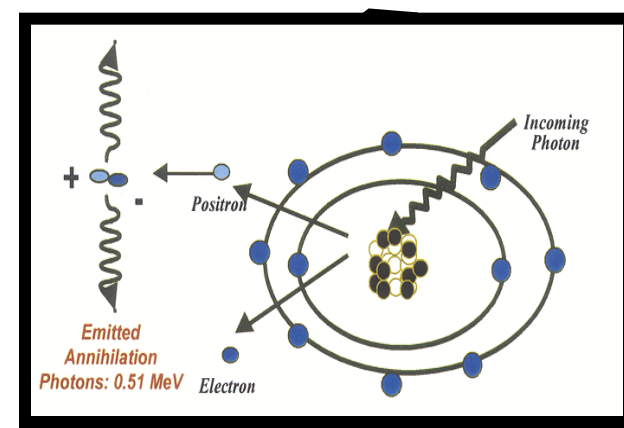
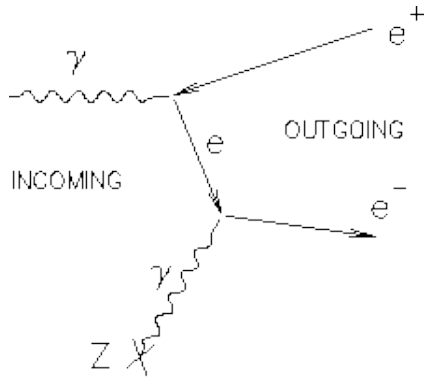
- Typically reported in terms of the mass attenuation coefficient

$$\sigma \frac{N_A}{A} = \frac{\mu}{\rho} [g/cm^2]$$



Produzione di coppie

Un fotone si materializza creando una coppia elettrone-positrone:



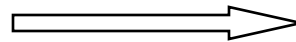
Tale processo può verificarsi solo se il fotone possiede un'energia maggiore della somma delle masse delle due particelle prodotte; ossia deve avere un'energia $E_\gamma \geq 1.022 \text{ MeV}$

L'eccesso di energia del fotone incidente verrà trasformato in energia cinetica del positrone e dell'elettrone:

$$E_{\text{cinetica}} = \frac{E_\gamma (\text{MeV}) - 1.022}{2}$$

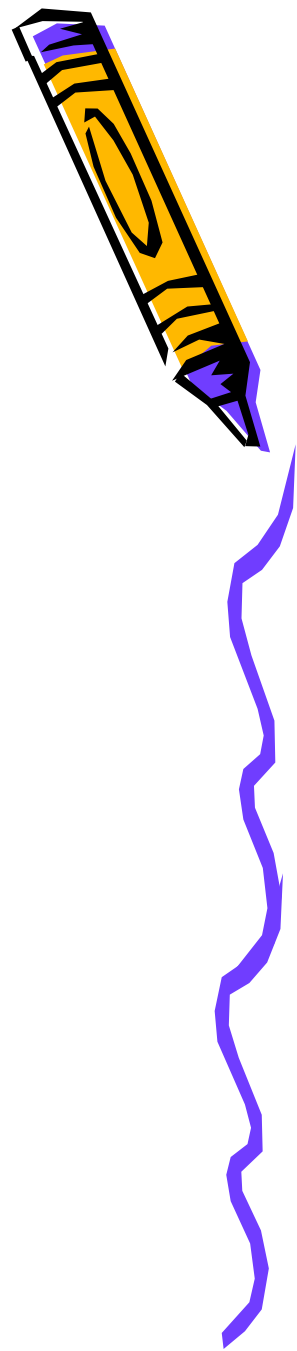
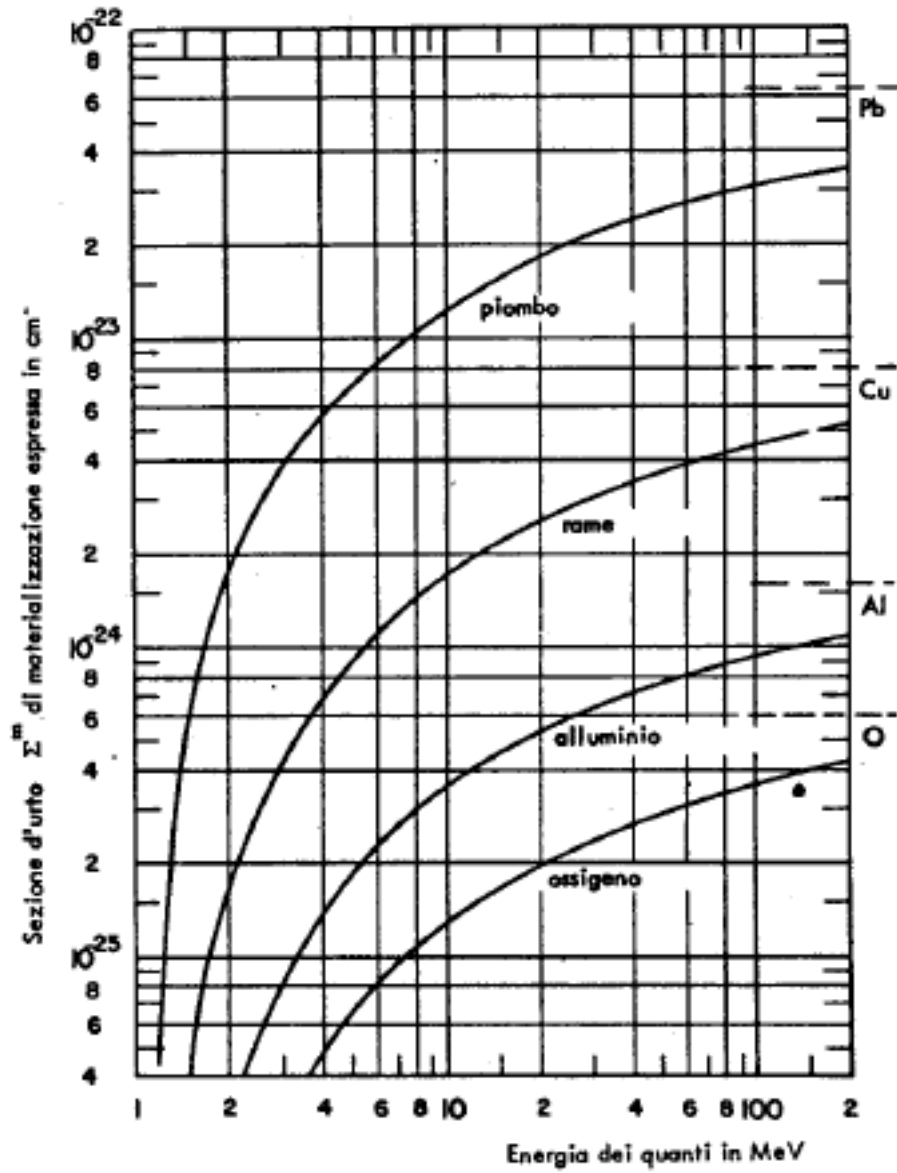
La sezione d'urto per produzione di coppie vale:

$$\sigma_{pp} \cong \frac{Z^2 \alpha^3}{(m_e c^2)^2}$$



$$\mu_{pp} = \frac{7}{9X_0} \approx \rho \frac{Z^2}{A}$$





Attenuazione dei fotoni

Il **coefficiente di attenuazione totale** μ_{tot} , è la somma dei coefficienti dei tre processi considerati, e cioè:

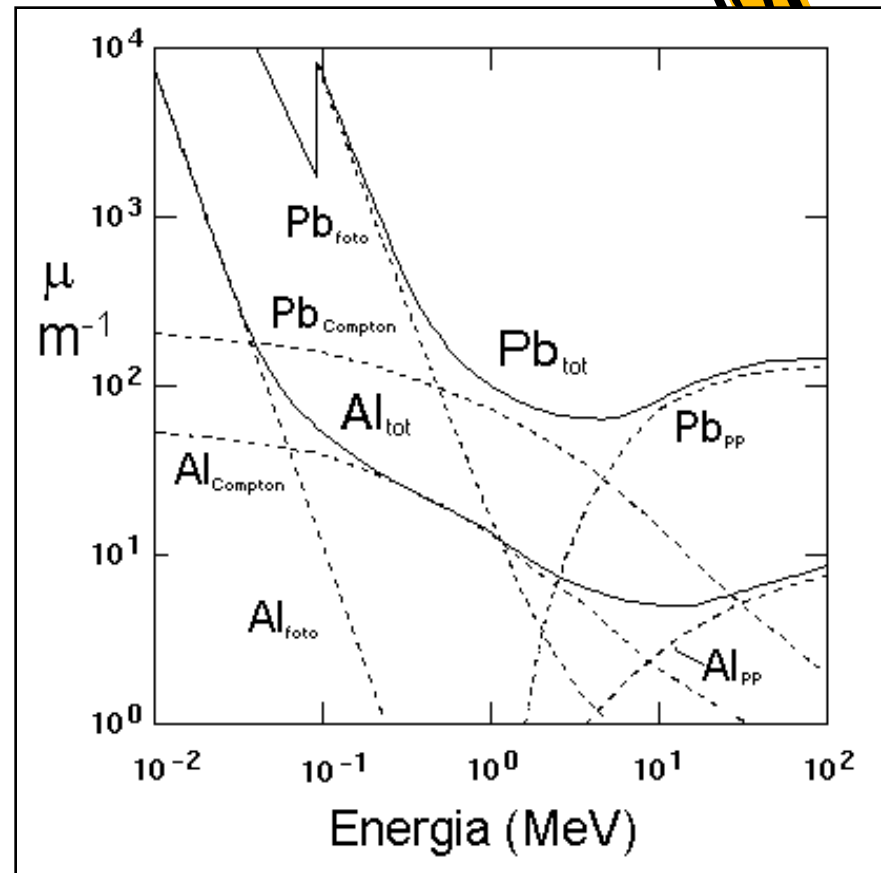
$$\mu_{tot} = \mu_{foto} + \mu_{Compton} + \mu_{pp}$$

Il numero di fotoni diffusi o assorbiti in uno spessore dx è proporzionale al flusso di fotoni incidenti $\Phi(x)$ e alla probabilità totale d'interazione μ_{tot} :

$$-d\phi(x) = \phi(x) \cdot \mu_{tot} \cdot dx$$

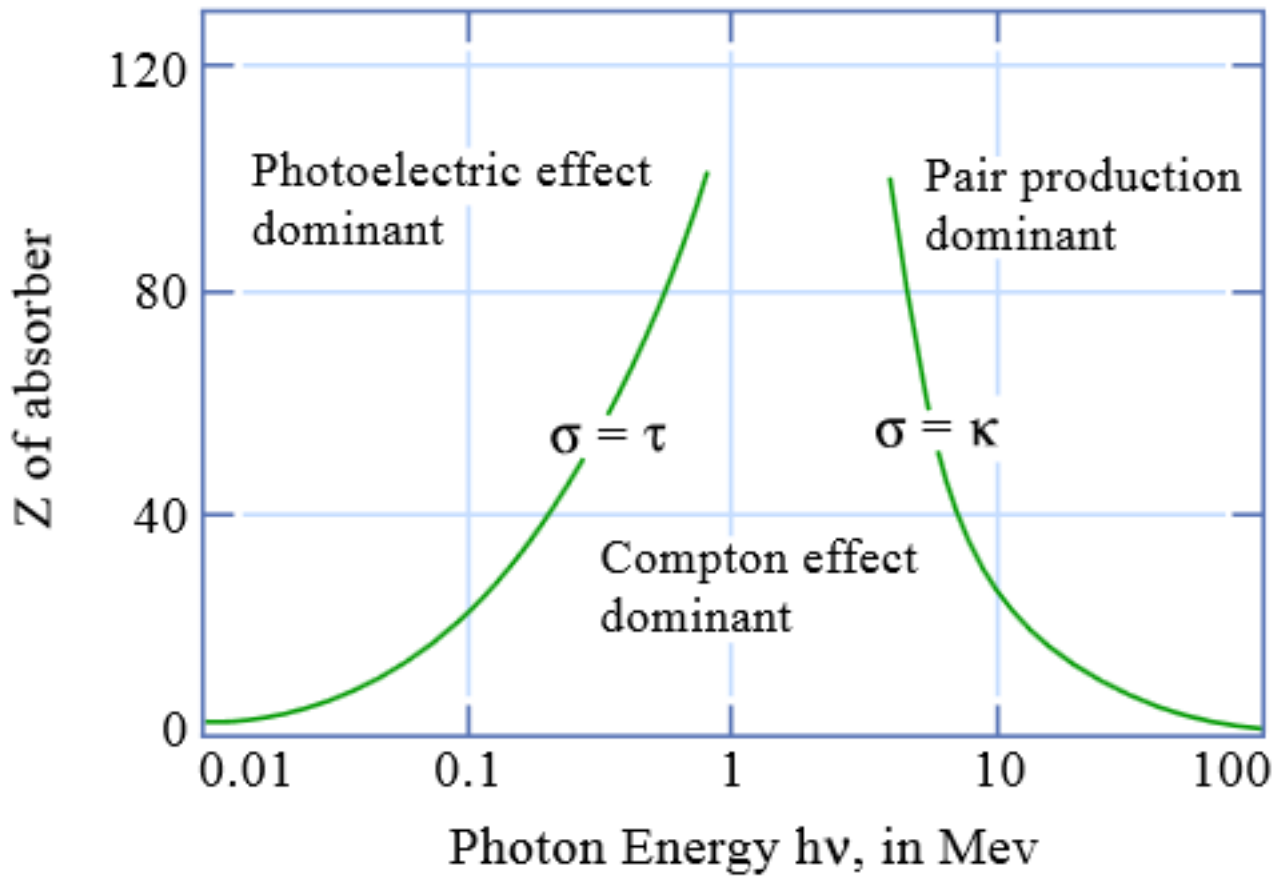
Dopo aver attraversato uno spessore x , l'intensità del fascio è:

$$\phi(x) = \phi(0) \cdot e^{-\mu_{tot} \cdot x}$$



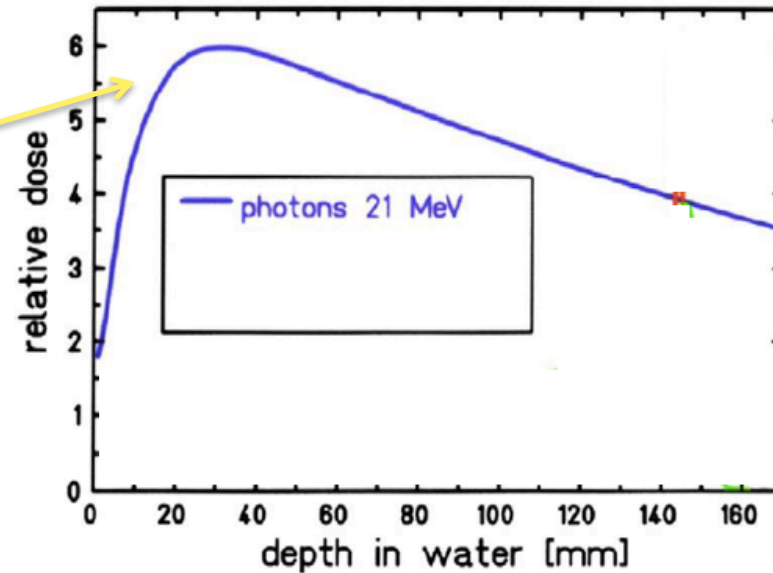
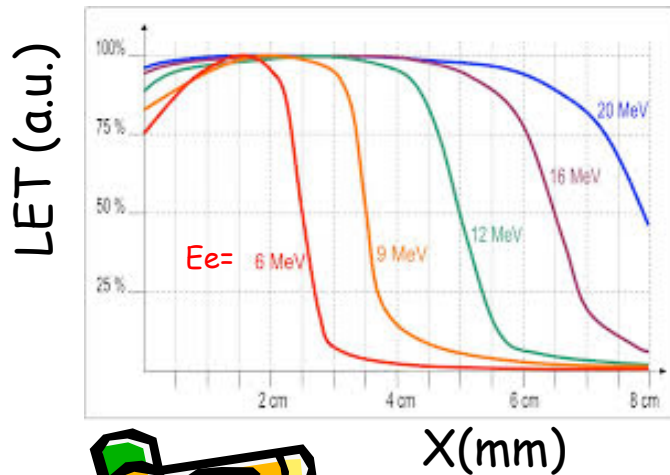
*Esempio: γ del ^{208}Tl (2.61 MeV).
Nel piombo $\mu_{tot} = 0.477$ 1/cm quindi $\lambda = 2.1$ cm*

Relative importance of processes



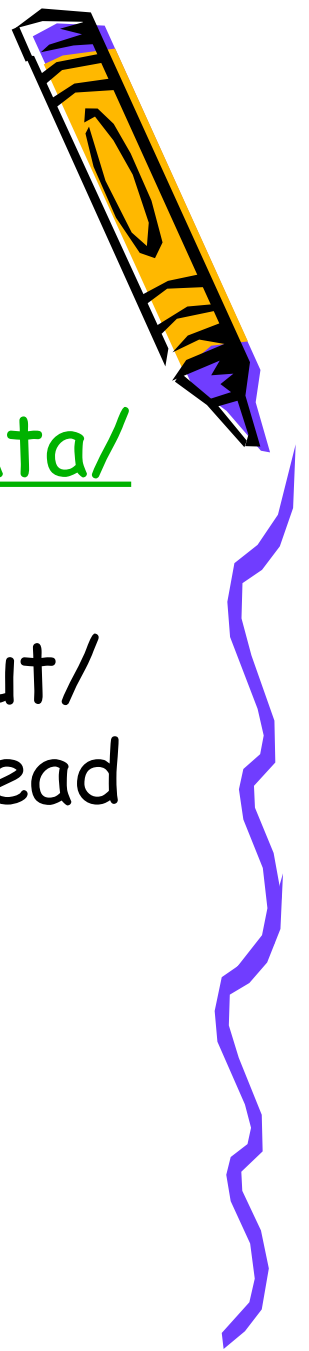
Photons LET

Build-up/Skin-Sparing effect



Kahoot!

Databases with mass attenuation coefficients



- <http://physics.nist.gov/PhysRefData/XrayMassCoef/tab3.html>
- <http://www.wolframalpha.com/input/?i=mass+attenuation+coefficient+lead+140+keV+photon>



Exercise Nr. 3

Estimate the amount of

- Lead
- plastic (Poly(methyl methacrylate-PMMA, $C_5O_2H_8$, $\rho=1.2 \text{ g/cm}^3$),
- paraffin ($C_{31}H_{64}$, $\rho=0.9 \text{ g/cm}^3$)
- iron

needed to attenuate by 10^{-4} photons of

- 100 keV
- 1 MeV
- 10 MeV

