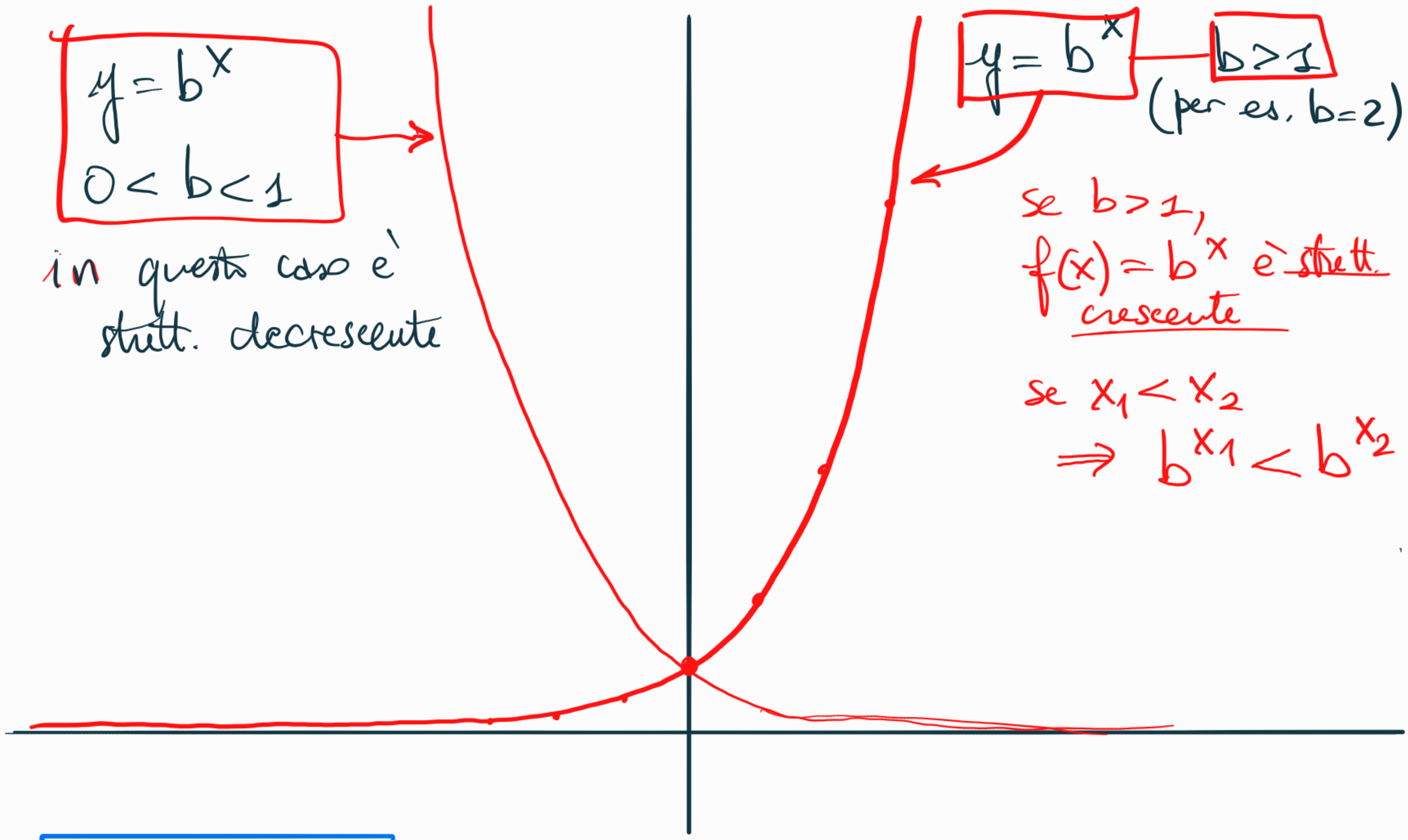


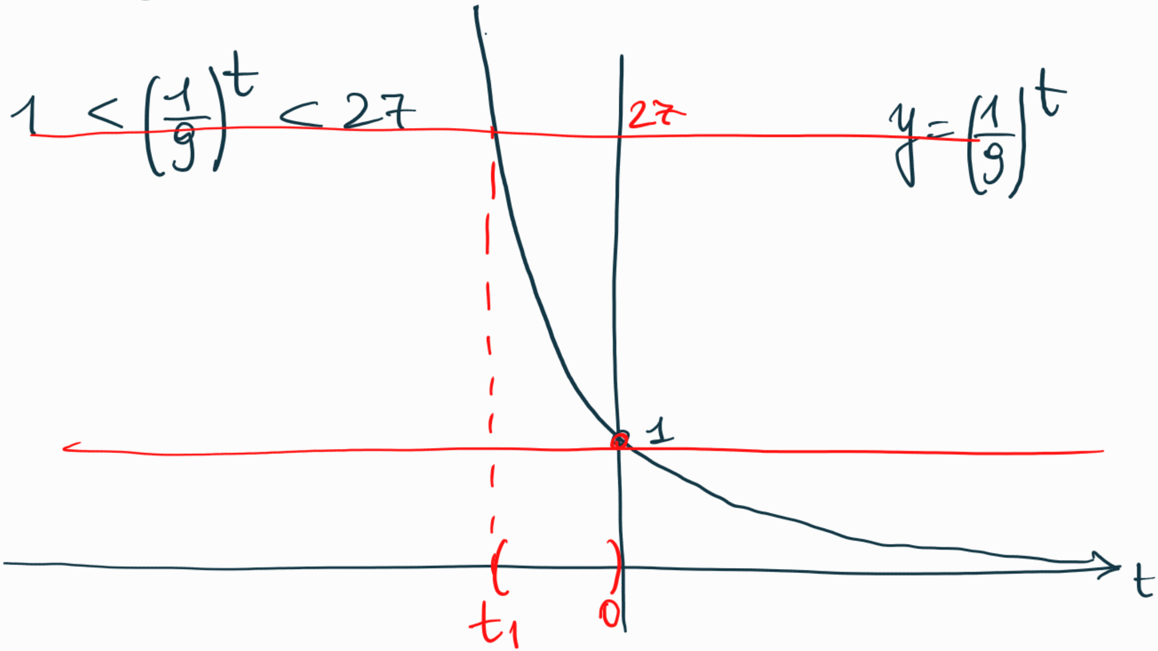
Esponenziali: sono le funzioni del tipo

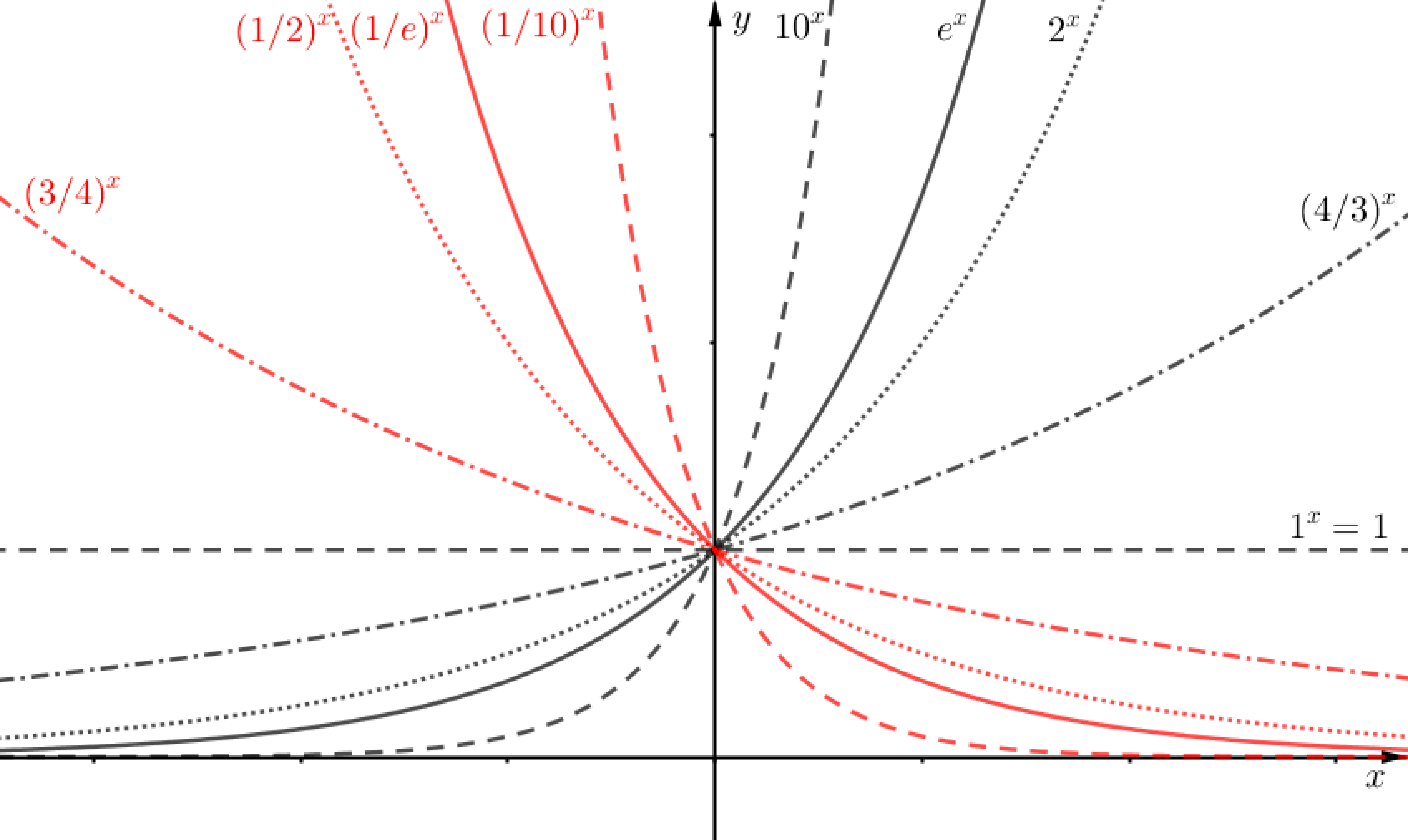
$$f(x) = b^x \qquad x \in \mathbb{R}$$

con $b > 0$

Risolvere la doppia diseq.ne

$$1 < \left(\frac{1}{9}\right)^{\overset{=t}{\boxed{2x-3}}} < 27.$$




$(1/2)^x$
$(1/e)^x$
$(1/10)^x$
y
10^x
e^x
2^x
$(3/4)^x$
$(4/3)^x$
$1^x = 1$
x

$$\left(\frac{1}{9}\right)^{t_1} = 27 \qquad \left(\frac{1}{9}\right)^{\frac{1}{2}} = \frac{1}{3}$$

$$\Updownarrow \qquad \left(\frac{1}{9}\right)^{-1/2} = 3$$

$$t_1 = -\frac{3}{2} \qquad \left(\frac{1}{9}\right)^{-3/2} = 27$$

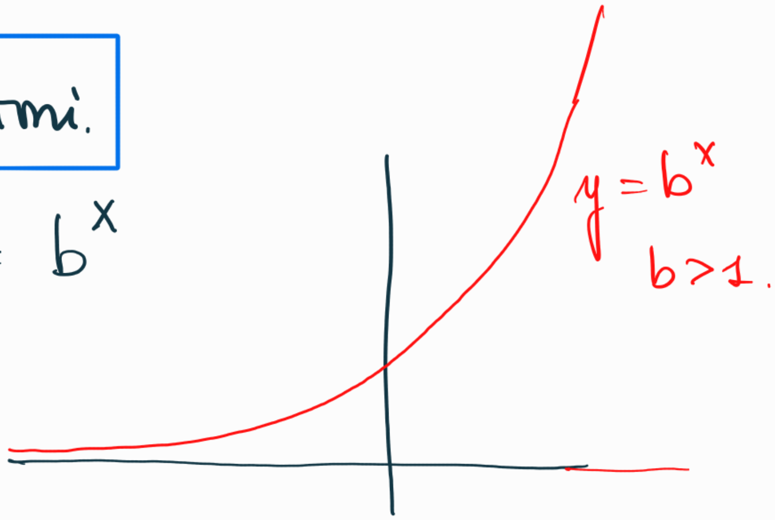
Sol^{ne} $\quad -\frac{3}{2} < \underset{\underset{2x-3}{\parallel}}{t} < 0 \qquad -\frac{3}{2} < 2x-3 < 0$

$$\frac{3}{2} < 2x < 3$$

$$\boxed{\frac{3}{4} < x < \frac{3}{2}}$$

Logaritmi.

$f(x) = b^x$



$b>1$

$$\begin{aligned} f:\ & \mathbb{R} \longrightarrow \mathbb{R} \\ & x \longmapsto f(x) = b^x \end{aligned}$$

Così non è suriettiva..

$$\boxed{\begin{aligned} f:\ & \mathbb{R} \longrightarrow (0,+\infty) \\ & x \longmapsto f(x) = b^x \end{aligned}}$$

1) è suriettiva? Sì, $f(x) = b^x$ è continua in un intervallo $(\mathbb{R})$

$\underset{\text{Teorema}}{\Longrightarrow}$ f assume tutti i valori tra $\underbrace{\inf_{\mathbb{R}} f}_{\overset{\parallel}{0}}$ e $\underbrace{\sup_{\mathbb{R}} f}_{\overset{\parallel}{+\infty}}$

2) è iniettiva? sì, perché è strett. crescente.

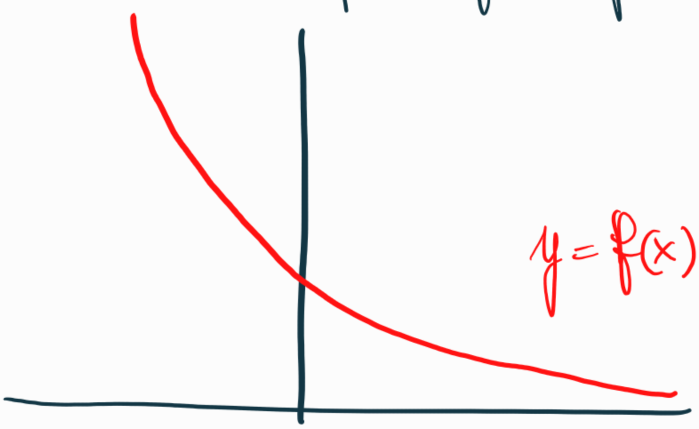
f è biiettiva da $\mathbb{R}$ in $(0,+\infty)$

$\forall y \in (0,+\infty)\ \exists!\ x \in \mathbb{R}$ t.c. $b^x = y$

$$f^{-1}: (0,+\infty) \longrightarrow \mathbb{R}$$
$$\begin{array}{ccc} \Psi & & \\ y & \longmapsto & f^{-1}(y) = \text{l'unico } x \in \mathbb{R} \text{ t.c. } b^x = y. \\ & & \| \\ & & \log_b y \end{array}$$

$$\boxed{\begin{array}{l} \forall y > 0,\ \log_b y \text{ è l'unico } x \in \mathbb{R} \quad \text{t.c. } b^x = y \\ \quad x = \log_b y \iff b^x = y \end{array}}$$

La stessa cosa si può fare per $\boxed{\begin{array}{l} f(x) = b^x \\ \quad \text{con } b \in (0,1) \end{array}}$



$$f(x): \mathbb{R} \to (0,+\infty) \qquad (b \in (0,1))$$
$$\quad x \mapsto b^x$$

1) suriettiva? sì, come prima
2) iniettiva? sì, perché strett. decrescente.
$\Big| \Rightarrow$ biiettiva

$$f^{-1}: (0,+\infty) \longrightarrow \mathbb{R}$$
$$\begin{array}{ccc} \Psi & & \\ y & \longmapsto & f^{-1}(y) = \text{l'unico } x \in \mathbb{R} \text{ t.c. } b^x = y \\ & & = \log_b y \end{array}$$

In entrambi i casi

$$x = \log_b y \iff b^x = y$$

Es $\log_3 81 = 4 \iff 3^4 = 81$

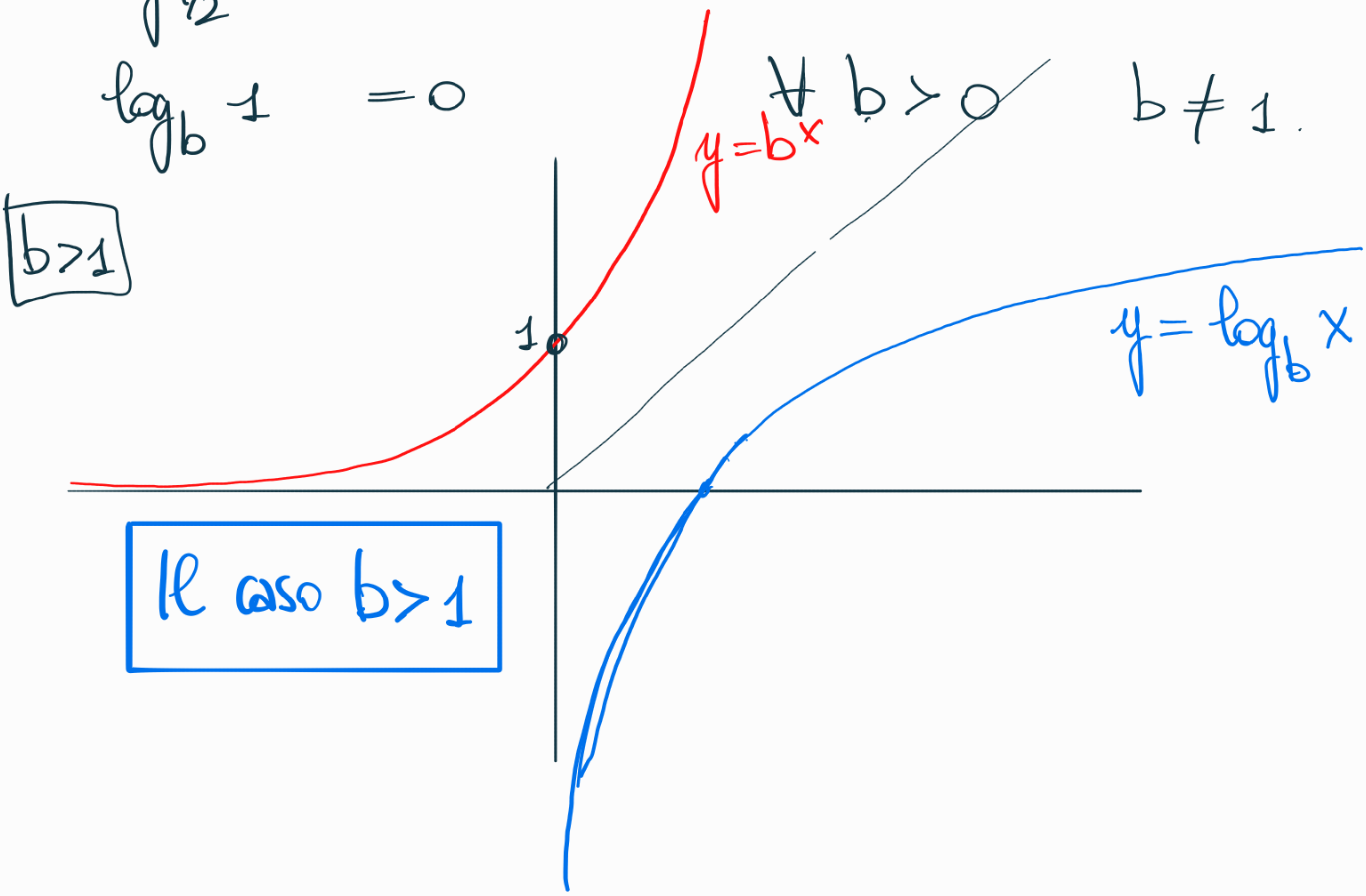
$\log_3 \sqrt{3} = \frac{1}{2} \qquad 3^{1/2} = \sqrt{3}$

$\log_{\sqrt{3}} 3 = 2 \qquad (\sqrt{3})^2 = 3$

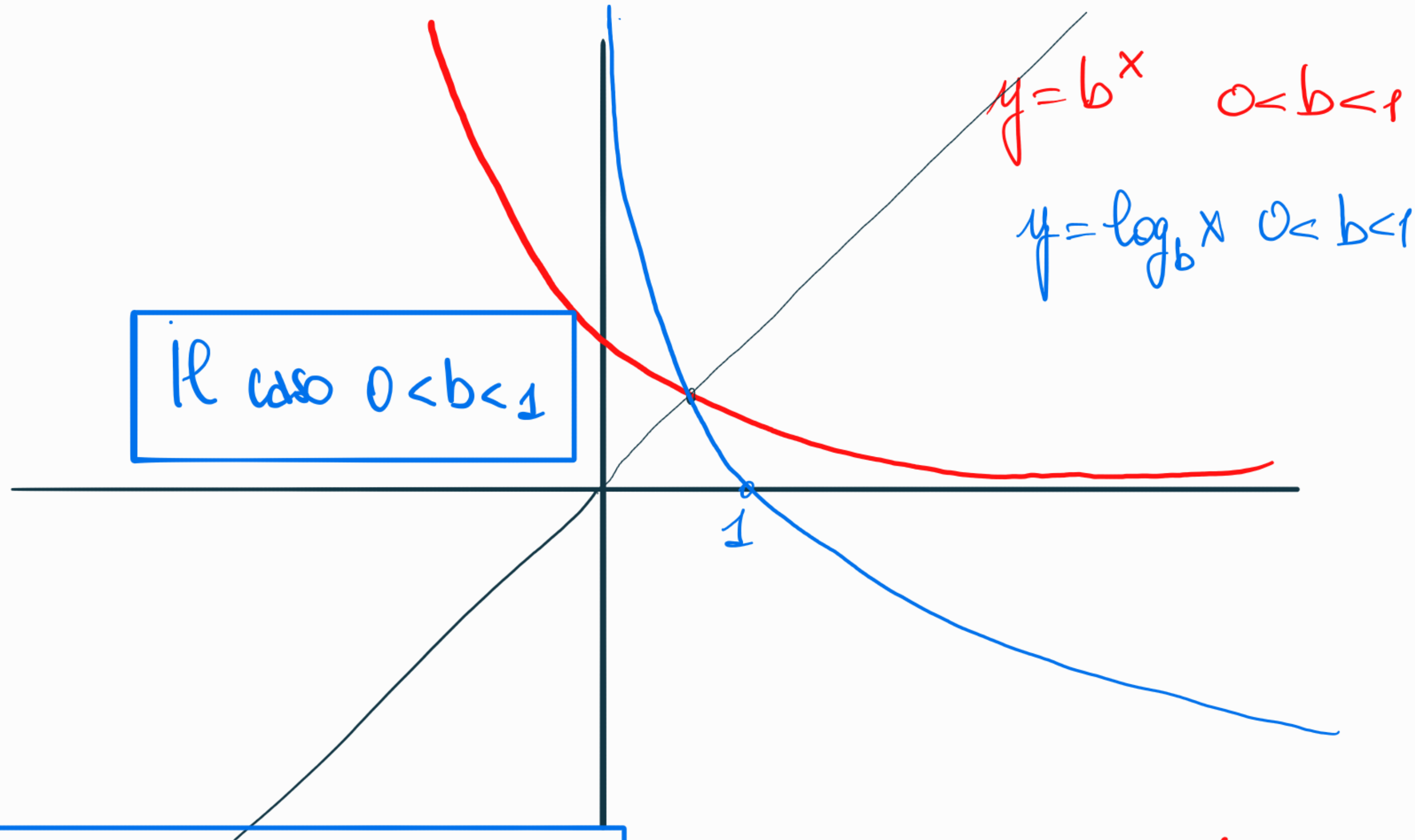
$\log_{1/2} 64 = -6 \qquad \left(\frac{1}{2}\right)^{-6} = 2^6 = 64$

$\log_{1/2} 1 = 0$

$\log_b 1 = 0 \qquad \forall\, b > 0 \quad b \neq 1.$



Il caso $0<b<1$



Proprietà dei logaritmi

$f(x) = b^x$

1) $\boxed{b^{\log_b x} = x} \iff f(f^{-1}(x)) = x \quad \forall x>0$

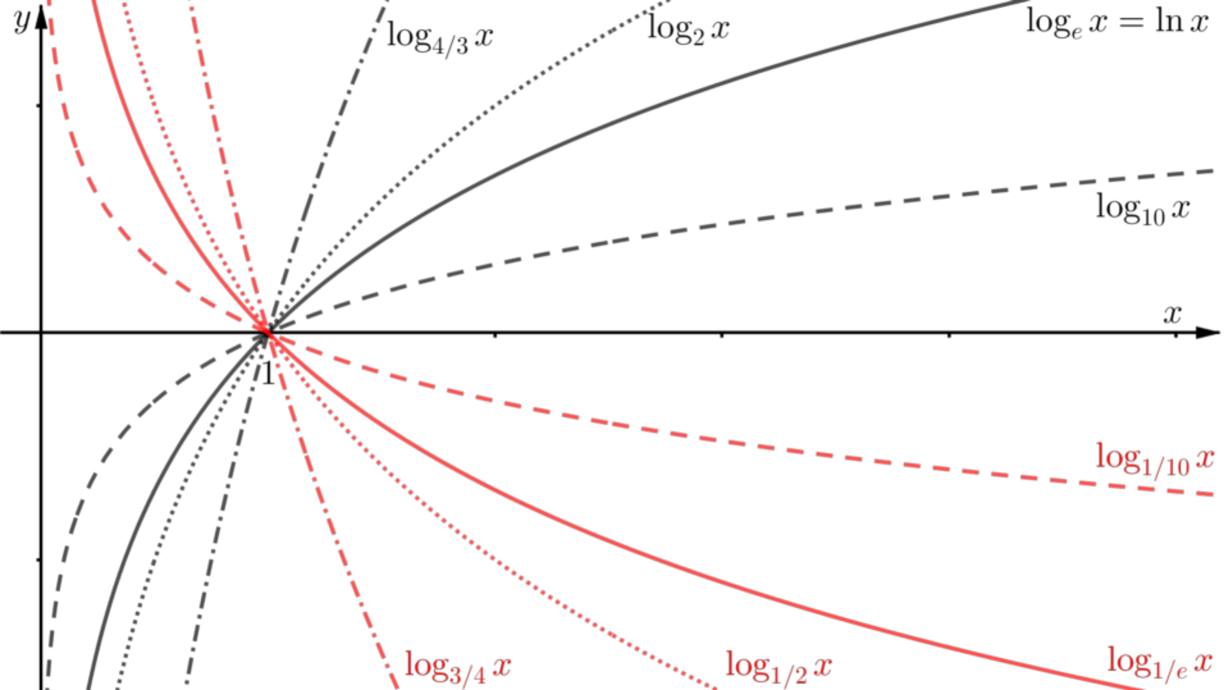
$\boxed{\log_b(b^x) = x} \iff f^{-1}(f(x)) = x \quad \forall x\in\mathbb{R}$

2) $$\boxed{\log_b(xy) = \underbrace{\log_b x + \log_b y}_{t} \quad \forall x,y>0}$$

$$\log_b(xy) = t \iff b^t \stackrel{?}{=} xy$$

$$b^t = b^{\log_b x + \log_b y} = \underbrace{b^{\log_b x}}_{x} \cdot \underbrace{b^{\log_b y}}_{y} = xy$$

$$\log(x^2) = \log(x\cdot x) = \log x + \log x = 2\log x$$



y
x
1
$\log_{4/3} x$
$\log_2 x$
$\log_e x = \ln x$
$\log_{10} x$
$\log_{1/10} x$
$\log_{3/4} x$
$\log_{1/2} x$
$\log_{1/e} x$

$$\underbrace{\log(x^2)}_{\text{def}^{ta}\ \forall x \neq 0} = \underbrace{2\log x}_{\text{def}^{ta}\text{ solo per } x>0}$$

$\log(x^2) = 2\log x$ è falsa per $x<0$
vera per $x>0$

$$\boxed{\log(x^2) = 2\log|x|}$$ vera $\forall x \neq 0$

$$\underbrace{\log(x(x-1))}_{\text{definita in } (-\infty,0)\cup(1,+\infty)} = \log x + \log(x-1)$$ vera se $x>1$.

$$\log(x\underset{\overset{\shortparallel}{-x(1-x)}}{(x-1)}) = \begin{cases} \log x + \log(x-1) & \text{se } x>1 \\ \log(-x) + \log(1-x) & \text{se } x<0 \end{cases}$$

$$\overset{?}{=} \log|x| + \log|x-1|$$ ~~ok~~ ma attenzione,

questa è def^{ta} anche in $(0,1)$