

$$2) \lim_{x \rightarrow \pm\infty} \left(1 + \frac{a}{x}\right)^x = e^a \quad \forall a \in \mathbb{R}$$

$$\left(1 + \frac{a}{x}\right)^x = \left[\underbrace{\left(1 + \frac{1}{\left(\frac{x}{a}\right)}\right)^{\frac{x}{a}}}_{\substack{\downarrow x/a \rightarrow \pm\infty \\ e}} \right]^a \rightarrow e^a \quad \forall a \neq 0$$

Per $a=0$ è banale.

$$\lim_{x \rightarrow +\infty} \left(1 + \frac{1}{x+1}\right)^{x^2} = (1^{+\infty}) = +\infty$$

$$\left(1 + \frac{1}{x+1}\right)^{x^2} = \left[\left(1 + \frac{1}{x+1}\right)^{x+1} \right]^{\frac{x^2}{x+1}} \rightarrow (e^{+\infty}) = +\infty$$

$$= e^{\frac{x^2}{x+1} \log \left(1 + \frac{1}{x+1}\right)^{x+1}}$$

