Equazioni e diseq^{ni} irrazionali.

$$\boxed{\sqrt{x^2+4}=2x} \Rightarrow x^2+4=4x^2$$
$$\not\Leftarrow$$

quadrando potrei aver aggiunto soluzioni.

$$3x^2=4 \qquad x^2=\frac{4}{3} \qquad x=\pm\frac{2}{\sqrt3}$$

Le verifico nell'eq^{ne} di partenza

$x=\frac{2}{\sqrt3}$
$$\sqrt{\frac43+4}\overset{?}{=}2\cdot\frac{2}{\sqrt3}$$
$$\sqrt{\frac{4+12}{3}} \qquad \frac{4}{\sqrt3}$$
$$\frac{4}{\sqrt3}$$

Ok.

$x=-\frac{2}{\sqrt3}$
$$\sqrt{\frac43+4}\overset{?}{=}-2\cdot\frac{2}{\sqrt3}$$

NO

$$\boxed{\text{Unica sol}^{ne}\text{ dell'eq}^{ne}\text{ è } \quad x=\frac{2}{\sqrt3}}$$

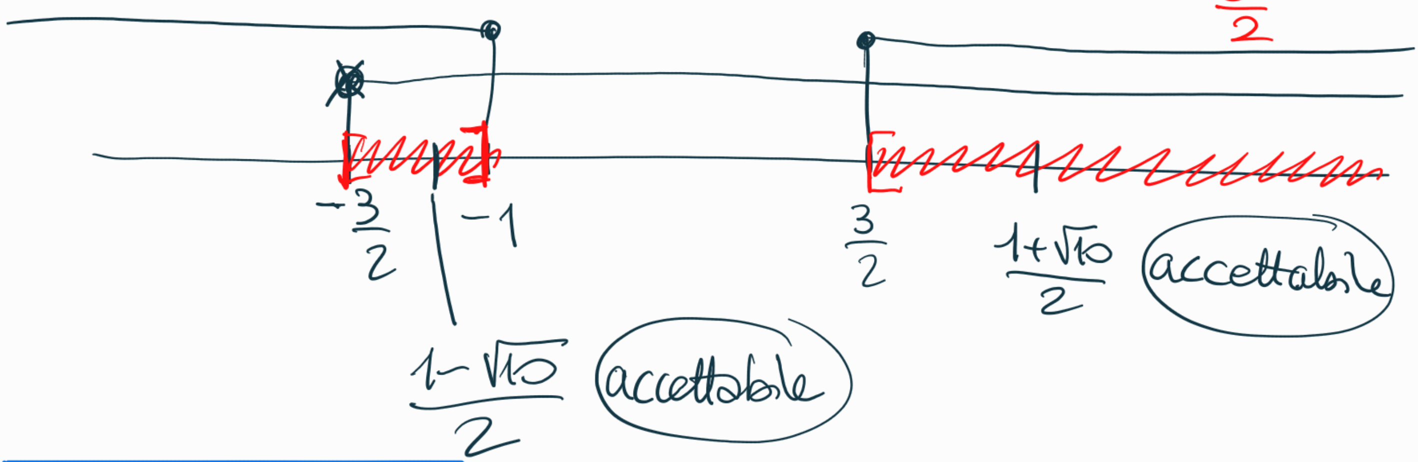
$$\boxed{\sqrt{2x+3}=\sqrt{4x^2-2x-6}}$$

$$\begin{cases} 2x+3\geq 0 & x\geq-\frac32 \\ 4x^2-2x-6\geq 0 \Leftrightarrow & 2x^2-x-3\geq 0 \Leftrightarrow (x\leq -1)\vee\left(x\geq\frac32\right) \\ 2x+3=4x^2-2x-6 & \Leftrightarrow 4x^2-4x-9=0 \end{cases}$$

$$x=\frac{2\pm\sqrt{4+36}}{4}=\frac{2\pm2\sqrt{10}}{4}$$

$$\boxed{x = \frac{1 \pm \sqrt{10}}{2}}$$

$$x_{1,2} = \frac{1 \pm \sqrt{1+24}}{4} = \frac{1 \pm 5}{4} = \begin{cases} -1 \\ \frac{3}{2} \end{cases}$$



$$\boxed{\text{sol}^{ni}\quad x = \frac{1 \pm \sqrt{10}}{2}}$$

$$\boxed{\underbrace{\sqrt[n]{P(x)}}_{\geq 0} < Q(x) \quad (\leq)}$$

dove $\boxed{n \text{ pari}}$

$P(x), Q(x)$ polinomi.

1) imponiamo $P(x) \geq 0$.

2) imponiamo $Q(x) > 0$

3) adesso posso elevare alla n.

La diseq^{ne} $\sqrt[n]{P(x)} < Q(x)$ equivale al sistema.

$$\begin{cases} P(x) \geq 0 \\ Q(x) > 0 \\ P(x) < (Q(x))^n \end{cases}$$

Esempio

$$\boxed{\sqrt{x^2+3x+9} \le x+3}$$

$$\begin{cases} \cancel{x^2+3x+9 \ge 0} \\ x+3 \ge 0 \\ x^2+3x+9 \le (x+3)^2 \end{cases}$$

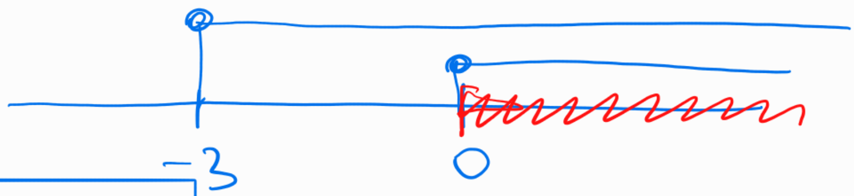
$x_{1,2} = \frac{-3 \pm \sqrt{9-36}}{2}$ $\quad \Delta < 0$

verificata $\forall x \in \mathbb{R}$ (la butto)

$x \ge -3$

$$\cancel{x^2}+3x+\cancel{9} \le \cancel{x^2}+6x+\cancel{9} \qquad x \ge 0.$$

$$\begin{cases} x \ge -3 \\ x \ge 0 \end{cases}$$



$$\boxed{\text{Sol}^{\text{ni}}: \quad x \ge 0}$$

$$\boxed{\sqrt[n]{P(x)} > Q(x)}$$

$\boxed{n \text{ pari}}$

$P(x), Q(x)$ polinomi.

1) Imponiamo $P(x) \ge 0$

2) se $Q(x) < 0$, l'eq^{ne} è automaticamente vera.

Se invece $Q(x) \ge 0$, posso quadrare.

$$\begin{cases} P(x) \ge 0 \\ Q(x) < 0 \end{cases} \quad \vee \quad \begin{cases} \cancel{P(x) \ge 0} \\ Q(x) \ge 0 \\ P(x) > (Q(x))^n \ge 0 \end{cases}$$

oss la III implica la I

$$\boxed{\sqrt{3(x+1)} > 2x-1}$$

$$A\begin{cases} \cancel{3}(x+1) \geq 0 \\ 2x-1<0 \end{cases} \quad \vee \quad B\begin{cases} 2x-1 \geq 0 \\ \underbrace{3(x+1)}_{3x+3} > (2x-1)^2 = 4x^2-4x+1 \end{cases}$$

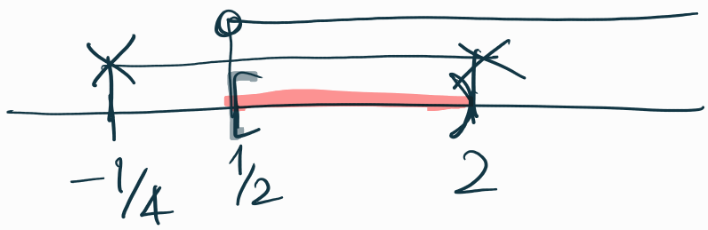
$$\begin{cases} x \geq -1 \\ x < \frac{1}{2} \end{cases} \qquad \begin{cases} x \geq \frac{1}{2} \\ 4x^2-7x-2<0 \end{cases}$$

$$x_{1,2} = \frac{7 \pm \sqrt{49+32}}{8} = \frac{7 \pm 9}{8} = \begin{cases} -1/4 \\ 2 \end{cases}$$

sol^{ni} di A

$$\boxed{-1 \leq x < \frac{1}{2}}$$

$$B \begin{cases} x \geq 1/2 \\ -1/4 < x < 2 \end{cases}$$

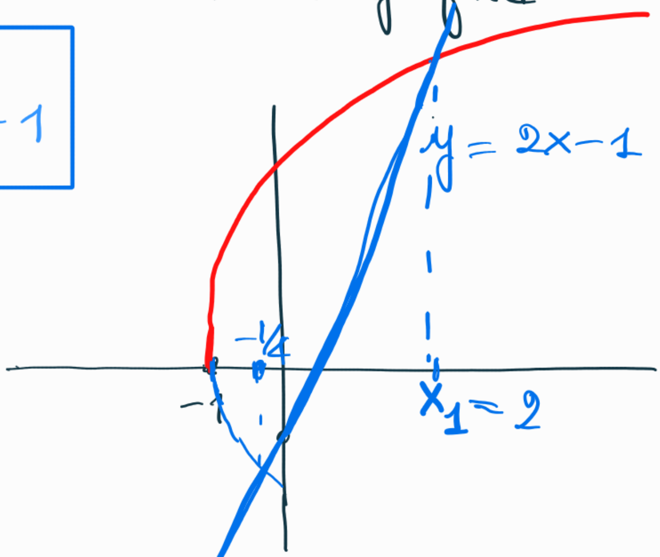


sol^{ni} B $\quad \frac{1}{2} \leq x < 2$

$$\boxed{\text{sol}^{ni} \text{ di } B \cup \text{sol}^{ni} \text{ di } A = [-1, 2)}$$

Si può sempre tentare la risoluzione grafica.

$$\boxed{\sqrt{3(x+1)} > 2x-1}$$



$y = \sqrt{x}$

$y = \sqrt{x+1} \qquad y = \sqrt{3(x+1)}$

Sol^{ni} della diseq^{ne}: $[-1, x_1) = [-1, 2)$

Troviamo x_1 $\quad \sqrt{3(x+1)} = 2x - 1$

quadriamo $\quad 3x+3 = (2x-1)^2$

$x = \underset{\text{NO}}{-1/4}, \quad \boxed{x=2}$

$\Rightarrow$ $\boxed{\text{Sol}^{ni} \text{ della diseq}^{ne}: \quad x \in [-1, 2)}$