Risolvere la diseq^{ne}

$$|x-2| \le |x-3|$$

1° modo: "spezzando" i valori assoluti.

$$\begin{cases} x \geq 3 \\ \cancel{x}-2 \leq \cancel{x}-3 \end{cases} \quad \vee \quad \begin{cases} 2 \leq x < 3 \\ x-2 \leq 3-x \end{cases} \quad \vee \quad \begin{cases} x < 2 \\ 2-\cancel{x} \leq 3-\cancel{x} \end{cases}$$

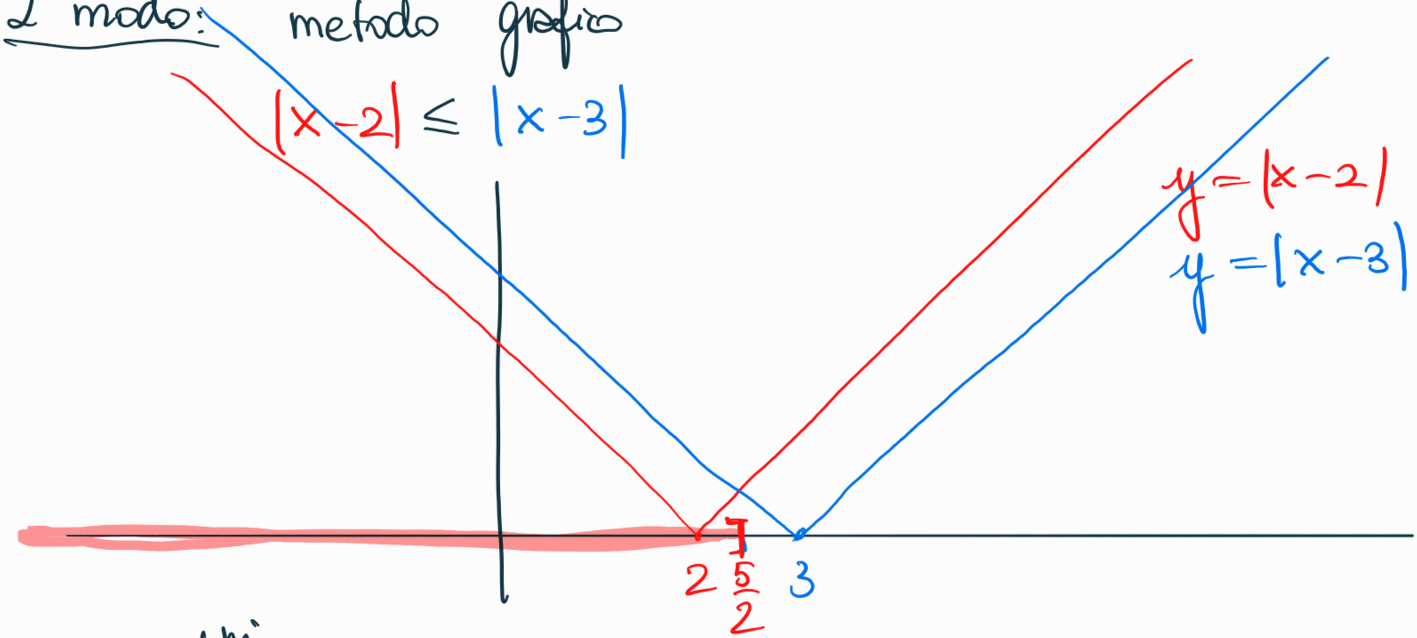
falso $\qquad$ $\cancel{2}x \leq \frac{5}{2}$ $\qquad$ sempre vera

nessuna soluzione $\cup$ sol^{ni} $2 \leq x \leq \frac{5}{2}$ $\cup$ sol^{ni}: $x < 2$.

sol^{ni} della diseq^{ne}: $\qquad x \leq \frac{5}{2} \qquad \left(x \in \left(-\infty, \frac{5}{2}\right] \right)$

2° modo: metodo grafico

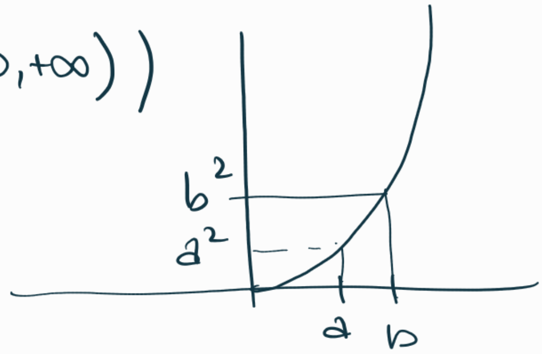
$|x-2| \leq |x-3|$



sol.^{ni}: $\quad x \leq \frac{5}{2}$.

3° modo. i) se $a, b \geq 0$, allora $\quad a \leq b \iff a^2 \leq b^2$

$\left(f(x) = x^2 \text{ è strettam. crescente in } [0, +\infty) \right)$



$$|x-2| \le |x-3| \iff |x-2|^2 \le |x-3|^2 \iff (x-2)^2 \le (x-3)^2$$

ii) $|a|^2 = a^2 \quad \forall a \in \mathbb{R}$

$$\Updownarrow$$

$$\cancel{x^2} - 4x + 4 \le \cancel{x^2} - 6x + 9$$

$$\Updownarrow$$

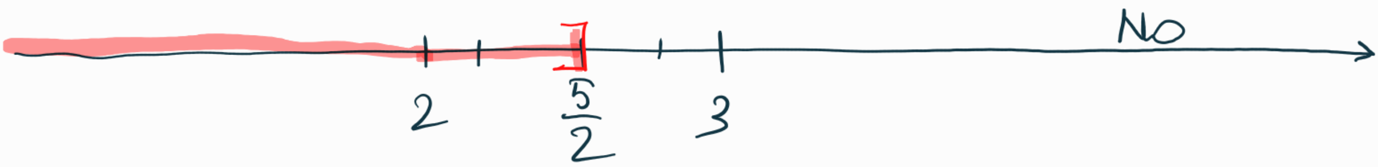
$$2x \le 5$$

$$\Updownarrow$$

$$x \le \frac{5}{2}.$$

4° modo. $|x-2| \le |x-3|$ equivale a:

"Quali x distano da 2 non più di quanto distano da 3"?



ovviamente (→ figura) tutti i punti $x \le \frac{5}{2}$.

Disegnare il grafico di $f(x) = \big|\, |x^2-4| - 1 \,\big|$

1° modo: spezzando i valori assoluti.

$$f(x) = \begin{cases} |x^2-4| - 1 & \text{se } |x^2-4| \ge 1 \\ 1 - |x^2-4| & \text{se } |x^2-4| < 1 \end{cases}$$

$$\Updownarrow$$

$$3 = 4-1 < x^2 < 4+1 = 5$$

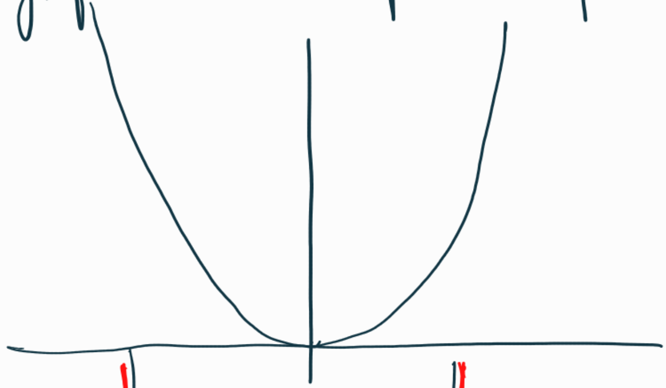
$$\Updownarrow$$

$$-\sqrt{5} < x < -\sqrt{3} \ \vee \ \left(\sqrt{3} < x < \sqrt{5}\right)$$

(impegnativo)

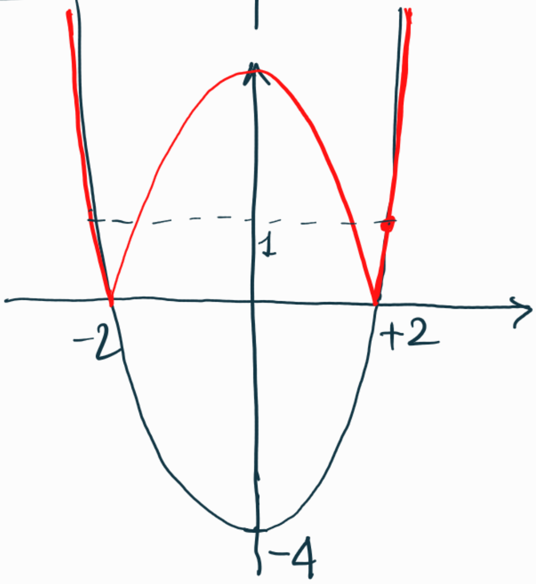
2° modo: costruire il grafico con semplici trasformazioni.

$y = x^2$



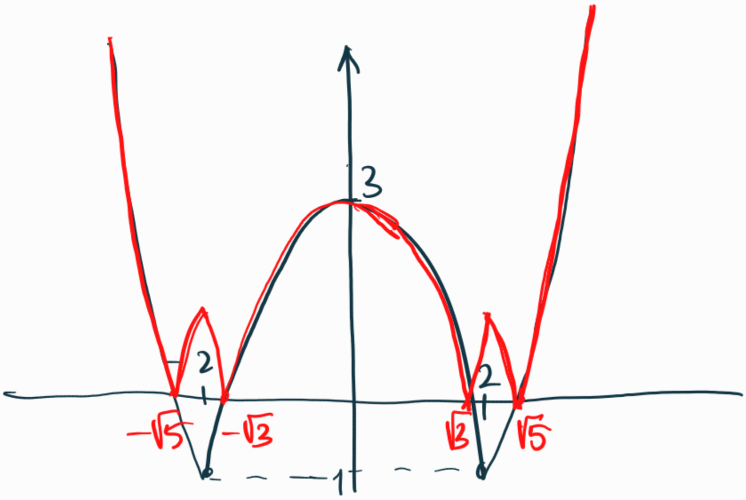
$y = x^2 - 4$

$y = |x^2 - 4|$



$y = |x^2 - 4| - 1$

$y = \left||x^2 - 4| - 1\right|$



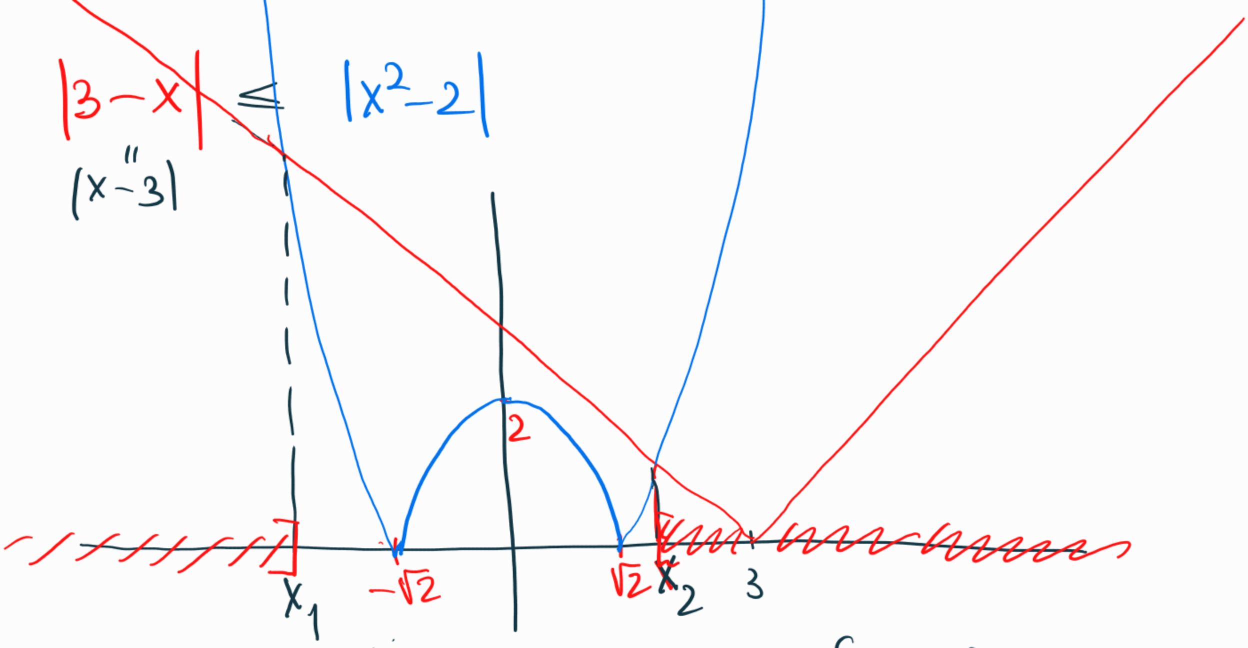
Trovare il dominio di $f(x) = \sqrt{|x^2 - 2| - |3 - x|}$

dom $f = \{x \in \mathbb{R} : \ |x^2 - 2| \geq |3 - x| \}$

risoluzione grafica.

$|3-x| \leq |x^2-2|$

$\overset{\shortparallel}{|x-3|}$



Per trovare x_1 e x_2 risolvo il sistema $\begin{cases} y = 3-x \\ y = x^2-2 \end{cases}$

$$x^2-2 = 3-x \iff x^2+x-5=0 \iff$$

$$\iff x_{1,2} = \frac{-1 \pm \sqrt{1+20}}{2} = \frac{-1\pm\sqrt{21}}{2}$$

$$x_1 = \frac{-1-\sqrt{21}}{2} \qquad x_2 = \frac{-1+\sqrt{21}}{2}$$

$$\text{dom } f = \left(-\infty, \frac{-1-\sqrt{21}}{2}\right] \cup \left[\frac{-1+\sqrt{21}}{2}, +\infty\right)$$