10) $E \subseteq \mathbb{R}$ è limitato $\iff \exists M \geq 0$ t.c. $|x| \leq M \ \forall x \in E$

$\boxed{\Leftarrow}$ ovvia: se $|x| \leq M \quad \forall x \in E$

$$\Updownarrow$$

$$-M \leq x \leq M \quad \forall x \in E$$

$\boxed{\Rightarrow}$ vediamo un esempio: supponiamo che $-8 \leq x \leq 3 \quad \forall x \in E$

$$\Rightarrow \quad |x| \leq \max\{3, 8\} = 8$$

Vediamo la dim. generale.

Supponiamo che E sia limitato, dunque $\exists\, a, b \in \mathbb{R}$ t.c.

$$a \leq x \leq b \quad \forall x \in E$$

$$\Rightarrow \quad \min\{-|a|, -|b|\} \leq -|a| \leq a \leq x \leq b \leq |b| \leq \underbrace{\max\{|a|, |b|\}}_{M} \quad \forall x \in E$$

$$\min\{-|a|, -|b|\} = \underbrace{-\max\{|a|, |b|\}}_{-M}$$

cioè $|x| \leq M \quad \forall x \in E$

Studiare la funzione $f(x) = \dfrac{|x^2 - 3x|}{3 - |x|}$

I primi passi sono: 1) trovare dom f; 2) "spezzare" i val. assoluti

1) dom. f.. Deve essere $3-|x| \neq 0$

Risolvo $3-|x|=0 \iff |x|=3 \iff x=\pm 3$

$$\text{dom } f = \mathbb{R} \setminus \{\pm 3\} = (-\infty,-3) \cup (-3,3) \cup (3,+\infty).$$

2) $|x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$

$$|x^2-3x| = |x|\,|x-3|$$

$$|x-3| = \begin{cases} x-3 & \text{se } x > 3 \\ -x+3 & \text{se } x < 3 \end{cases}$$

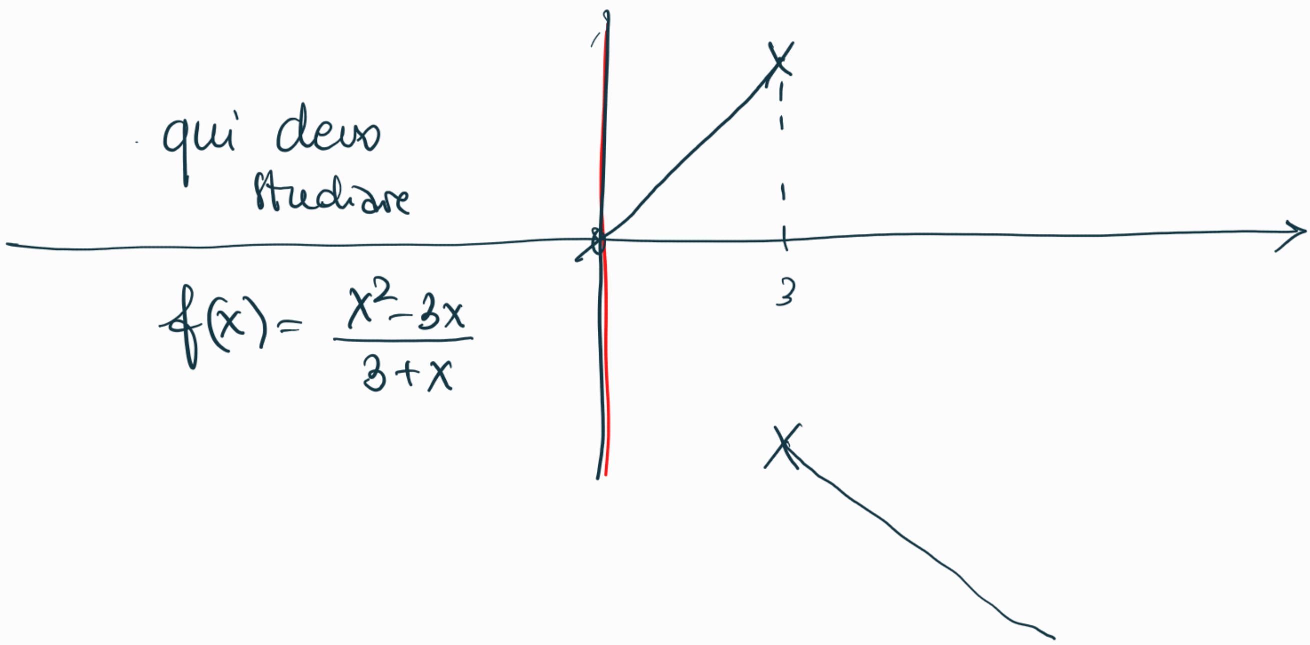
Tre casi

$x<0$ | $0 \leq x < 3$ | $x>3$ (punti 0 e 3 sulla retta)

$$x>3 \Rightarrow f(x) = \frac{|x|\,|x-3|}{3-|x|} = \frac{x\cancel{(x-3)}^{-1}}{\cancel{3-x}} = -x$$

$$0 \leq x < 3 \Rightarrow f(x) = \frac{x(3-x)}{3-x} = x$$

$$x<0 \; (x \neq -3) \Rightarrow f(x) = \frac{-x(3-x)}{3+x} = \frac{x^2-3x}{3+x}$$



Esercizio: Risolvere la diseq^{ne}.

$$|x+2| \leq |2x-3| + 1.$$

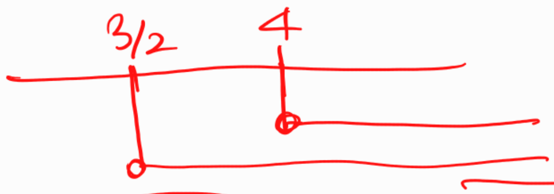
1° modo: spezzando i valori assoluti.

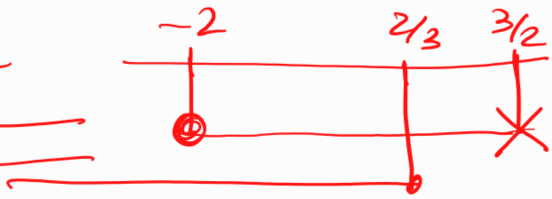
$$|x+2| = \begin{cases} x+2 & \text{se } x \geq -2 \\ -x-2 & \text{se } x \leq -2 \end{cases}$$

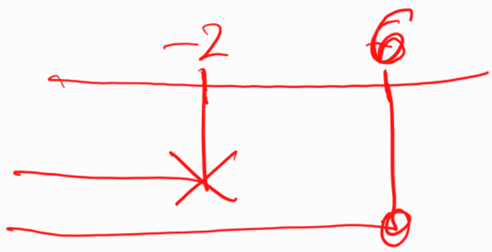
$$|2x-3| = \begin{cases} 2x-3 & \text{se } x \geq 3/2 \\ 3-2x & \text{se } x < 3/2 \end{cases}$$

$$\begin{cases} x \geq \frac{3}{2} \\ x+2 \leq 2x-3+1 \end{cases} \vee \begin{cases} -2 \leq x < 3/2 \\ x+2 \leq 3-2x+1 \end{cases} \vee \begin{cases} x < -2 \\ -x-2 \leq 3-2x+1 \end{cases}$$

$$\begin{cases} x \geq \frac{3}{2} \\ x \geq 4 \end{cases} \qquad \begin{cases} -2 \leq x < 3/2 \\ \cancel{3}x \leq \frac{2}{3} \end{cases} \qquad \begin{cases} x < -2 \\ x \leq 6 \end{cases}$$







$x \geq 4$ $\qquad$ $-2 \leq x \leq \frac{2}{3}$ $\qquad$ $x < -2$

$x \in [4, +\infty)$ $\qquad$ $x \in \left[-2, \frac{2}{3}\right]$ $\qquad$ $x \in (-\infty, -2)$

Sol^{ne} della diseq^{ne}:

$$x \in [4, +\infty) \cup \left[-2, \frac{2}{3}\right] \cup (-\infty, -2) =$$

$$= \left(-\infty, \frac{2}{3}\right] \cup [4, +\infty)$$

$(x \le \frac{2}{3}) \vee (x \ge 4)$

---

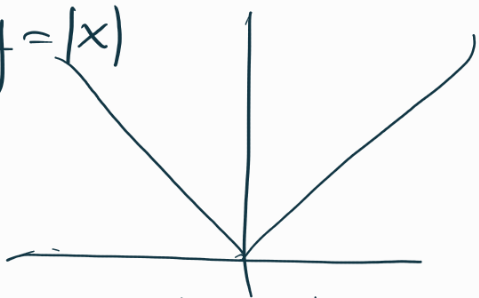
2) Discussione grafica

$$|x+2| \le |2x-3| + 1$$

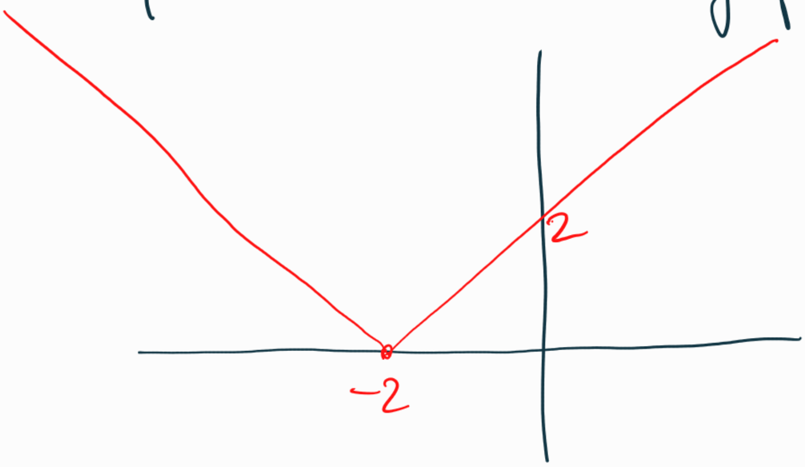
$$y = |x+2| = \begin{cases} x+2 & \text{se } x \ge -2 \\ -x-2 & \text{se } x < -2 \end{cases}$$

e disegnare le due semirette.

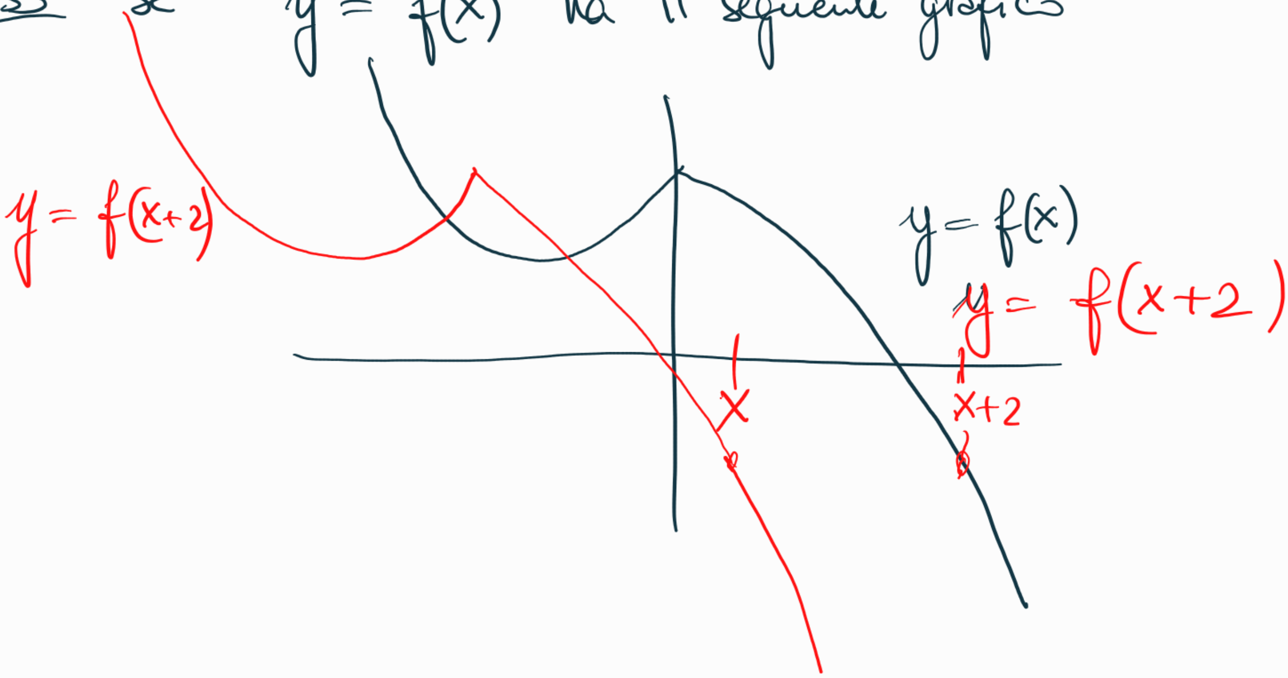
oppure partire dal grafico di $y = |x|$



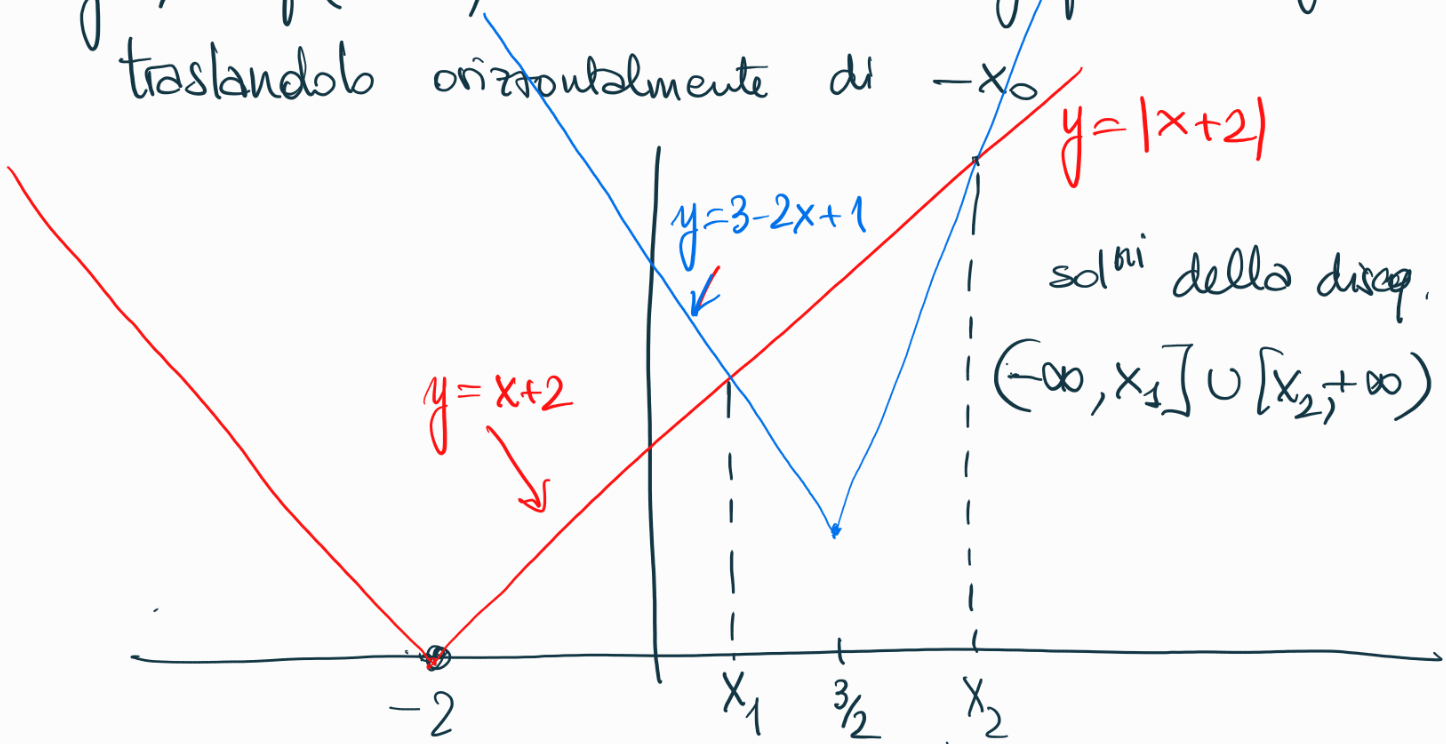
Da questo si ottiene il grafico di $y = |x+2|$



OSS se $y = f(x)$ ha il seguente grafico



In generale, se $f(x)$ ha un certo grafico, $g(x) = f(x+x_0)$ si ottiene dal grafico di f traslandolo orizzontalmente di $-x_0$

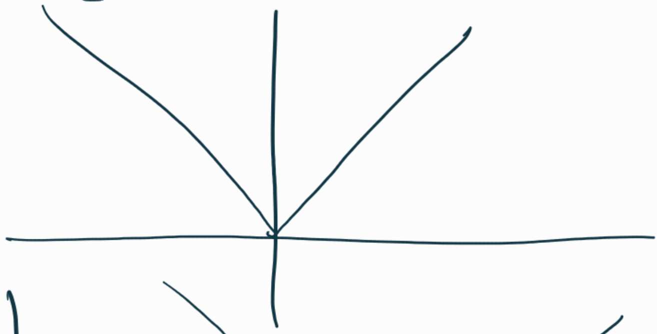


sol^{ni} della disequ.

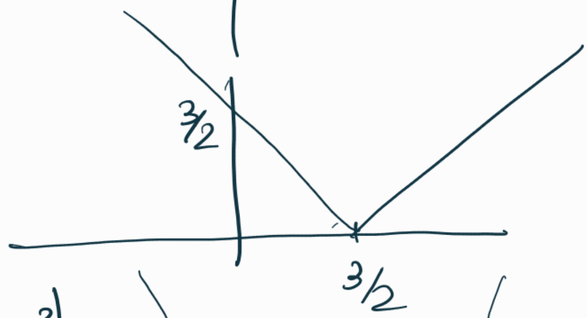
$(-\infty, x_1] \cup [x_2, +\infty)$

$y = |2x-3|+1$

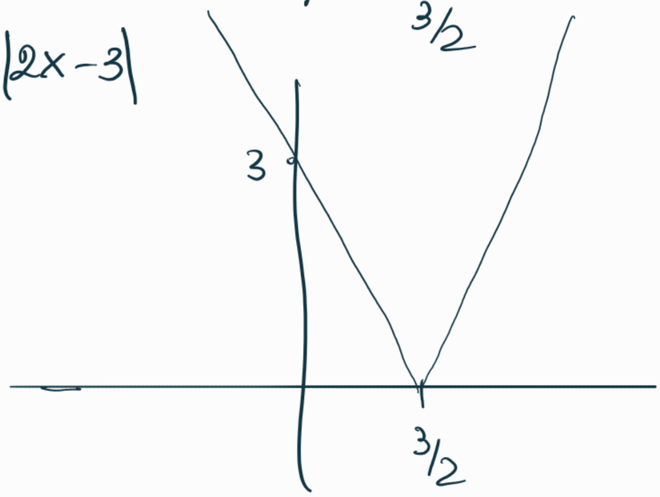
$y = |x|$



$y = \left| x - \frac{3}{2} \right|$



$y = 2\left| x - \frac{3}{2} \right| = |2x-3|$



Trovo x_1

$$\begin{cases} y = x+2 \\ y = 3-2x+1 \end{cases}$$

$x+2 = 3-2x+1$

$3x = 2 \qquad x = x_1 = 2/3$