Formulario

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Costanti Fisiche

Velocità della luce	$c = 2.99 \times 10^8\,\mathrm{m/s}$
Costante di Planck ridotta	$\hbar = 1.05 \times 10^{-34}\,\mathrm{J\ s} = 6.6 \times 10^{22}\,\mathrm{MeV\ s}$
	$\hbar c = 197\,\mathrm{MeV\ fm} = 197\ \mathrm{eV\ nm}, \quad hc = 1.23 \cdot 10^{-6}\ \mathrm{eV\ m}$
Massa dell'elettrone	$m_e c^2 = 0.511\,\mathrm{MeV}$
Massa del protone	$m_p c^2 = 938.3\,\mathrm{MeV}$
Massa del neutrone	$m_n c^2 = 939.6\,\mathrm{MeV}$
Raggio di Bohr	$a_0 = 0.53\,\mathrm{Å} = 0.53 \times 10^{-10}\,\mathrm{m}$ $\quad$ ($\hbar^2/(m_e e^2) =$ in unità di Gauss)
Energia di ionizzazione idrogeno	$E_I = 13.6\,\mathrm{eV}$ $\quad$ ($E_I = m_e e^4/(2\hbar^2)$ in unità di Gauss)

Atomo di idrogeno

Autofunzioni radiali $R_{nl}(r)$ per il problema Coulombiano (a_0 è il raggio di Bohr):

$$R_{10}(r) = 2\frac{1}{a_0^{3/2}}\, e^{-\frac{r}{a_0}} \qquad R_{30}(r) = 2\frac{1}{(3a_0)^{3/2}} \left(1 - \frac{2r}{3a_0} + \frac{2r^2}{27a_0^2}\right) e^{-\frac{r}{3a_0}}$$

$$R_{20}(r) = \frac{1}{(2a_0)^{3/2}} \left(2 - \frac{r}{a_0}\right) e^{-\frac{r}{2a_0}} \qquad R_{31}(r) = \frac{4\sqrt{2}}{9}\, \frac{1}{(3a_0)^{3/2}}\, \frac{r}{a_0} \left(1 - \frac{r}{6a_0}\right) e^{-\frac{r}{3a_0}}$$

$$R_{21}(r) = \frac{1}{\sqrt{3}}\, \frac{1}{(2a_0)^{3/2}}\, \frac{r}{a_0}\, e^{-\frac{r}{2a_0}} \qquad R_{32}(r) = \frac{2\sqrt{2}}{27\sqrt{5}}\, \frac{1}{(3a_0)^{3/2}} \left(\frac{r}{a_0}\right)^2 e^{-\frac{r}{3a_0}}\,.$$

Oscillatore Armonico

Autofunzioni dell'oscillatore armonico unidimensionale:

$$\psi_n(x) = \left(\frac{m\omega}{\hbar\pi}\right)^{1/4} \frac{1}{\sqrt{2^n n!}}\, H_n(\xi) e^{-\xi^2/2}, \qquad \xi = \frac{x}{\beta}, \quad \beta \equiv \sqrt{\frac{\hbar}{m\omega}}$$

con $H_0(\xi) = 1$, $H_1(\xi) = 2\xi$, $H_2(\xi) = 4\xi^2 - 2$.

Autofunzioni radiali dell'oscillatore armonico isotropo tridimensionale:

$$R_{00}(r) = 2\,\frac{\beta^{3/2}}{\pi^{1/4}}\, e^{-\beta^2 r^2/2} \qquad R_{11}(r) = \sqrt{\frac{8}{3}}\, \frac{\beta^{3/2}}{\pi^{1/4}}\, \beta r e^{-\beta^2 r^2/2}$$

$$R_{20}(r) = \sqrt{6}\, \frac{\beta^{3/2}}{\pi^{1/4}} \left(1 - \frac{2}{3}\beta^2 r^2\right) e^{-\beta^2 r^2/2} \qquad R_{22}(r) = \sqrt{\frac{16}{15}}\, \frac{\beta^{3/2}}{\pi^{1/4}}\, \beta^2 r^2 e^{-\beta^2 r^2/2}$$

Integrali

$$\Gamma(n+1) = \int_0^{+\infty} t^n e^{-t} dt = n!$$

$$\int_{-\infty}^{+\infty} \frac{dx}{(1+x^2)^n} = \frac{\pi}{((n-1)!)^2} \frac{(2n-2)!}{2^{2n-2}} \qquad n \in \mathbb{N}^+$$

$$I_{n,\ell_1,m,\ell_2} = \int_0^{\infty} dr\, r^2 (r/a_0)^2 R_{n\ell_1}(r) R_{m\ell_2}(r);$$

$I_{1,0,1,0} = 3$, $I_{1,0,2,0} = -512\sqrt{2}/243$, $I_{2,0,2,0} = 42$, $I_{1,0,2,1} = 1280\sqrt{2}/(243\sqrt{3})$, $I_{2,0,2,1} = -20\sqrt{3}/243$, $I_{2,1,2,1} = 30$.

$$\int_{-\infty}^{+\infty} e^{-ax^2+bx+c}\, dx = \sqrt{\frac{\pi}{a}}\, e^{\frac{b^2}{4a}+c} \qquad \text{con } \mathrm{Re}\, a > 0$$

$$I_n = \int_{-\pi/2}^{\pi/2} dx\, x^n \cos x, \qquad J_n = \int_{-\pi/2}^{\pi/2} dx\, x^n \sin x,$$

$I_1 = 0$, $I_2 = \frac{1}{2}(\pi^2 - 8)$, $I_3 = 0$, $I_4 = \frac{1}{8}(\pi^4 - 48\pi^2 + 384)$, $J_1 = 2$, $J_2 = 0$, $J_3 = \frac{3}{2}(\pi^2 - 8)$, $J_4 = 0$.

$$I_{m,n} = \int_{-\pi/2}^{\pi/2} dx\, x \cos(mx) \sin(nx)$$

$I_{1,1} = \frac{\pi}{4}$, $I_{1,2} = \frac{8}{9}$, $I_{1,3} = \frac{\pi}{8}$, $I_{2,1} = -\frac{10}{9}$, $I_{2,2} = -\frac{\pi}{8}$, $I_{2,3} = \frac{26}{25}$.

$$I_{mn} = \int_{-\pi/4}^{\pi/4} dx \cos mx\, \cos nx$$

$I_{11} = (2+\pi)/4$, $I_{12} = 2\sqrt{2}/3$, $I_{13} = 1/2$, $I_{22} = \pi/4$, $I_{23} = 2\sqrt{2}/5$, $I_{33} = (3\pi - 2)/12$.

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