Introduzione alla Fisica degli Acceleratori di Particelle
1

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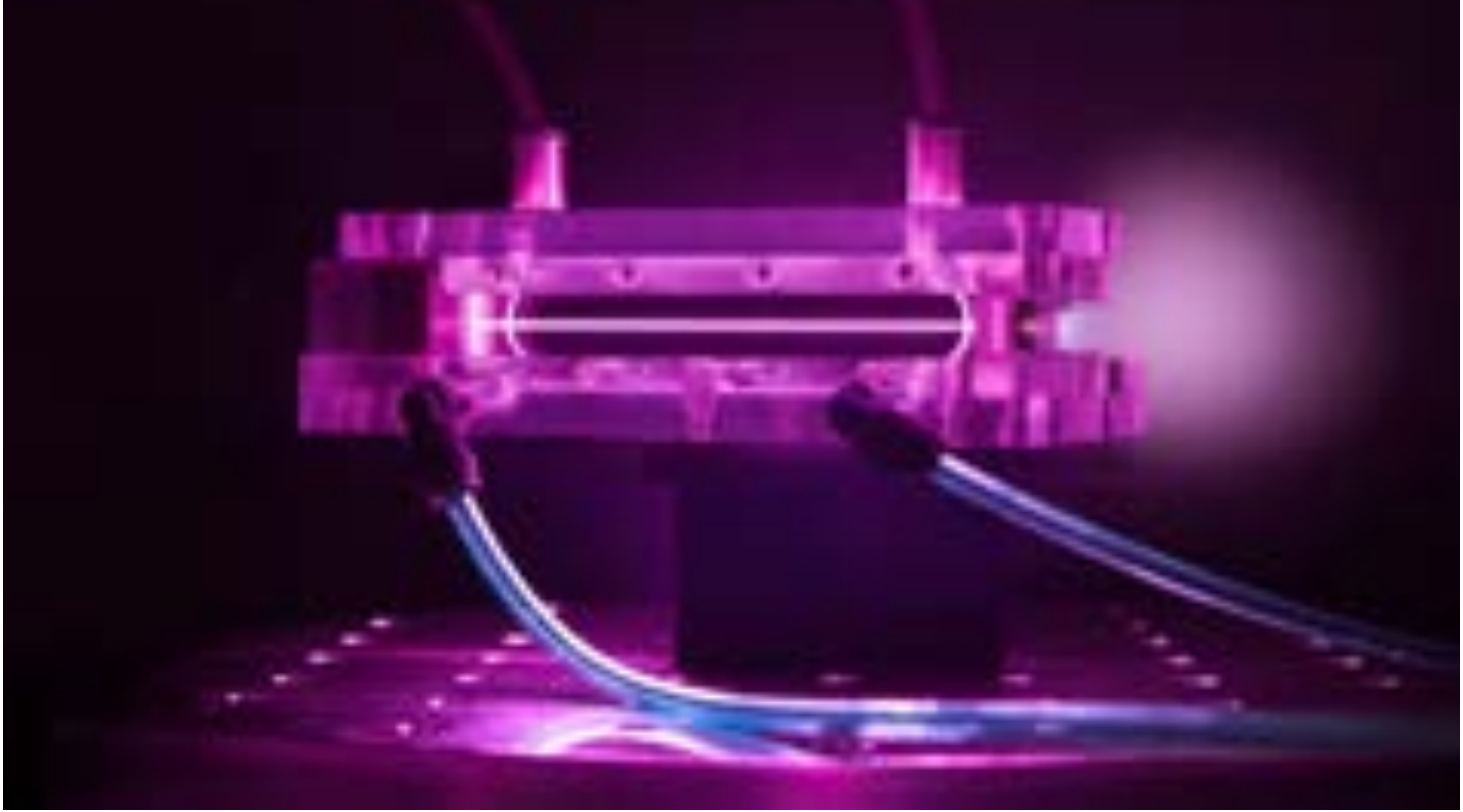
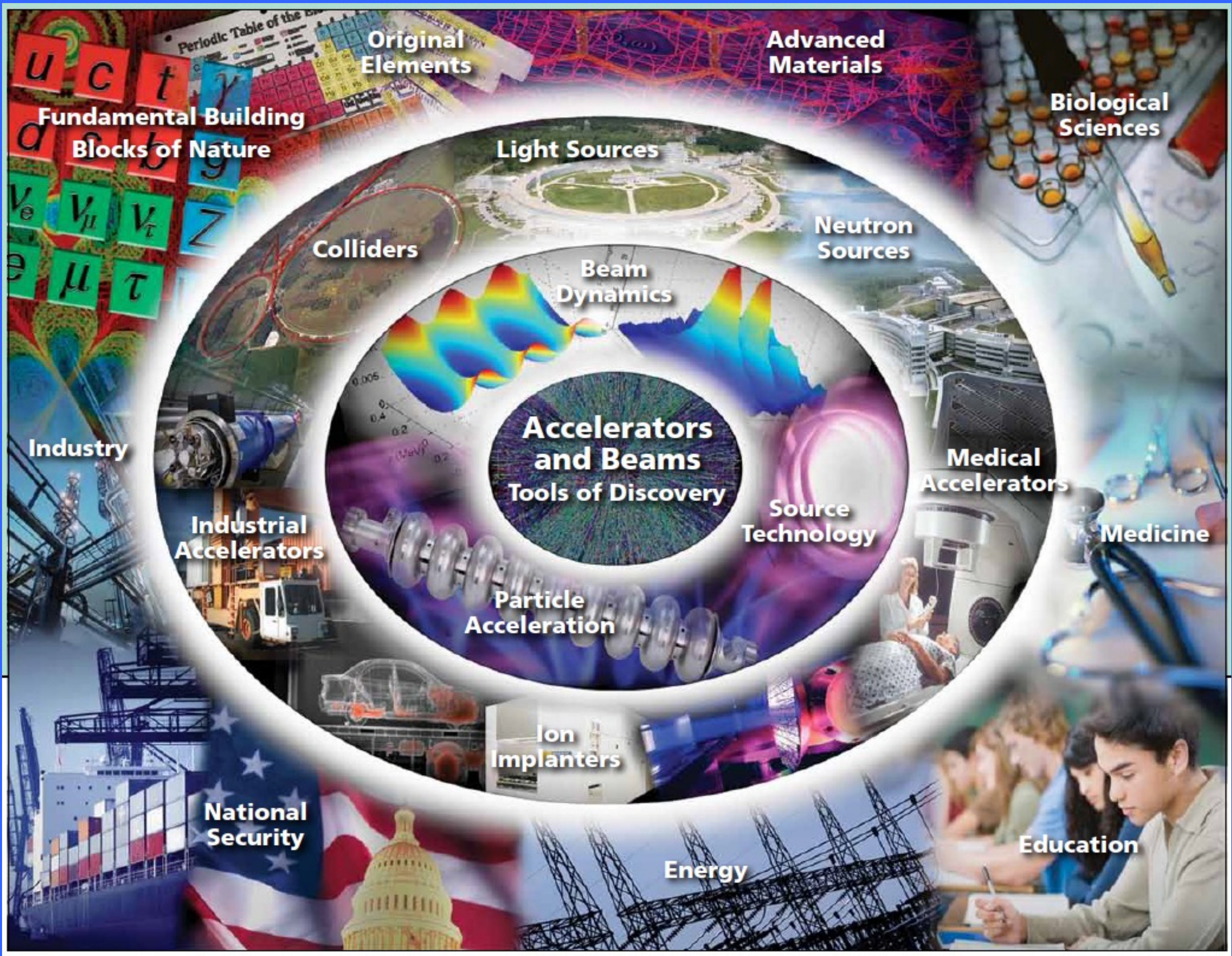
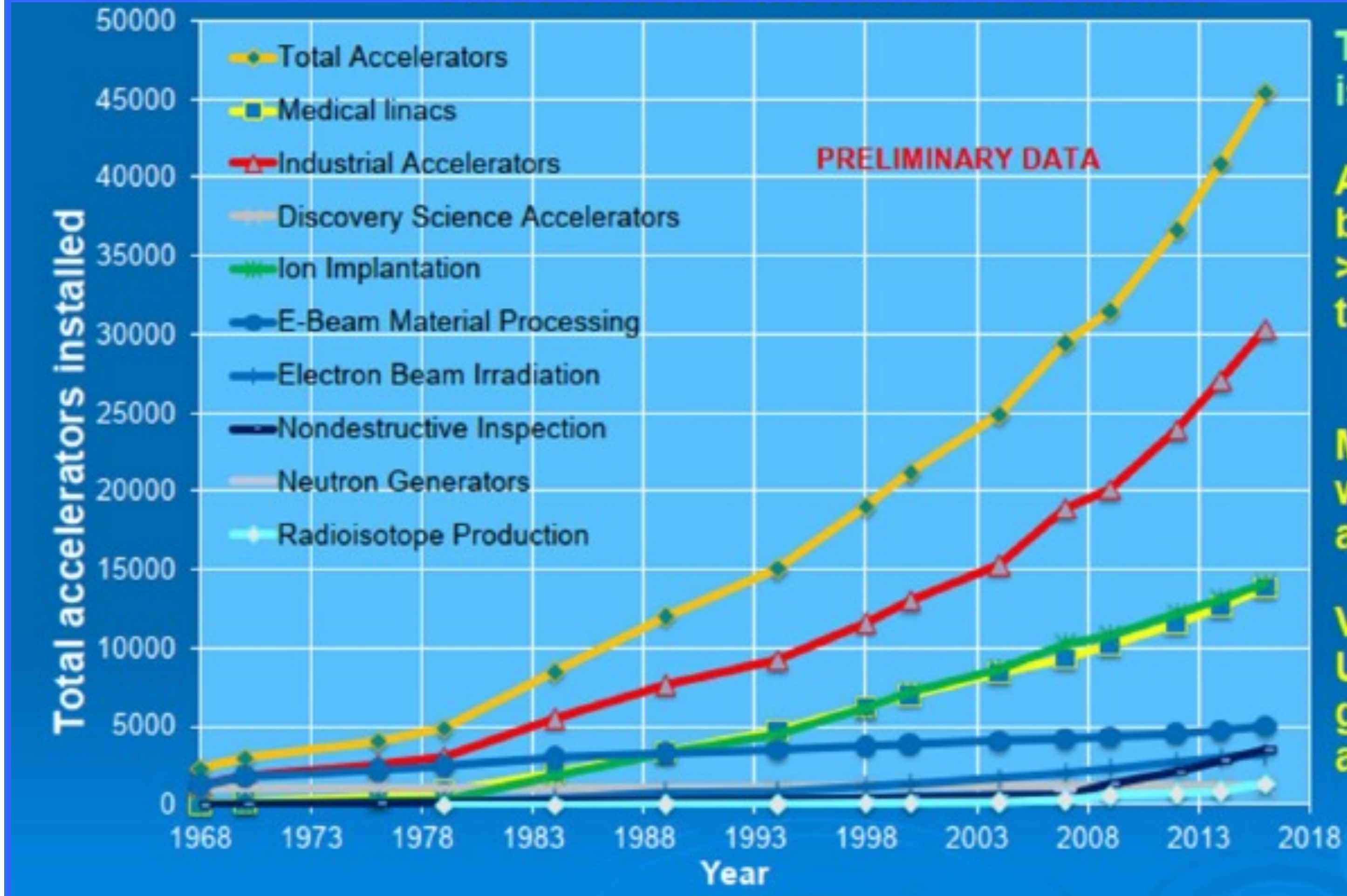



Accelerators
and Beams
Tools of Discovery
Beam
Dynamics
Particle
Acceleration
Source
Technology
Light Sources
Colliders
Neutron
Sources
Medical
Accelerators
Industrial
Accelerators
Ion
Implanters
Fundamental Building
Blocks of Nature
Original
Elements
Advanced
Materials
Biological
Sciences
Industry
Medicine
National
Security
Energy
Education

Accelerators installed worldwide



Total sales of accelerators is ~US$5B annually

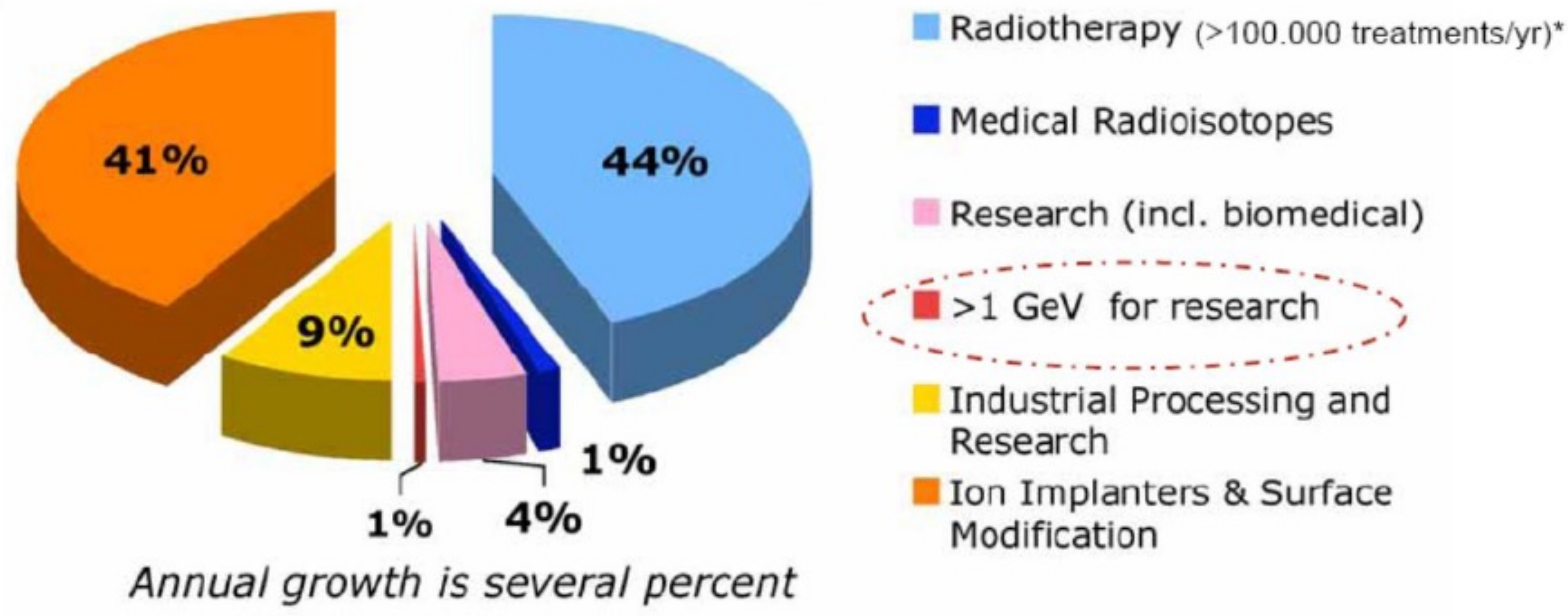
About 47,000 systems have been sold,
> 40,000 still in operation today

More than 100 vendors worldwide are in the accelerator business.

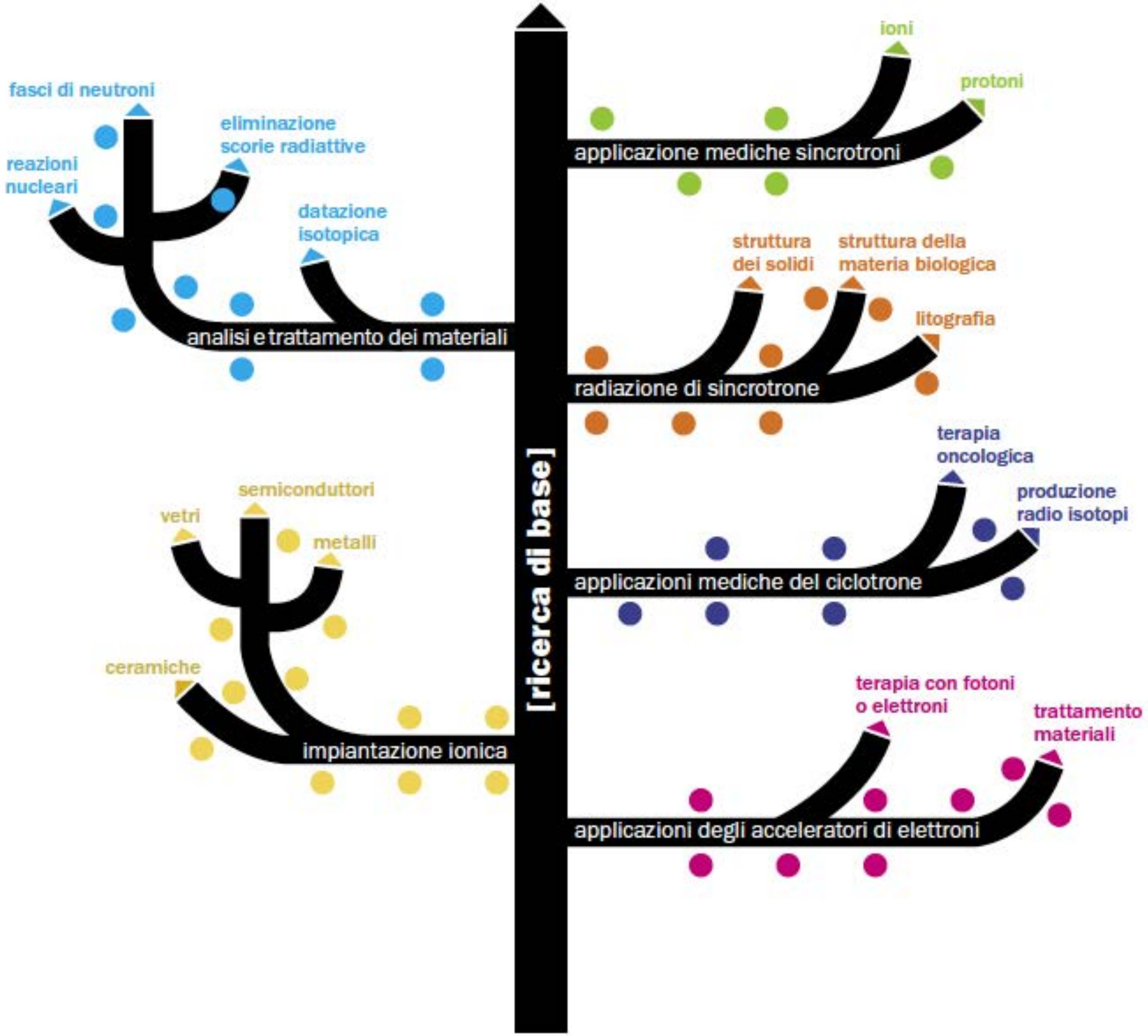
Vendors are primarily in US, Europe and Japan, but growing in China, Russia and India

-Accelerators for Americas Future Report, pp. 4, DoE, USA, 2011

Accelerators installed worldwide



-Accelerators for Americas Future Report, pp. 4, DoE, USA, 2011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eliminazione
scorie radiattive
reazioni
nucleari
datazione
isotopica
analisi e trattamento dei materiali
ioni
protoni
applicazione mediche sincrotroni
struttura
dei solidi
struttura della
materia biologica
litografia
radiazione di sincrotrone
terapia
oncologica
produzione
radio isotopi
applicazioni mediche del ciclotrone
semiconduttori
vetri
metalli
ceramiche
impiantazione ionica
terapia con fotoni
o elettroni
trattamento
materiali
applicazioni degli acceleratori di elettroni

LORENTZ FORCE: ACCELERATION AND FOCUSING

The basic equation that describes the acceleration/bending/focusing processes is the **Lorentz Force**. Particles are **accelerated through electric** fields and are **bended and focused through magnetic** fields.

$\vec{p} = momentum$

$m = mass$

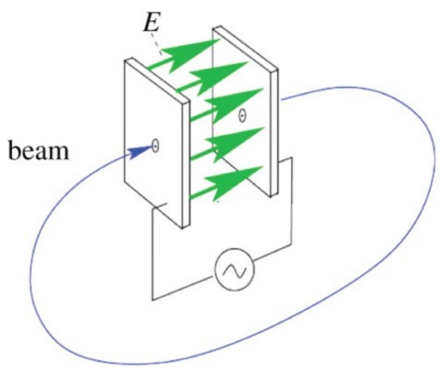
$\vec{v} = velocity$

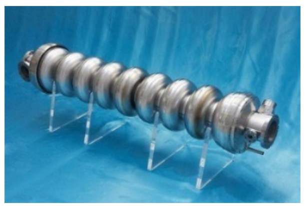
$q = charge$

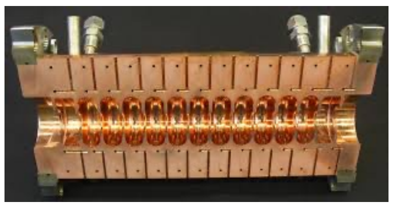
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

ACCELERATION

To accelerate, we need a force in the direction of motion



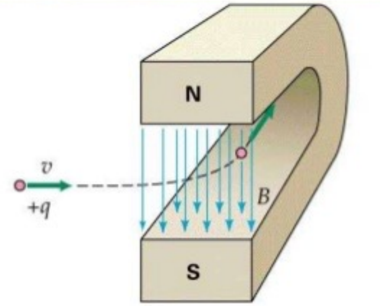


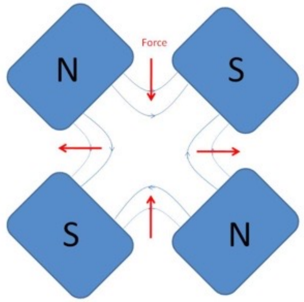


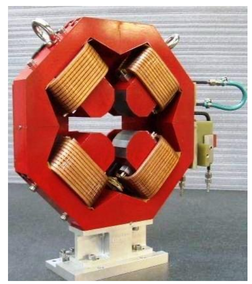
Longitudinal Dynamics

BENDING AND FOCUSING

2nd term always perpendicular to motion => no energy gain



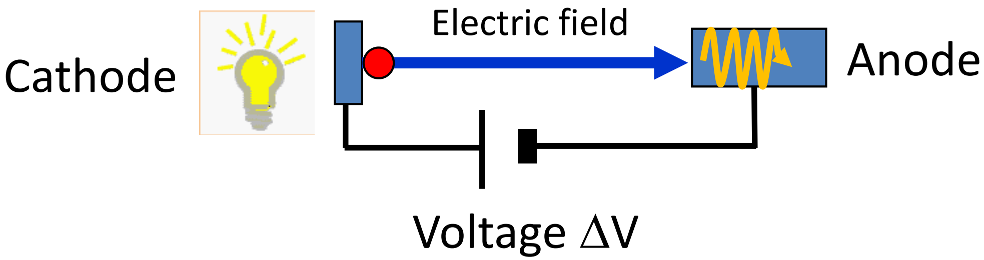




Transverse Dynamics

ACCELERATION: SIMPLE CASE

The **first historical linear particle accelerator** was built by the Nobel prize Wilhelm Conrad Röntgen (1900). It consisted in a vacuum tube containing a cathode connected to the negative pole of a DC voltage generator. **Electrons emitted by the heated cathode** were accelerated while flowing to another electrode connected to the positive generator pole (anode). Collisions between the energetic electrons and the anode produced **X-rays**.



Bertha Röntgen's Hand 8 Nov, 1895

The **energy gained** by the electrons travelling from the cathode to the anode is equal to their charge multiplied the DC voltage between the two electrodes.

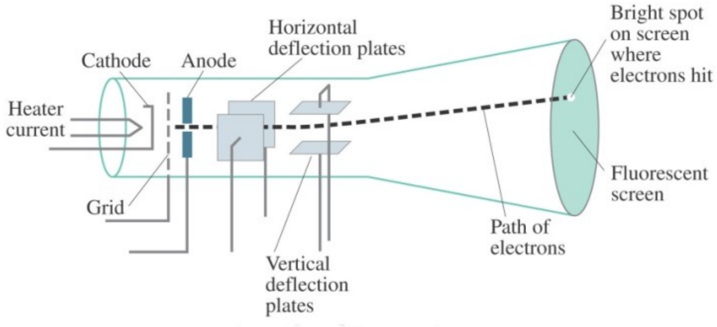
$$\frac{d\vec{p}}{dt} = q\vec{E} \quad \Rightarrow \quad \Delta E = q\Delta V$$

$$\vec{p} = momentum$$

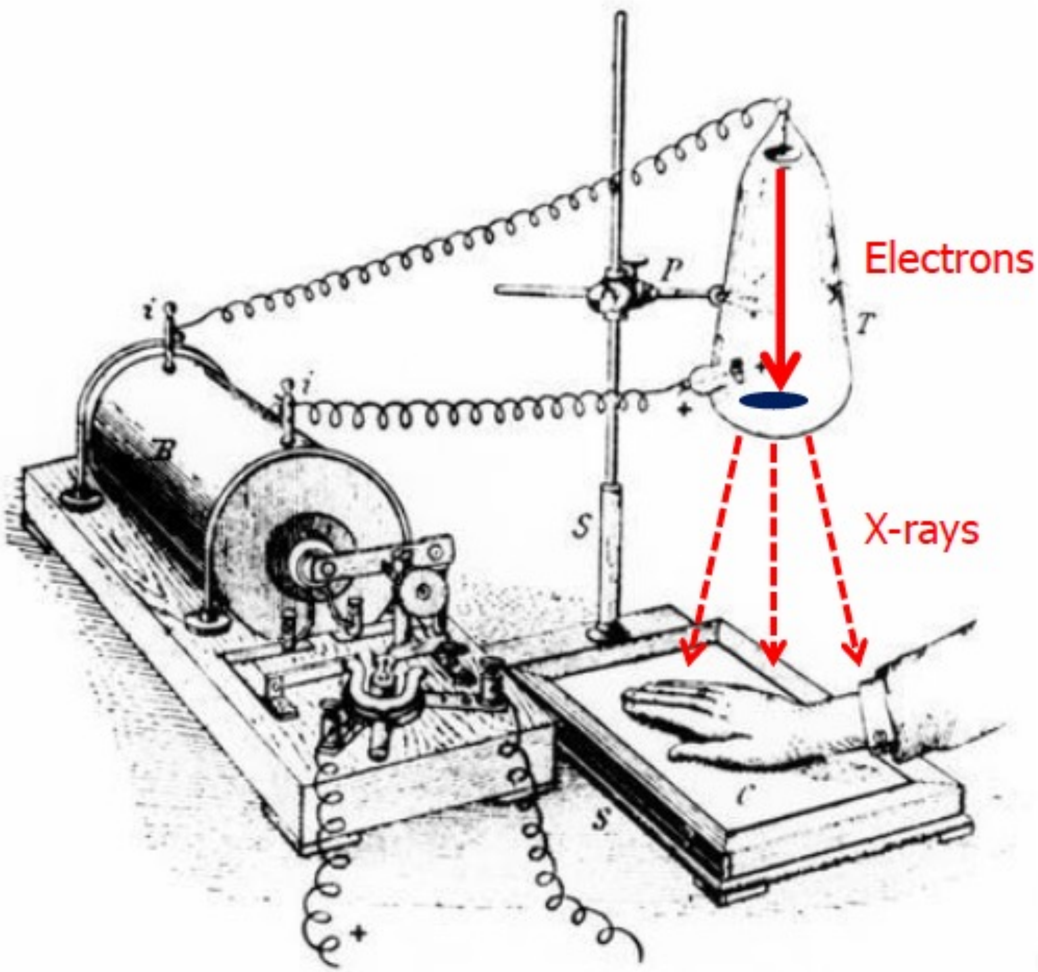
$$q = charge$$

$$E = energy$$

Particle energies are typically expressed in electron-volt [eV], equal to the energy gained by 1 electron accelerated through an electrostatic potential of 1 volt:
1 eV=1.6x10^{-19} J

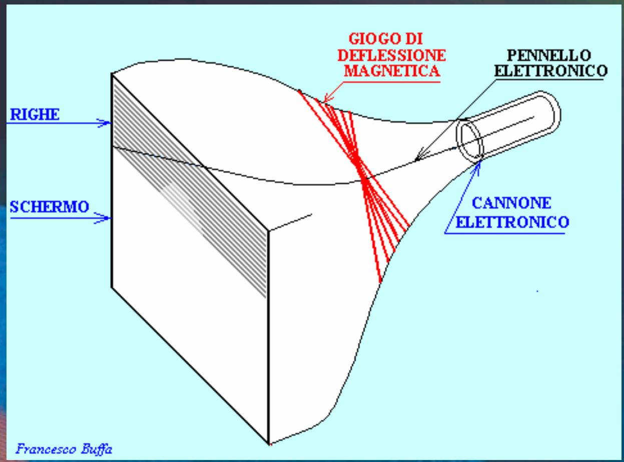


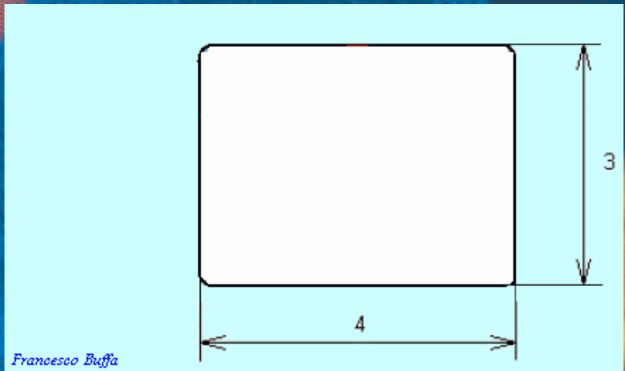
Roentgen 1896 – First radiograph of a hand

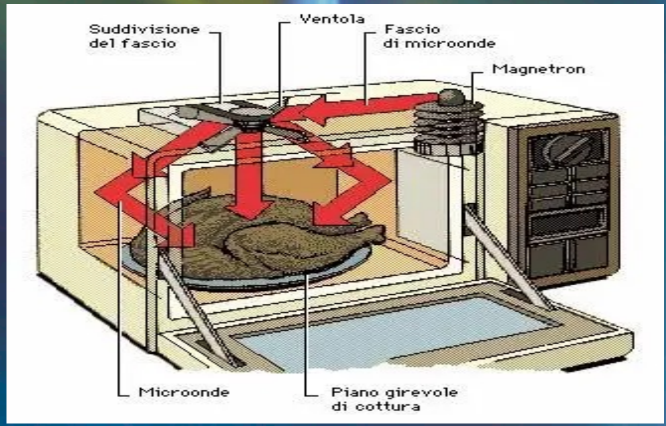


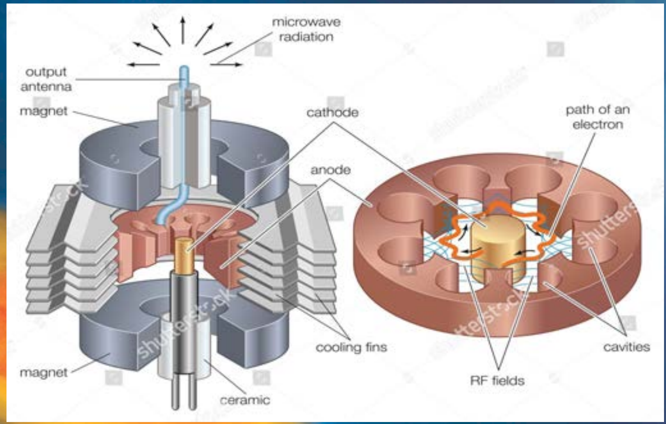
Acceleratori Domestici

FIGURE 3. ERNEST LAWRENCE, EDWIN MCMILLAN, AND LUIS ALVAREZ (left to right) admire a finished Chromatron. (Ernest O. Lawrence papers, BANC MSS 2005/200c, oversize box 3. Courtesy of the Bancroft Library, University of California, Berkeley.)

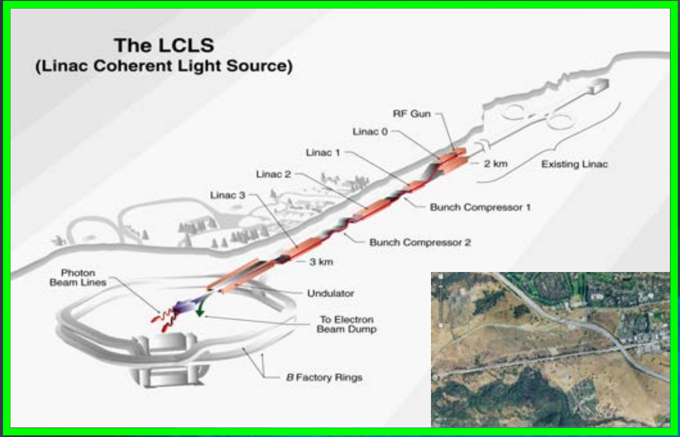


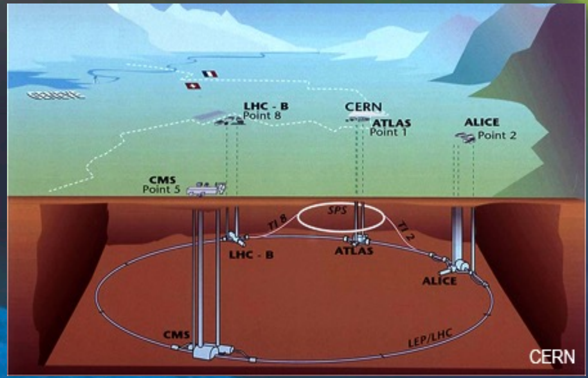


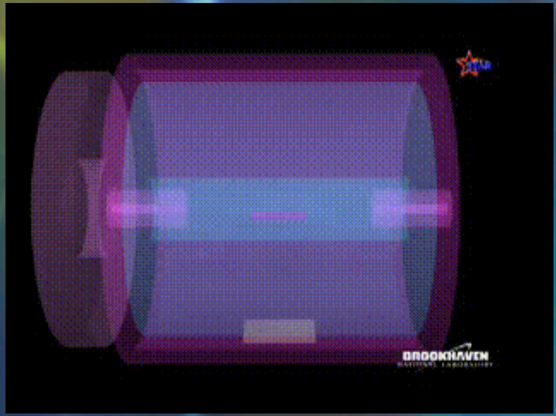


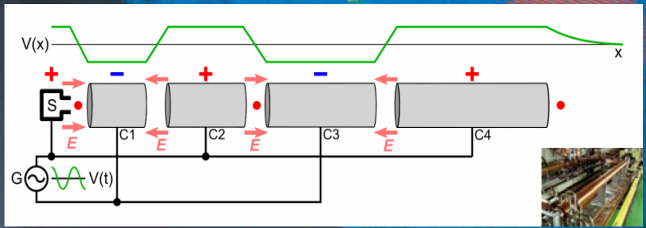


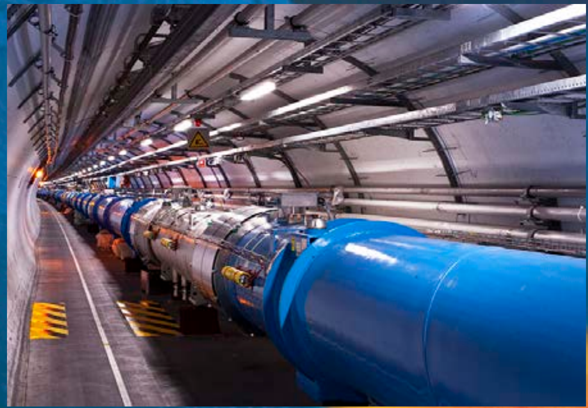
High Energy accelerators

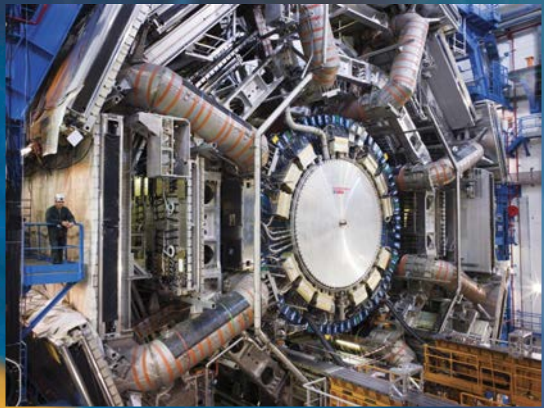












$$F_{Lorentz} = q\,v\,B = F_{centripital} = \frac{mv^2}{\rho}$$

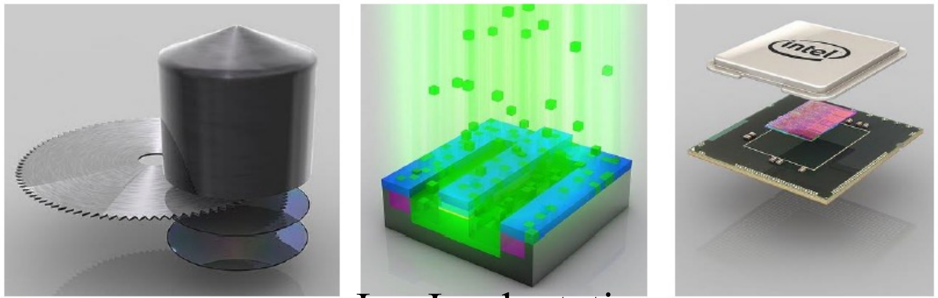
$$\Rightarrow \rho = \frac{mv}{qB} = \frac{p}{qB}$$

Equilibrium radius

$$\boxed{U_0 = \frac{e^2}{3\varepsilon_0}\frac{\beta^3\gamma^4}{\rho}}$$

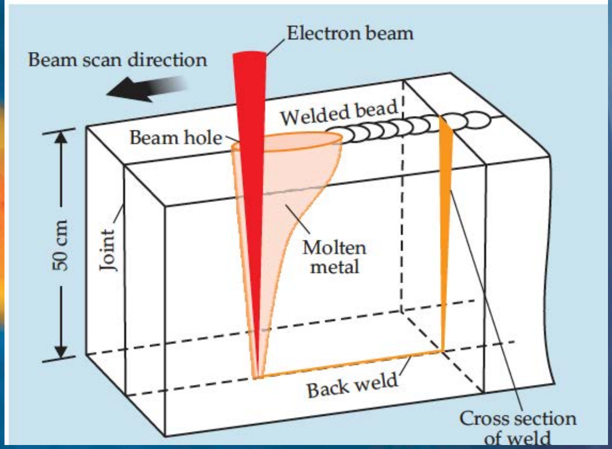
Energy lost by emission of radiation

Acceleratori per l'Industria



Images courtesy of Intel

Ion Implantation

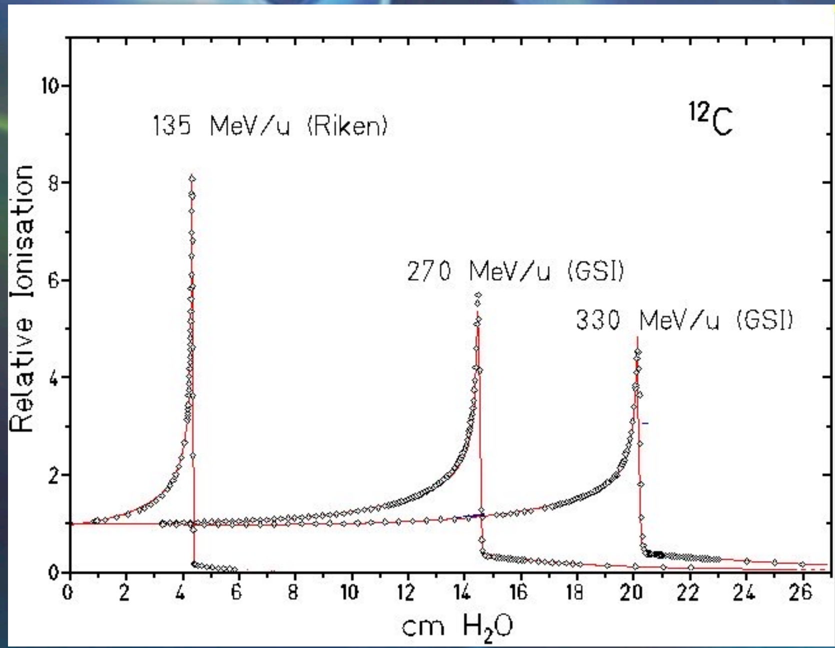


The beam business: Accelerators in industry

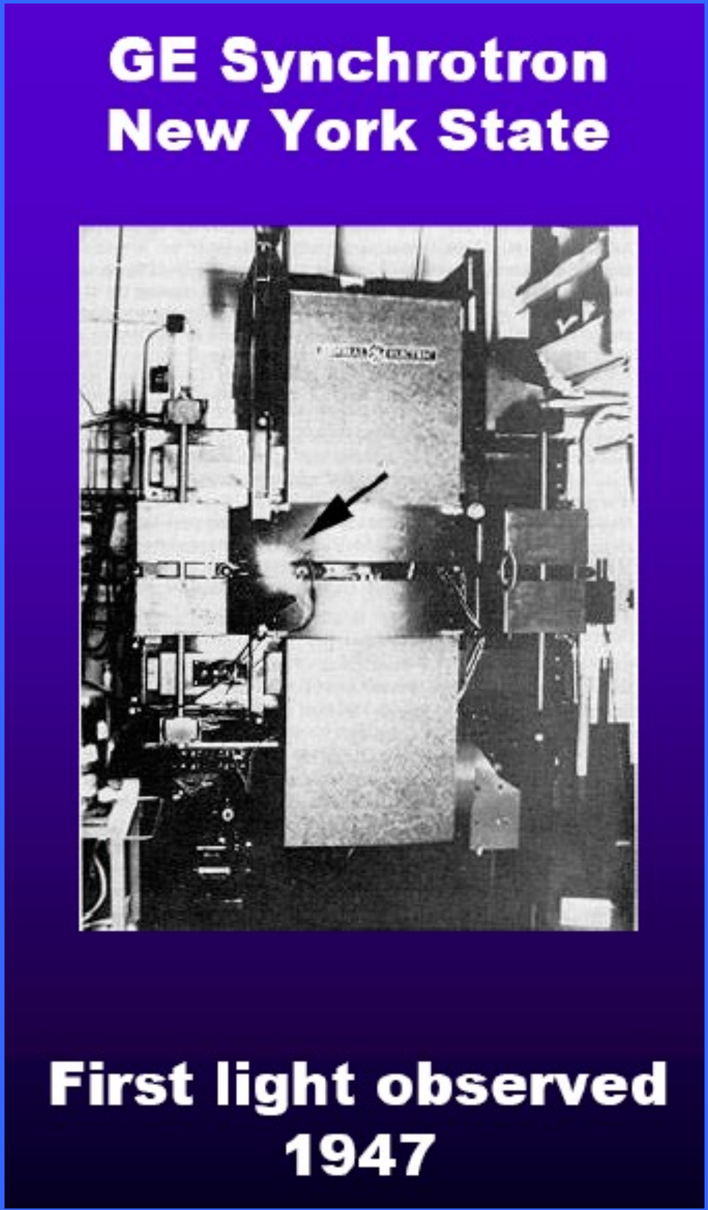
Robert W. Hamm, and Marianne E. Hamm

Citation: Physics Today **64**, 6, 46 (2011); doi: 10.1063/1.3603918

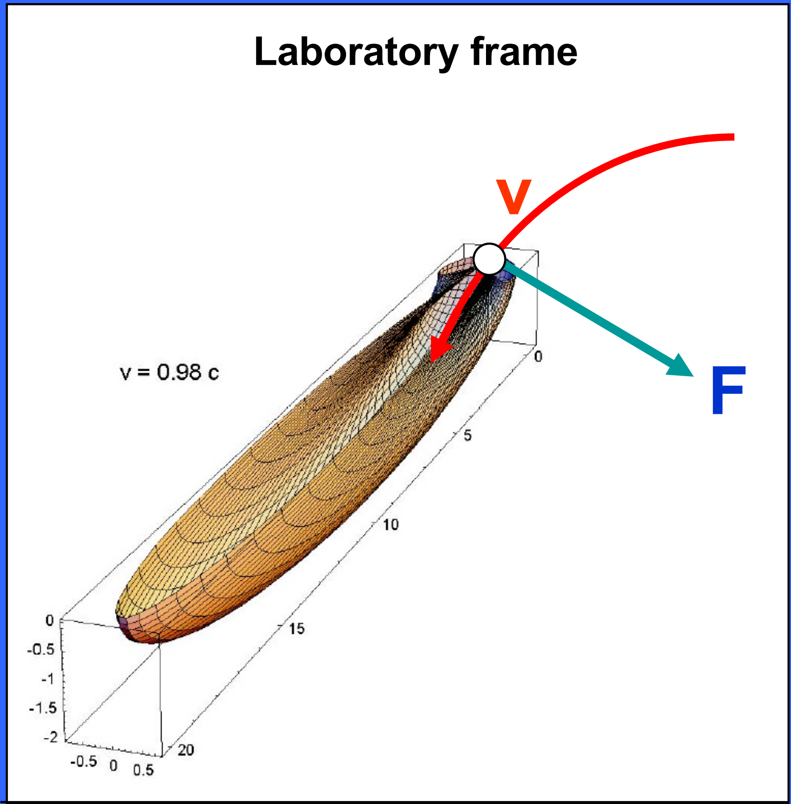
Medical Accelerators



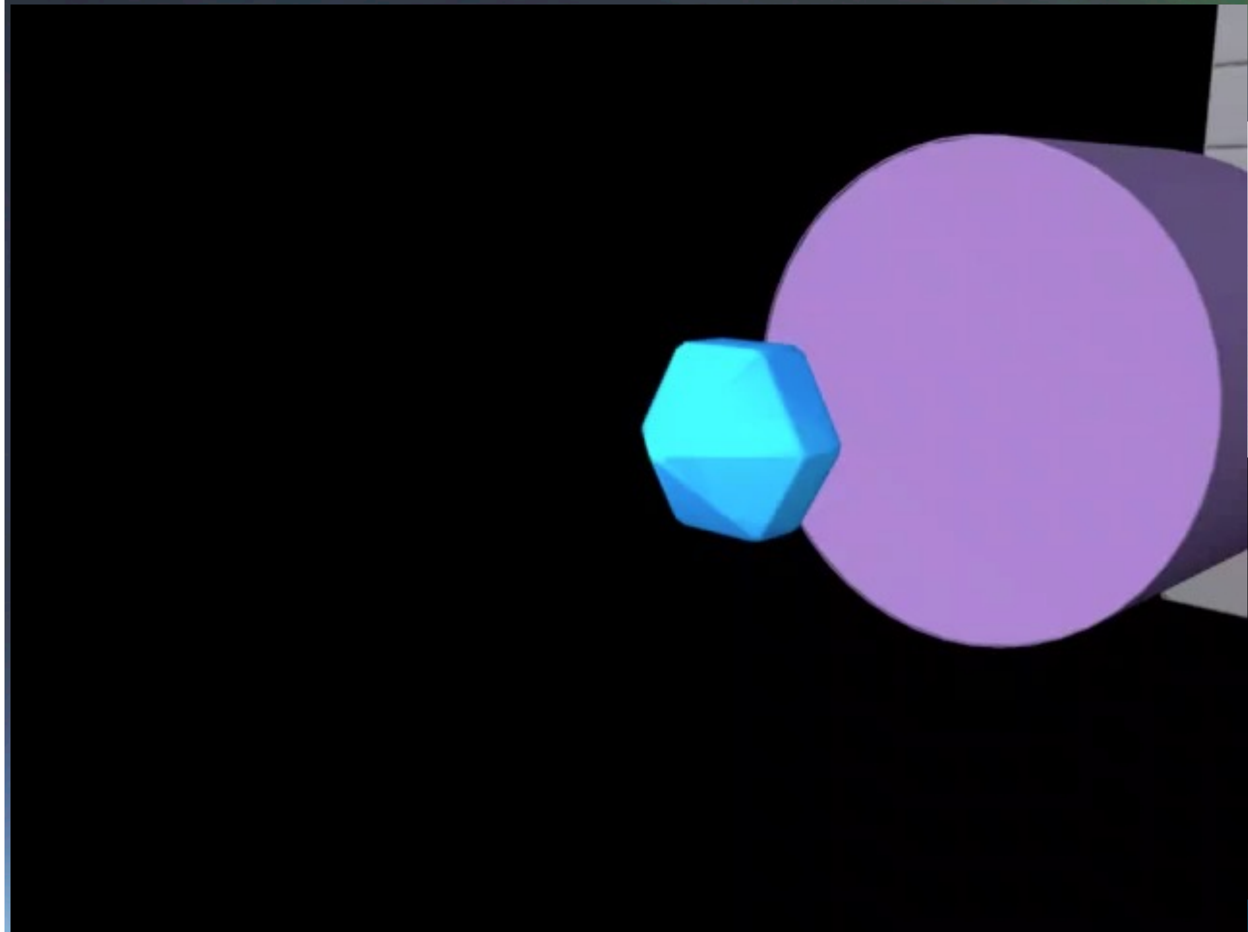
Synchrotron Radiation

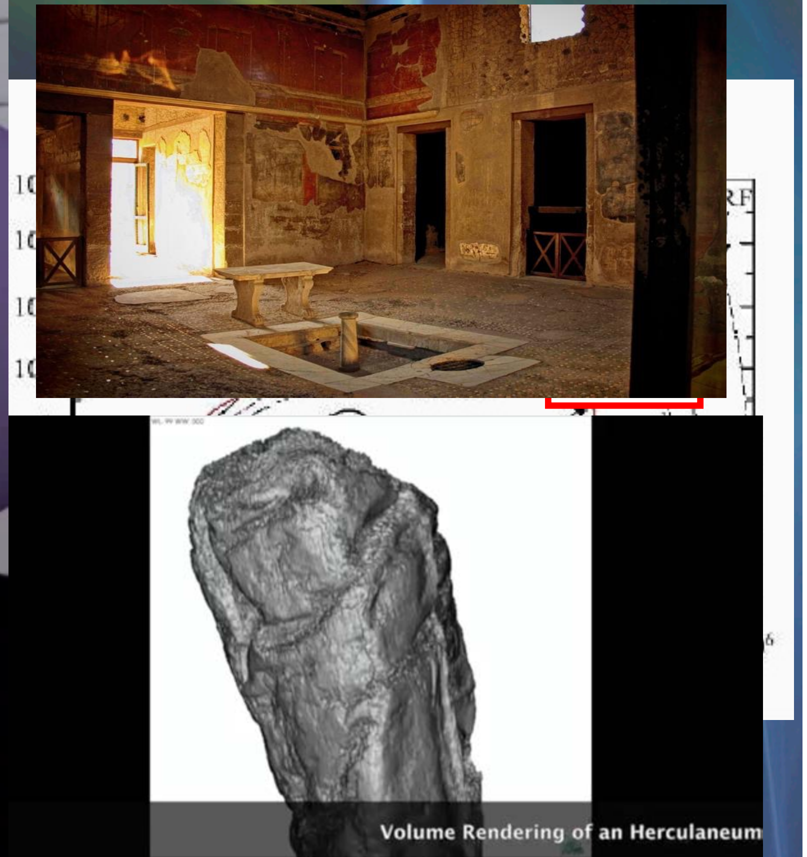


$$P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}$$

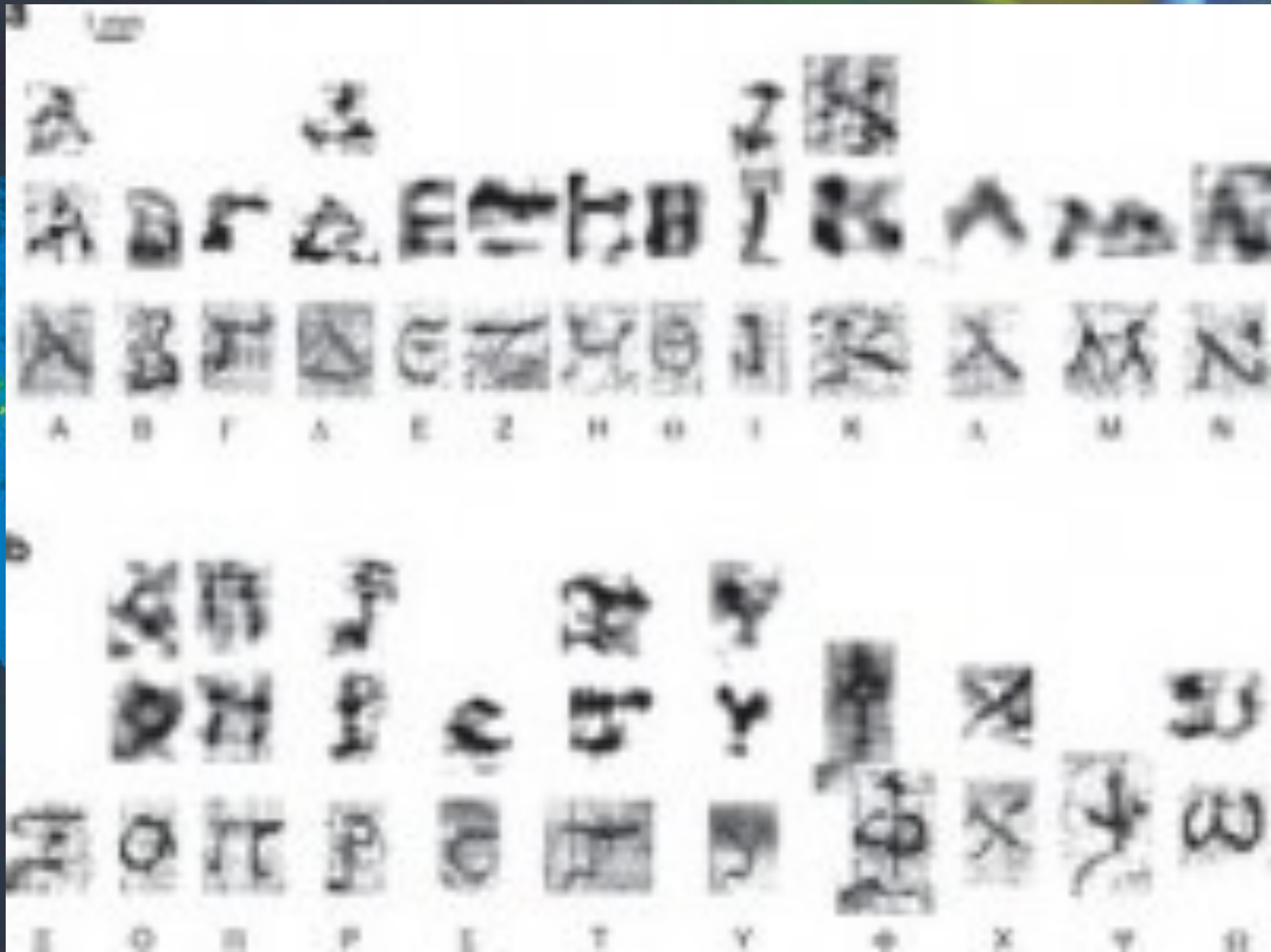


Synchrotron light sources





$$\langle P_s\,(\mathrm{MW})\rangle_{\mathrm{iso}} = 0.088463\,\frac{E^4\,(\mathrm{GeV})}{\rho\,(\mathrm{m})}\,I\,(\mathrm{A})\;.$$




Disney
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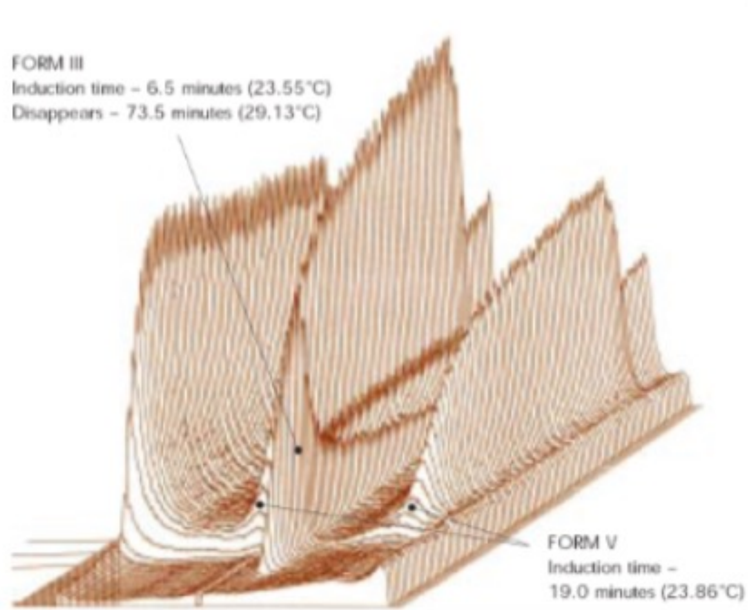
"... IL CENTRO DI EMINENTI RICERCHE PAPEROPOLESE!"
VI PRESENTO IL SINCROTRONE, UN TIPO PARTICOLARE DI ACCELERATORE DI PARTICELLE!
GENERA UN FASCIO DI PARTICELLE ELETTRICAMENTE CARICHE, SPINTE A VIAGGIARE IN TRAIETTORIA CURVA A UNA VELOCITÀ PROSSIMA A QUELLA DELLA LUCE!
COSÌ FACENDO, ESSE PRODUCONO UNA RADIAZIONE ELETTROMAGNETICA...
"... CONSISTENTE IN SPECIALI RAGGI X COLLIMATI!"

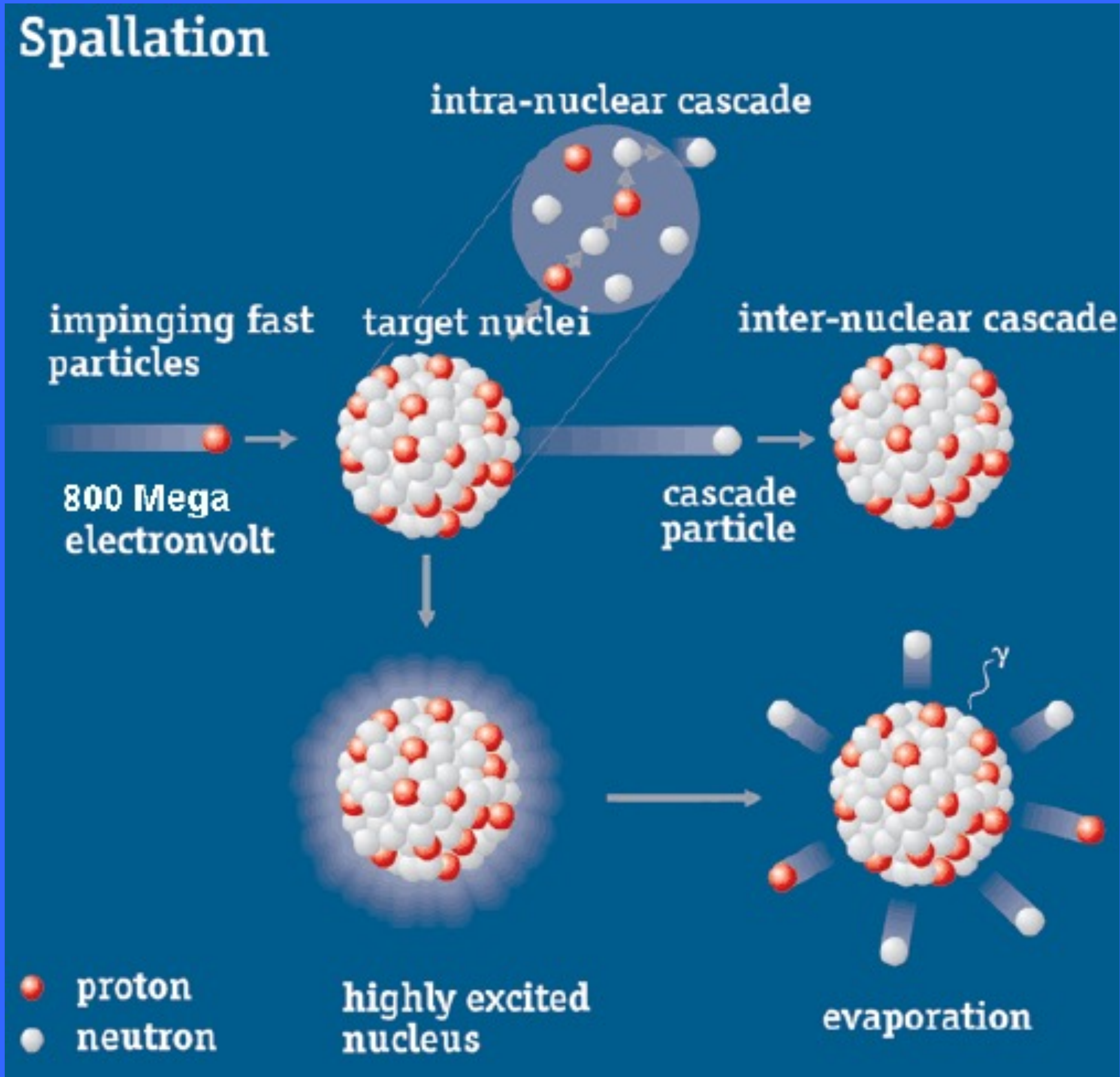
Prevent Chocolate Melting

Cadbury used X-rays from a particle accelerator to study how cocoa crystallises

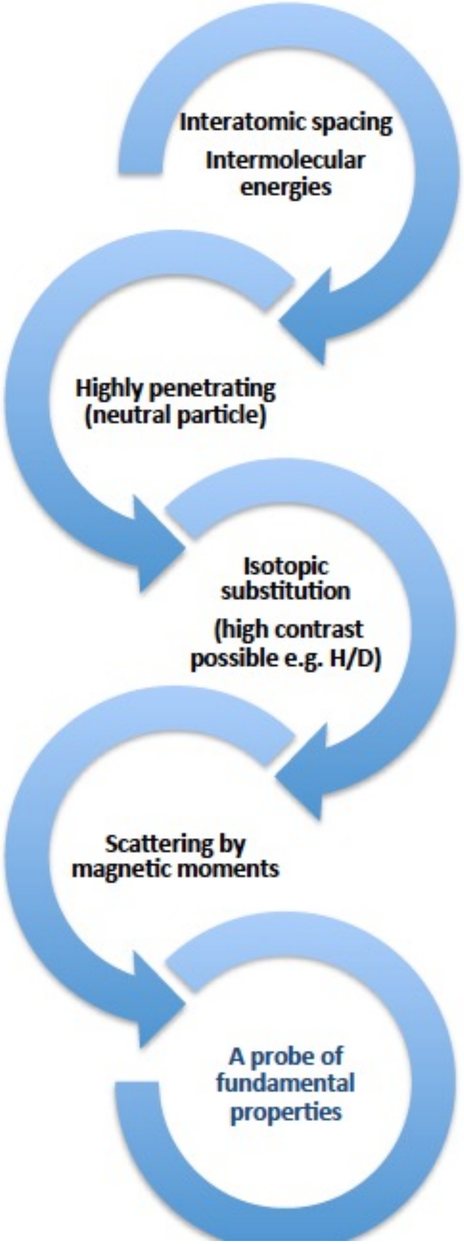
Of the six possible crystal forms, the fifth (form V) produces the best quality chocolate

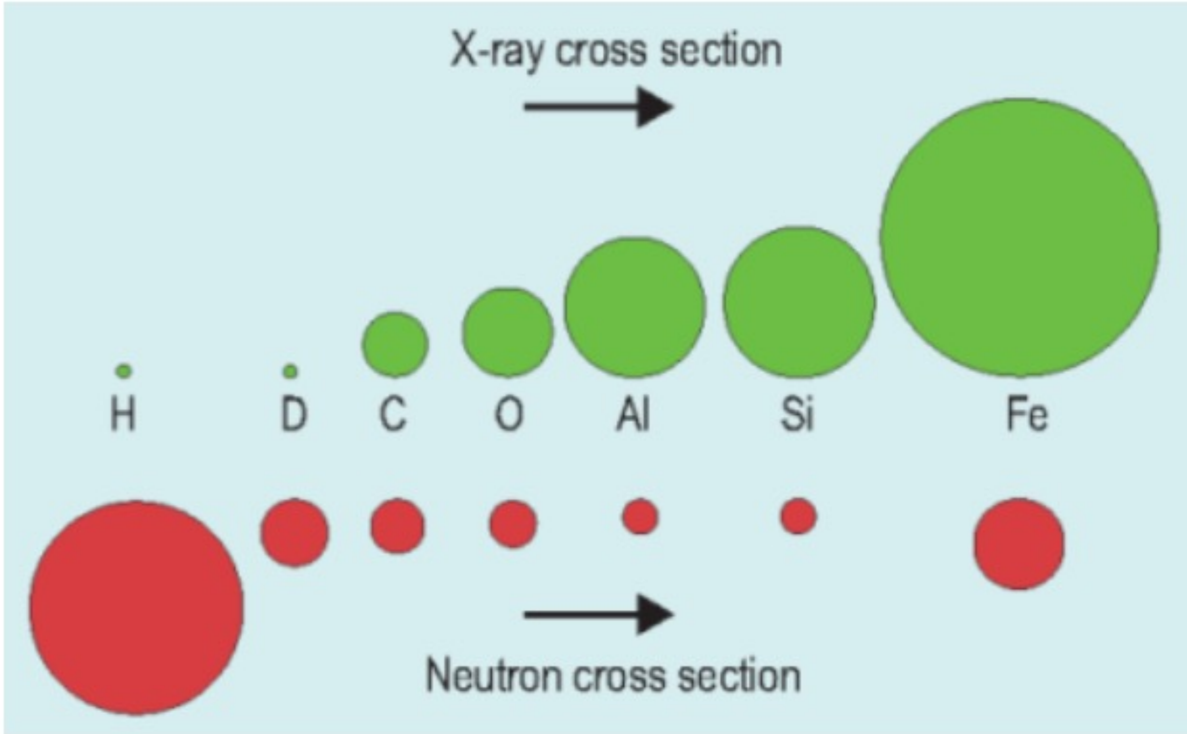


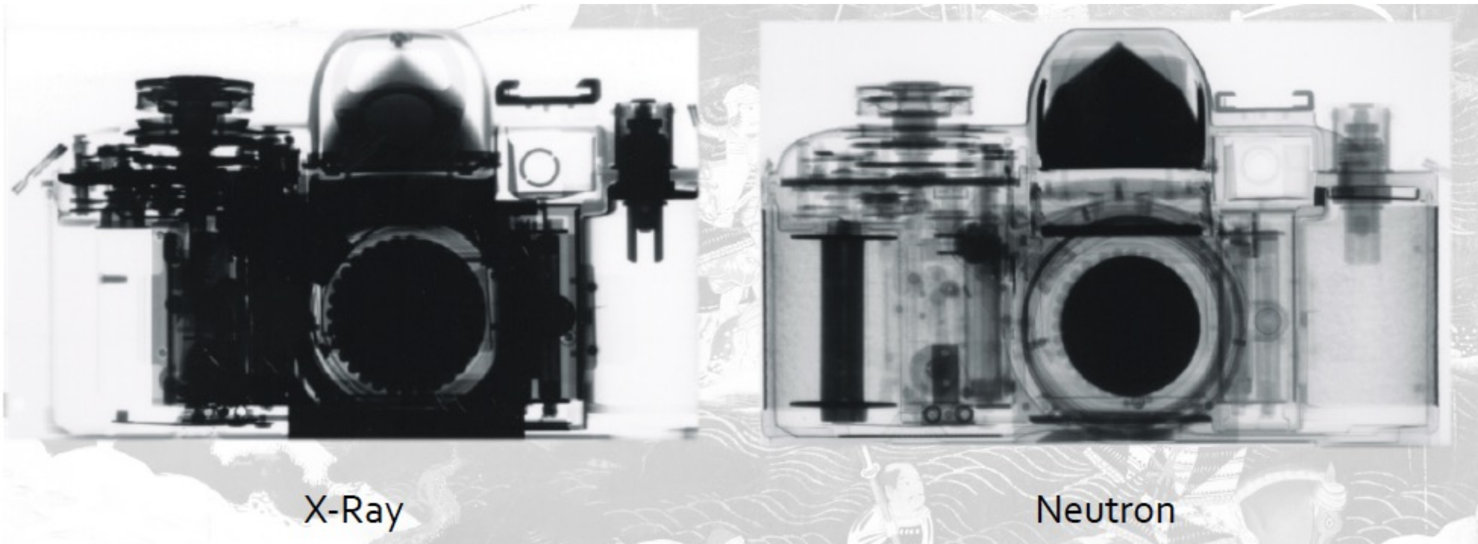
Neutron Sources



Relevant Neutron Properties







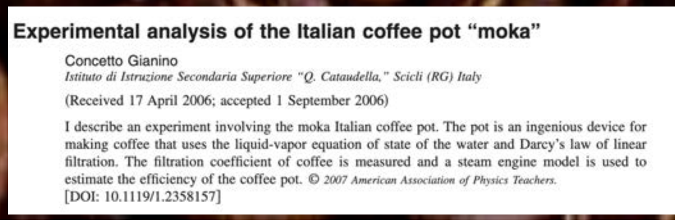
The special feature of Neutrograph is it's intensity together with a moderate collimation.

These properties allow the investigation of dynamic processes with an excellent time resolution and the transmittance through strongly absorbing and bulky materials.

A totally new spectrum of scientific and engineering applications could be developed.

Among the experiments are investigations of heat exchangers and combustion engines, parts from aircrafts, fossils and historical heritage.

Institut Laue-Langevin (ILL) in Grenoble



Experimental analysis of the Italian coffee pot "moka"

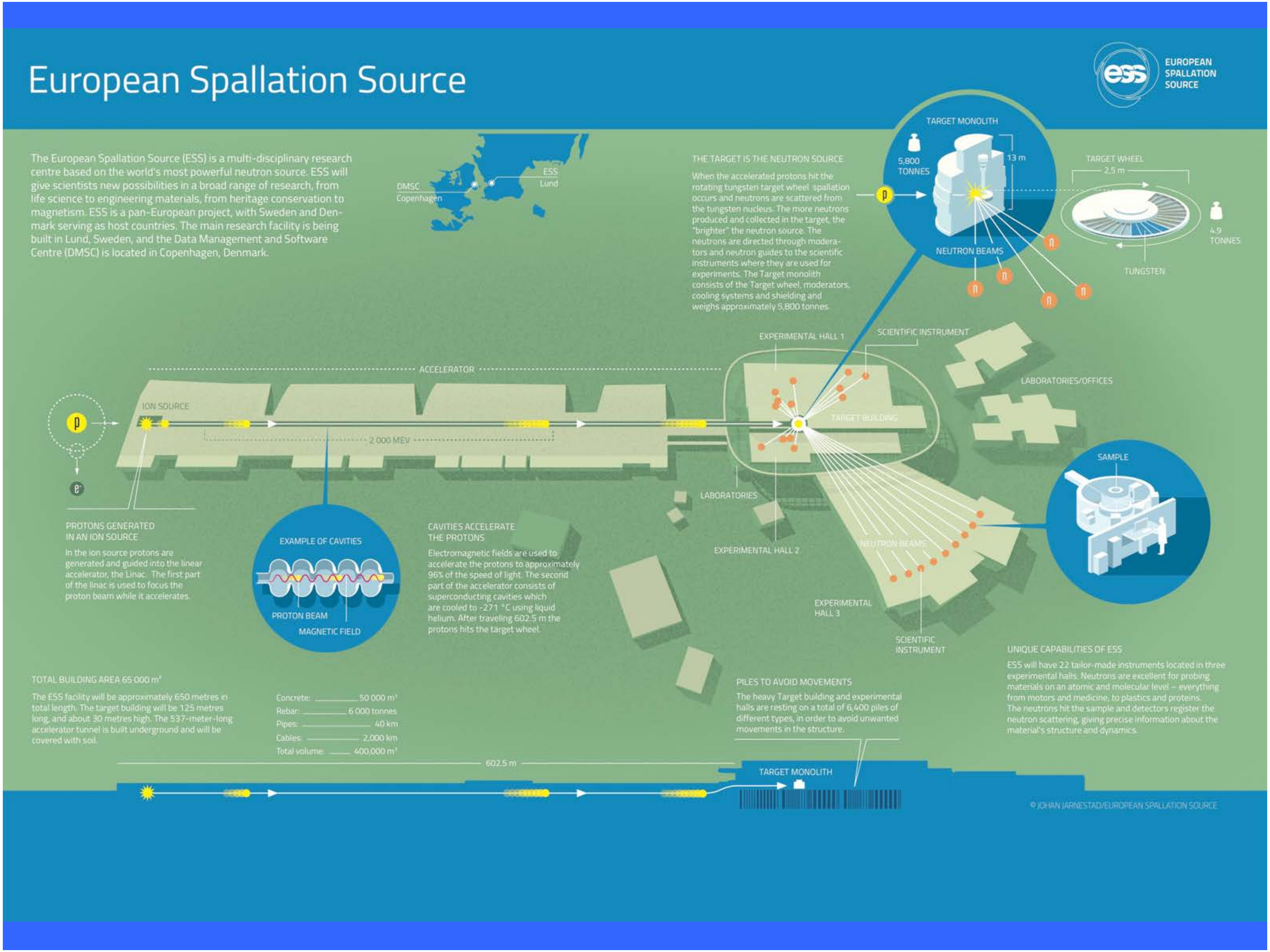
Concetto Gianino

Istituto di Istruzione Secondaria Superiore "Q. Cataudella," Scicli (RG) Italy

(Received 17 April 2006; accepted 1 September 2006)

I describe an experiment involving the moka Italian coffee pot. The pot is an ingenious device for making coffee that uses the liquid-vapor equation of state of the water and Darcy's law of linear filtration. The filtration coefficient of coffee is measured and a steam engine model is used to estimate the efficiency of the coffee pot. © *2007 American Association of Physics Teachers.*

[DOI: 10.1119/1.2358157]



European Spallation Source
EUROPEAN SPALLATION SOURCE
The European Spallation Source (ESS) is a multi-disciplinary research centre based on the world's most powerful neutron source. ESS will give scientists new possibilities in a broad range of research, from life science to engineering materials, from heritage conservation to magnetism. ESS is a pan-European project, with Sweden and Denmark serving as host countries. The main research facility is being built in Lund, Sweden, and the Data Management and Software Centre (DMSC) is located in Copenhagen, Denmark.
DMSC Copenhagen
ESS Lund
THE TARGET IS THE NEUTRON SOURCE
When the accelerated protons hit the rotating tungsten target wheel spallation occurs and neutrons are scattered from the tungsten nucleus. The more neutrons produced and collected in the target, the "brighter" the neutron source. The neutrons are directed through moderators and neutron guides to the scientific instruments where they are used for experiments. The Target monolith consists of the Target wheel, moderators, cooling systems and shielding and weighs approximately 5,800 tonnes.
TARGET MONOLITH
5,800 TONNES
13 m
NEUTRON BEAMS
TARGET WHEEL
2,5 m
4,9 TONNES
TUNGSTEN
EXPERIMENTAL HALL 1
SCIENTIFIC INSTRUMENT
ACCELERATOR
ION SOURCE
2 000 MEV
TARGET BUILDING
LABORATORIES/OFFICES
SAMPLE
LABORATORIES
EXPERIMENTAL HALL 2
NEUTRON BEAMS
EXPERIMENTAL HALL 3
SCIENTIFIC INSTRUMENT
PROTONS GENERATED IN AN ION SOURCE
In the ion source protons are generated and guided into the linear accelerator, the Linac. The first part of the linac is used to focus the proton beam while it accelerates.
EXAMPLE OF CAVITIES
PROTON BEAM
MAGNETIC FIELD
CAVITIES ACCELERATE THE PROTONS
Electromagnetic fields are used to accelerate the protons to approximately 96% of the speed of light. The second part of the accelerator consists of superconducting cavities which are cooled to -271 °C using liquid helium. After traveling 602.5 m the protons hits the target wheel.
TOTAL BUILDING AREA 65 000 m²
The ESS facility will be approximately 650 metres in total length. The target building will be 125 metres long, and about 30 metres high. The 537-meter-long accelerator tunnel is built underground and will be covered with soil.
Concrete: 50 000 m³
Rebar: 6 000 tonnes
Pipes: 40 km
Cables: 2,000 km
Total volume: 400,000 m³
PILES TO AVOID MOVEMENTS
The heavy Target building and experimental halls are resting on a total of 6,400 piles of different types, in order to avoid unwanted movements in the structure.
UNIQUE CAPABILITIES OF ESS
ESS will have 22 tailor-made instruments located in three experimental halls. Neutrons are excellent for probing materials on an atomic and molecular level – everything from motors and medicine, to plastics and proteins. The neutrons hit the sample and detectors register the neutron scattering, giving precise information about the material's structure and dynamics.
602.5 m
TARGET MONOLITH
© JOHAN JARNESTAD/EUROPEAN SPALLATION SOURCE

Area	Application	Beam	Accelerator	Beam energy/MeV	Beam current/ mA	Number
Medical	Cancer therapy	e	linac	4-20	10^{-2}	>14000
		p	cyclotron, synchrotron	250	10^{-6}	60
		C	synchrotron	4800	10^{-7}	10
	Radioisotope production	p	cyclotron	8-100	1	1600
Industrial	Ion implantation	B, As, P	electrostatic	< 1	2	>11000
	Ion beam analysis	p, He	electrostatic	<5	10^{-4}	300
	Material processing	e	electrostatic, linac, Rhodatron	≤10	150	7500
	Sterilisation	e	electrostatic, linac, Rhodatron	≤10	10	3000
Security	X-ray screening of cargo	e	linac	4-10	?	100?
	Hydrodynamic testing	e	linear induction	10-20	1000	5
Synchrotron light sources	Biology, medicine, materials science	e	synchrotron, linac	500-10000		70
Neutron scattering	Materials science	p	cyclotron, synchrotron, linac	600-1000	2	4
Energy – fusion	Neutral ion beam heating	d	electrostatic	1	50	10
	Heavy ion inertial fusion	Pb, Cs	Induction linac	8	1000	Under development
	Materials studies	d	linac	40	125	Under development
Energy – fission	Waste burner	p	linac	600-1000	10	Under development
	Thorium fuel amplifier	p	linac	600-1000	10	Under development
Energy – bio-fuel	Bio-fuel production	e	electrostatic	5	10	Under development
Environmental	Water treatment	e	electrostatic	5	10	5
	Flue gas treatment	e	electrostatic	0.7	50	Under development

Ultra-precise microscopy

- Probing particles are required for studies of the elementary constituents
- The associated de Broglie wavelength λ of a probing particle defines the minimum object size that can be resolved.

$$\lambda = \frac{h}{p} = h \times \frac{c}{E} \quad \text{with} \begin{cases} h = 4 \times 10^{-15} \text{ eVs} \quad \text{(Plank constant)} \\ p = \text{momentum}, \; E = \text{energy} \end{cases}$$

Resolving Smaller Objects Requires Higher Momentum Probe Particles

Example of probe wavelength

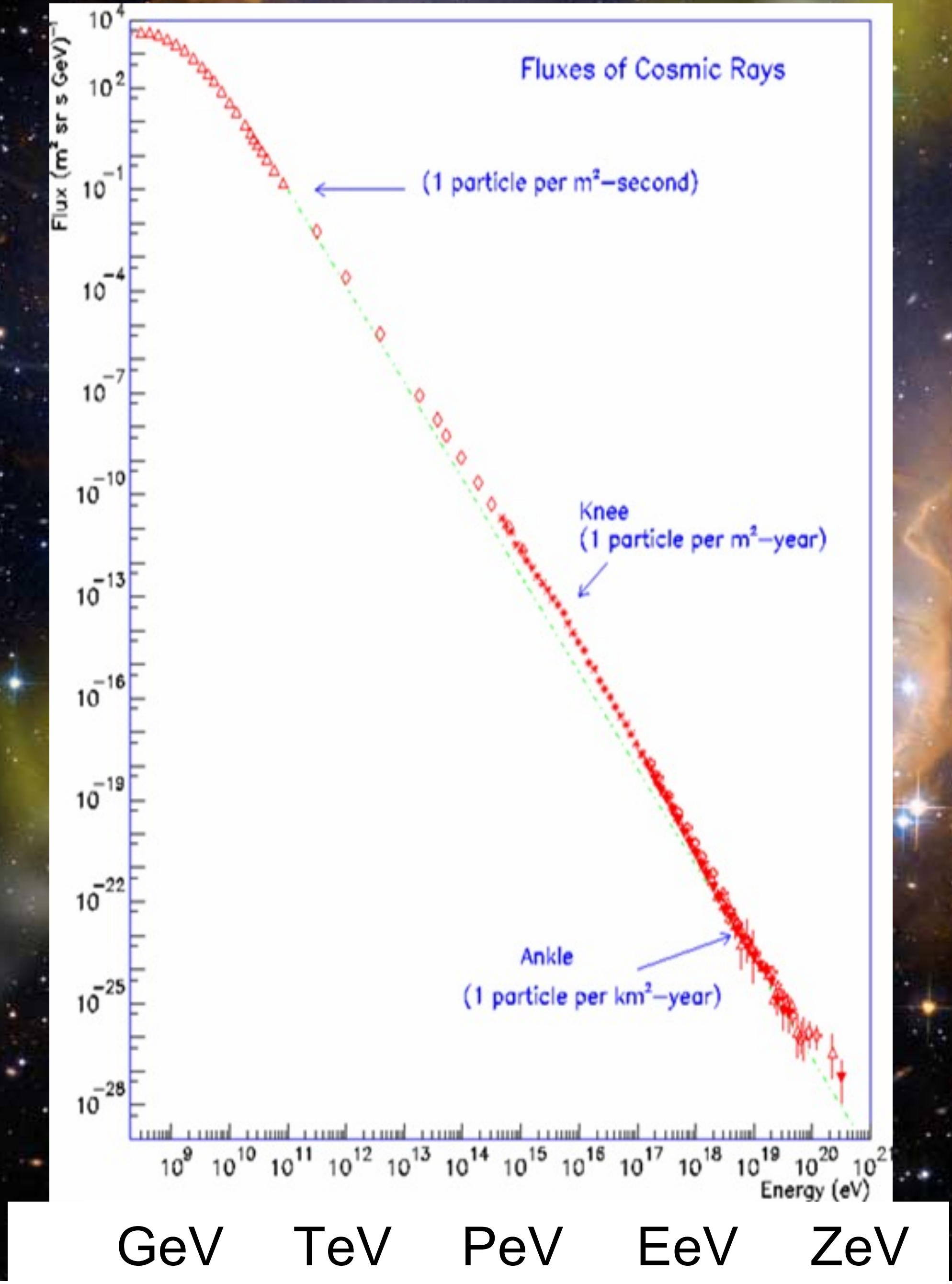
- electrons with p = 1 keV/c $\Rightarrow$ l = h/p = 4×10^{-12} m
- photons with E = 1 keV $\Rightarrow$ l = $h\times c/E \sim 1.2 \times 10^{-9}$ m.

➔ electrons have ~ 300 times better resolution than photons (electron-microscopy !)

Typical microscopic sizes

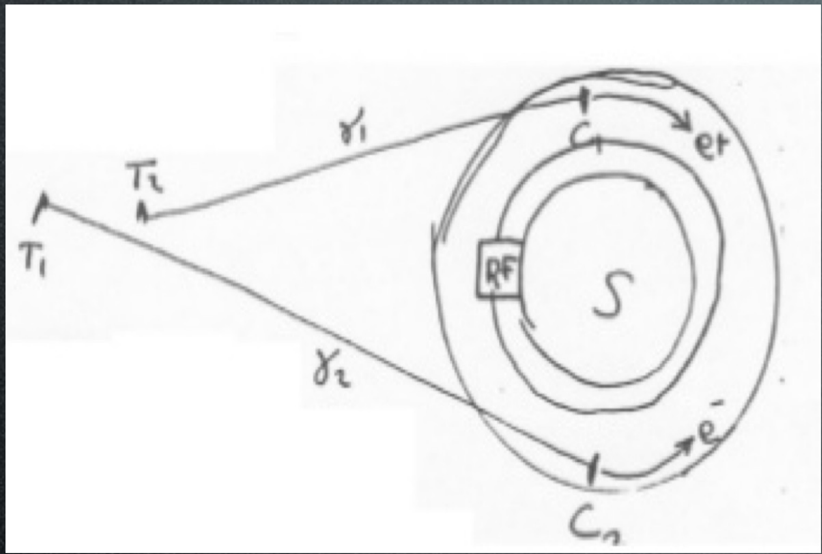
- Atom 10^{-10} m
- Nucleus 10^{-14} m
- Proton 10^{-15} m
- Quark 10^{-19} m

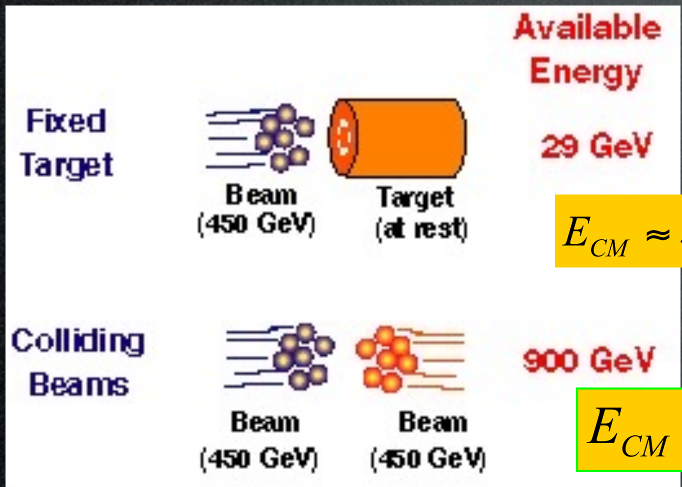
The Cosmos is a very large particle accelerator…but in uncontrolled conditions



Touschek's Anello Di Accumulazione (ADA)

1961 the first e+e- Collider

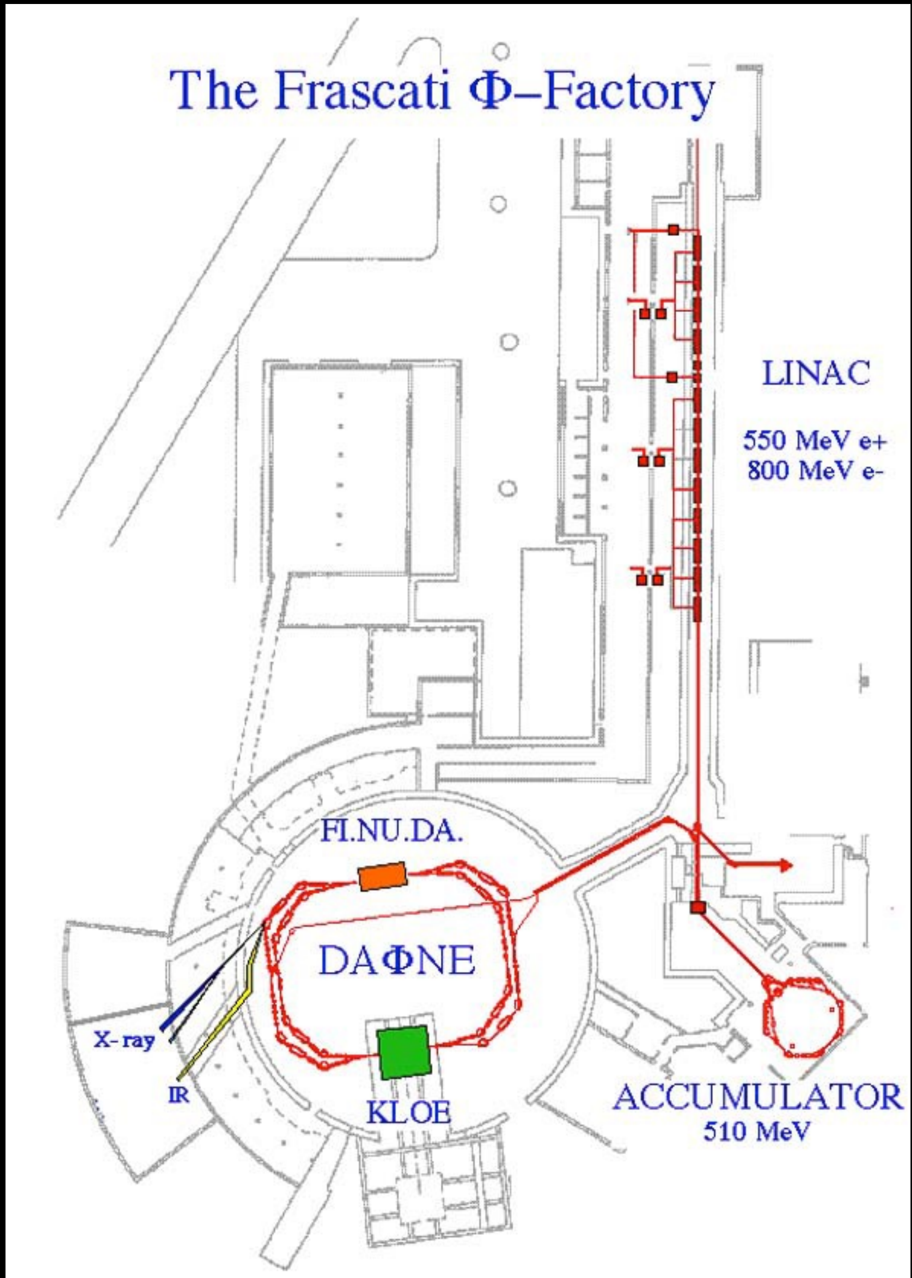


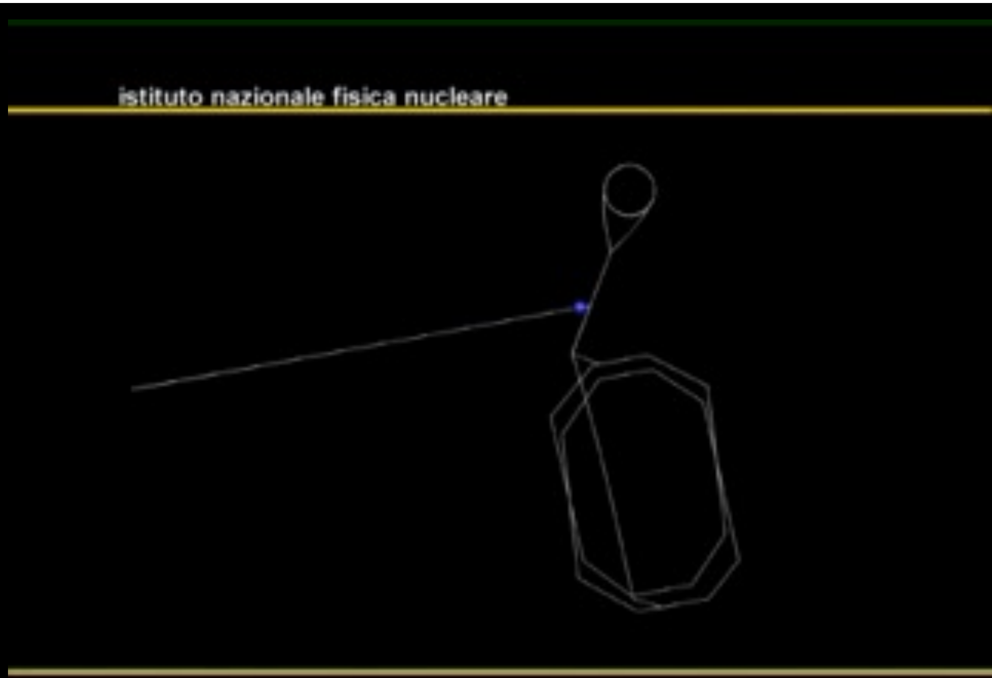


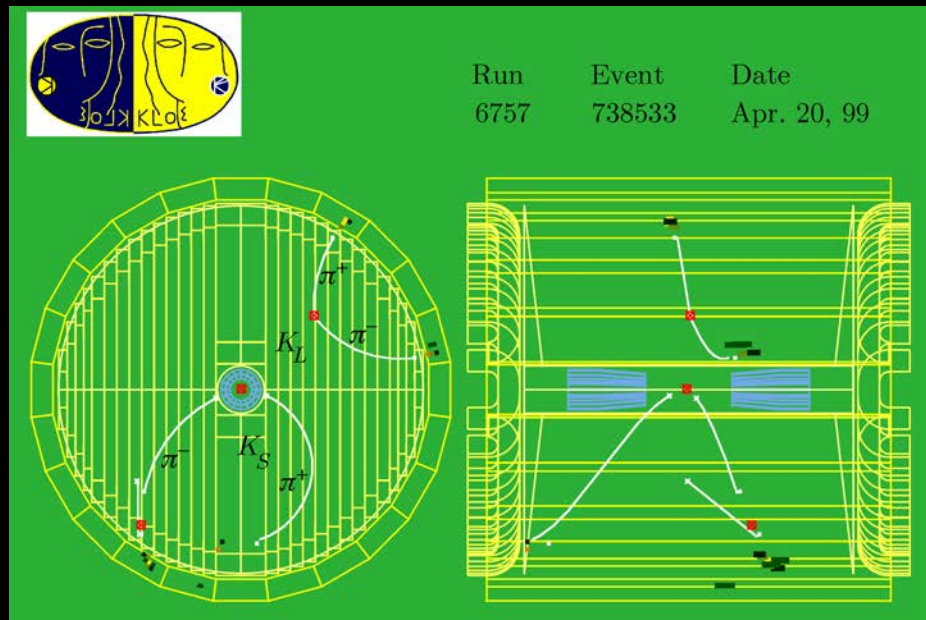
$$E_{CM} \approx \sqrt{2E_1 m_2}$$

$$E_{CM} \approx 2E$$

Collider e+ e- DAFNE (INFN)


The Frascati Φ–Factory
LINAC
550 MeV e+
800 MeV e-
FI.NU.DA.
DAΦNE
X- ray
IR
KLOE
ACCUMULATOR
510 MeV


istituto nazionale fisica nucleare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Historical Milestones

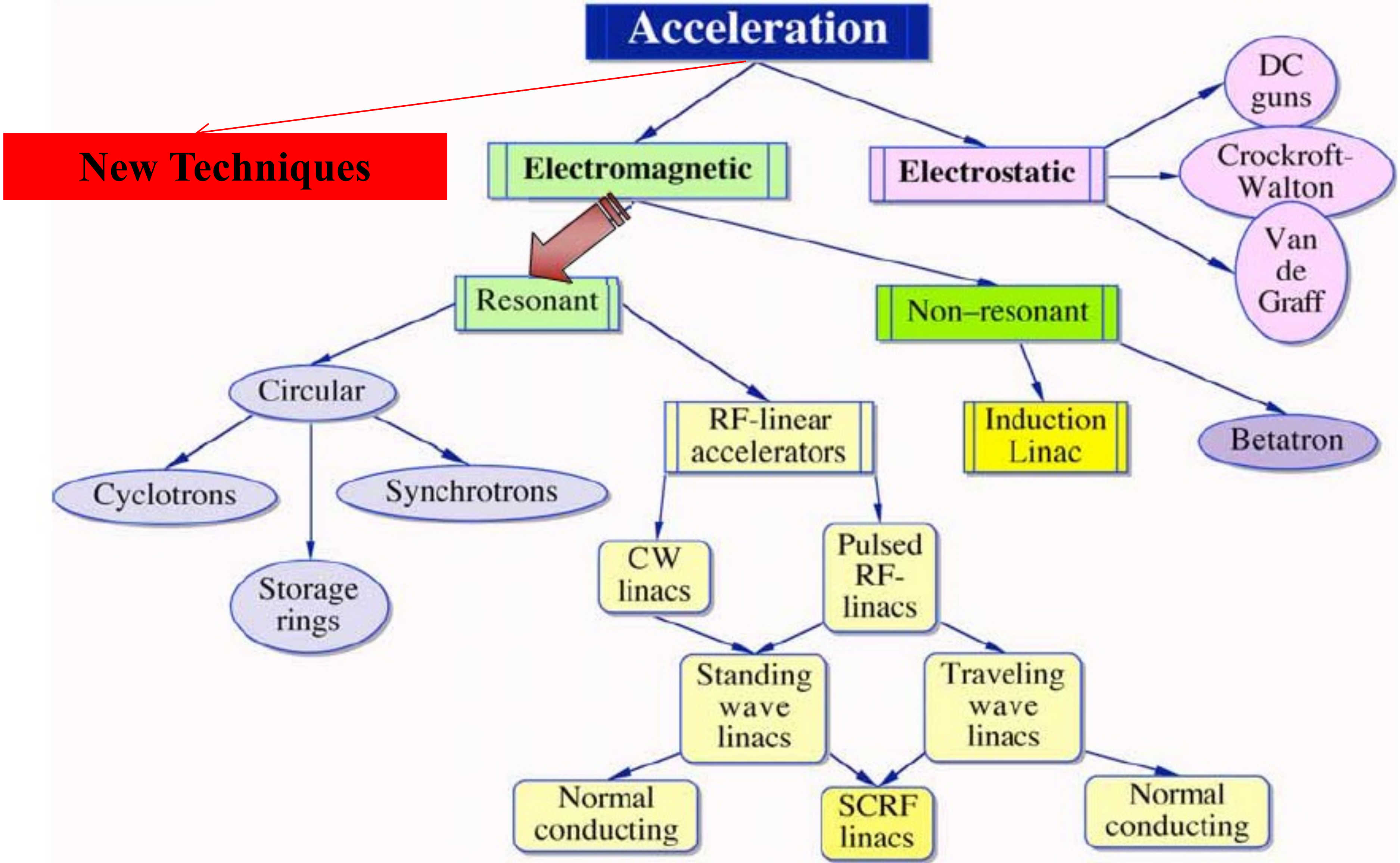
- 1900 to 1925 radioactive source experiments à la Rutherford -> request for higher energy beams;
- 1928 to 1932 electrostatic acceleration ->
  - Cockcroft & Walton * -> voltage multiplication using diodes and oscillating voltage (700 kV);
  - Van der Graaf -> voltage charging through mechanical belt (1.2 MV);
- 1928 resonant acceleration -> Ising establish the concept, Wideroe builds the first linac;
- 1929 cyclotron ->
  - small prototype by Livingstone (PhD thesis), large scale by Lawrence**;
- 1942 magnetic induction -> Kerst build the betatron;
- 1944 synchrotron -> MacMillan, Oliphant and Veksel invent the RF phase stability (longitudinal focusing);
- 1946 proton linac -> Alvarez build an RF structure with drift tubes (progressive wave in 2π mode);
- 1950 strong focusing -> Christofilos patent the alternate gradient concept (transverse strong focusing);
- 1951 tandem -> Alvarez upgrade the electrostatic acceleration concept and build a tandem;
- 1955 AGS -> Courant, Snider and Livingstone build the alternate gradient Cosmotron in Brookhaven;
- 1956 collider -> Kerst discuss the concept of colliding beams;
- 1961 e^+e^- collider -> Touschek invent the concept of particle-antiparticle collider;
- 1967 electron cooling -> Budker proposes the e-cooling to increase the proton beam density;
- 1968 stochastic cooling ->
  - Van der Meer*** proposes the stochastic cooling to compress the phase space;
- 1970 RFQ -> Kapchinski & Telyakov build the radiofrequency quadrupole;
- 1980 to now superconducting magnets -> developed in various laboratories to increase the beam energy;
- 1980 to now superconducting RF -> developed in various lab to increase the RF gradient.

* Nobel 1951

** Nobel 1939

*** Nobel 1984

Accelerator Configurations



Accelerators for Science Diplomacy

1954 CERN A small number of visionary scientists in Europe and North America identified the need for Europe to have a world-class physics research facility. Their vision was both to stop the brain drain to America that had begun during the Second World War, and to provide a force for unity in post-war Europe.
Today, CERN unites scientists from around the world in the pursuit of knowledge
CERN's convention states: "The Organization shall have no concern with work for military requirements and the results of its experimental and theoretical work shall be published or otherwise made generally available."

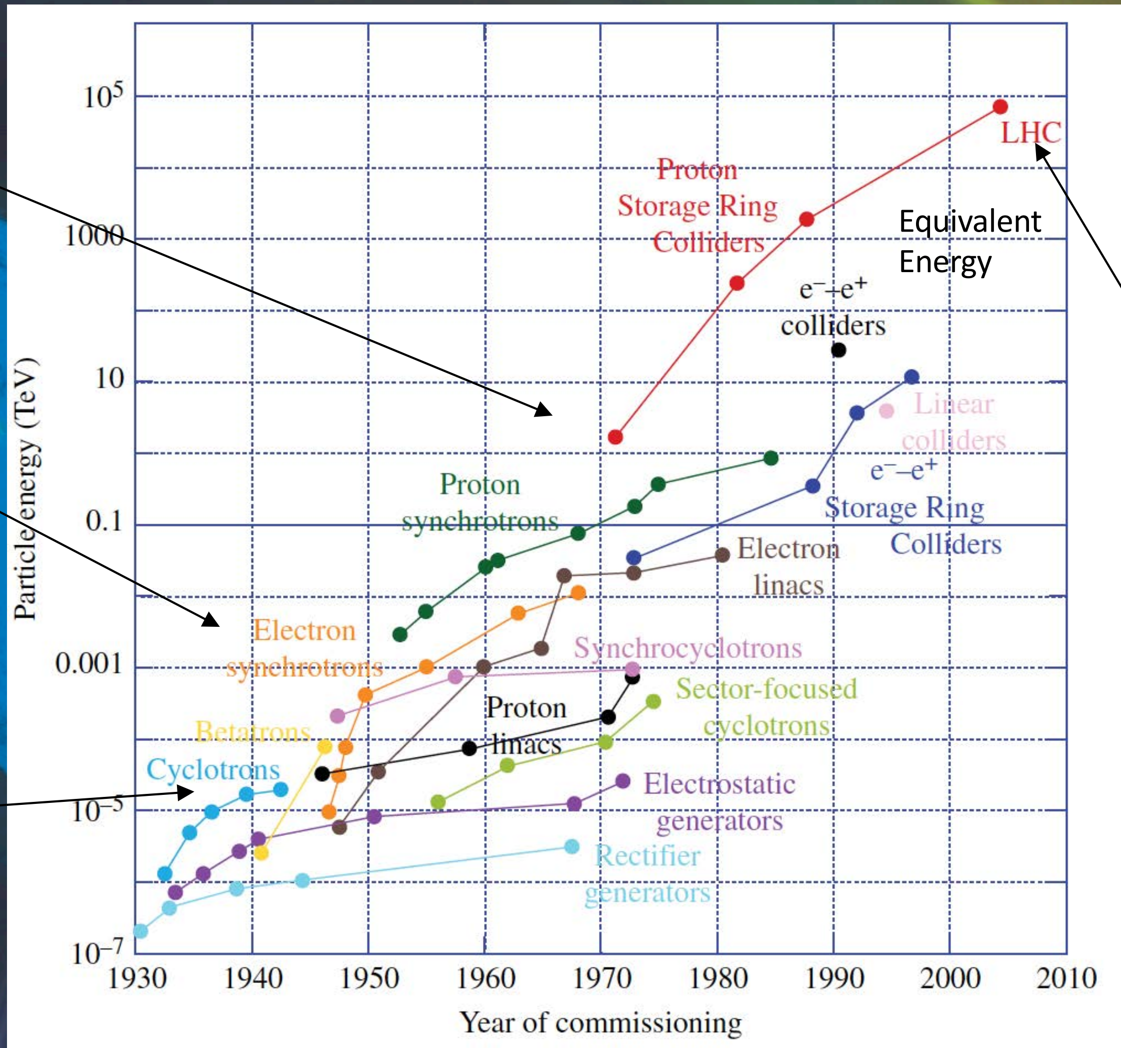
1964 ICTP Founded in 1964 by the late Nobel Laureate Abdus Salam, ICTP is a unique institution that explores fundamental scientific questions at the highest level, promotes active engagement with scientists in developing countries, and advances international cooperation through science. ELETTRA The access by researchers from developing countries has tripled over the last few years, and the Indian research community is one of the largest users.

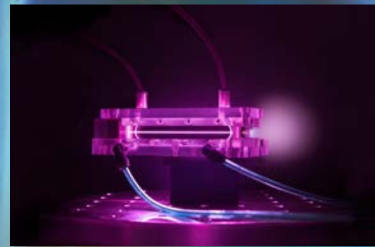
1983 SDI The Strategic Defense Initiative (SDI), derisively nicknamed the "Star Wars program", was a proposed missile defense system intended to protect the United States from attack by ballistic strategic nuclear weapons (intercontinental ballistic missiles and submarine-launched ballistic missiles). The concept was announced on March 23, 1983, by President Ronald Reagan,[1] a vocal critic of the doctrine of mutually assured destruction (MAD), which he described as a "suicide pact". Reagan called upon American scientists and engineers to develop a system that would render nuclear weapons obsolete.

2002 SESAME SESAME (Synchrotron-light for Experimental Science and Applications in the Middle East) is a "third-generation" synchrotron light source that was officially opened in Allan (Jordan) on 16 May 2017. It is the first synchrotron light source in the Middle East and neighbouring countries, and also the region's first major international centre of excellence.

African Light Source The African Light Source (AfLS) – as of December 2022 – is the initiative to build the first Pan-African synchrotron light source. The initiative is currently led – separately – by the African Light Source (AfLS) Foundation and the Africa Synchrotron Initiative (ASI).

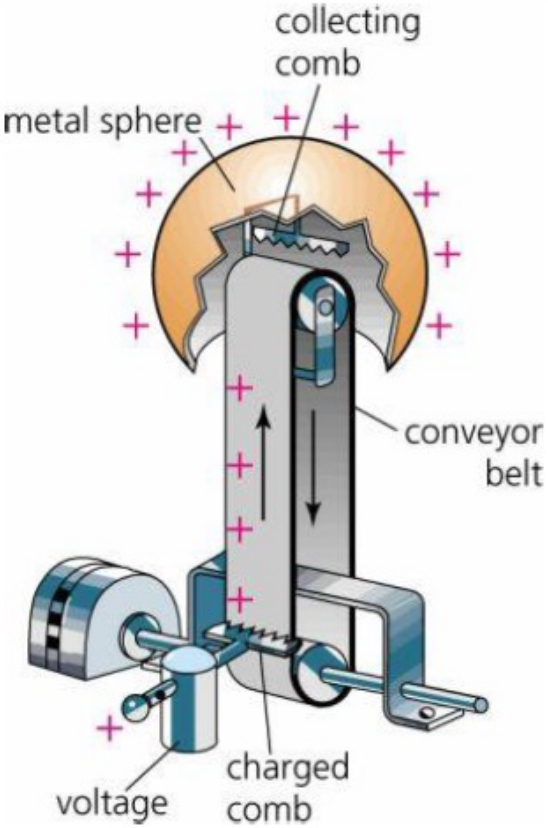
Il diagramma di Livingstone

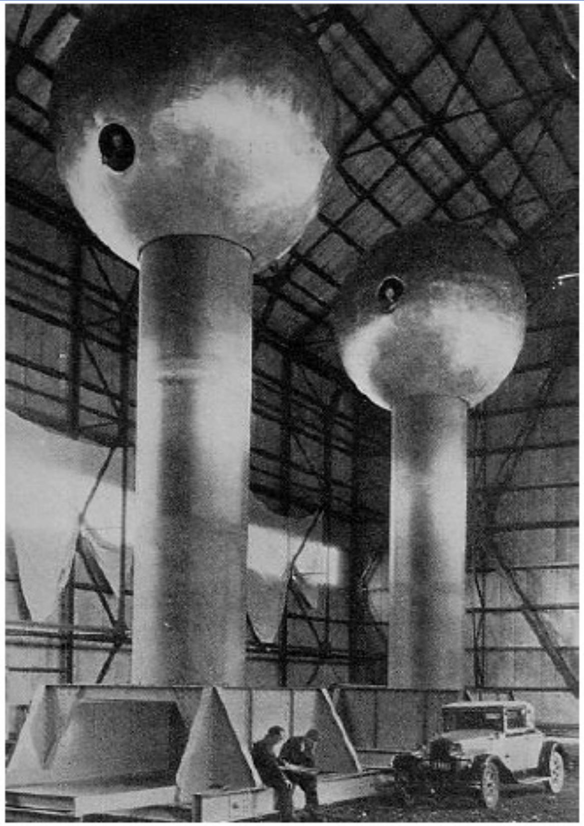




Energy of colliders is plotted in terms of the laboratory energy of particles colliding with a proton at rest to reach the same center of mass energy.

Electrostatic Accelerator: Van de Graff



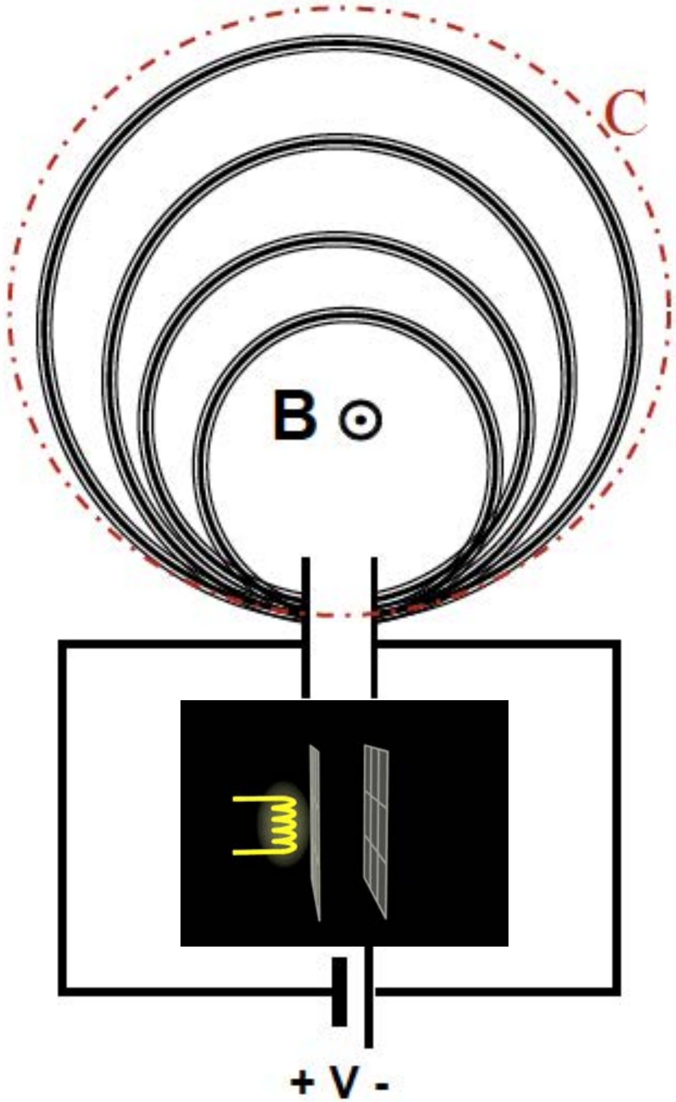


7 MV Van de Graaf at MIT
(1933)

R.J. Van de Graaf

- Electric charges are transported mechanically on an insulating belt
- Stable, continuous beams, practical limit 10 - 15 MV

Possible Higher energy DC accelerator?



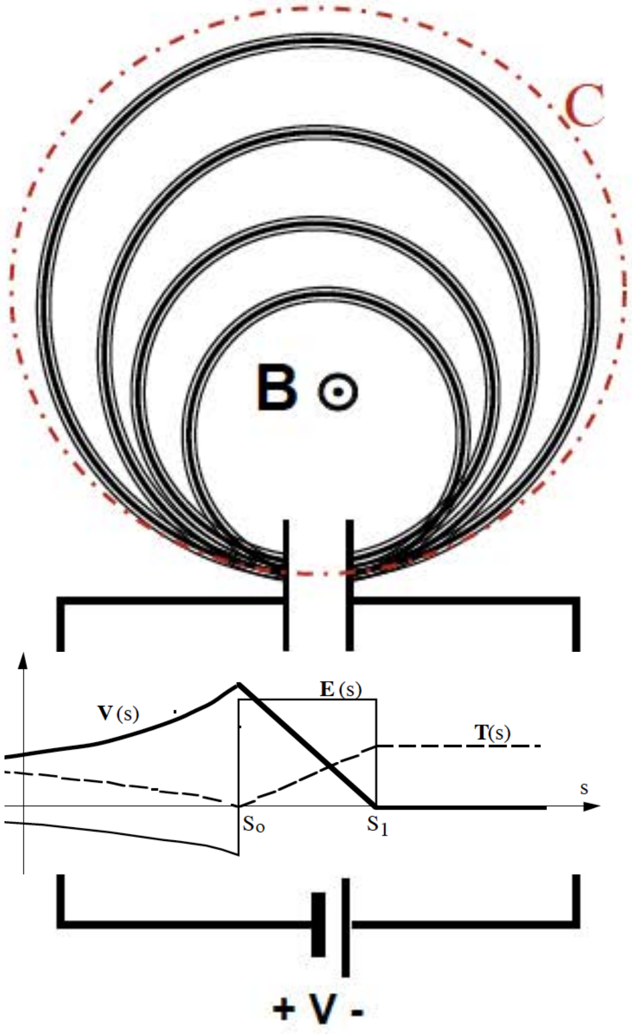
$$F_{Lorentz} = q\,v\,B = F_{centripital} = \frac{mv^2}{\rho}$$

$$\Rightarrow \rho = \frac{mv}{qB} = \frac{p}{qB}$$

$$\boxed{\rho(\mathrm{m}) = 3.34\left(\frac{p}{1\ \mathrm{GeV/c}}\right)\left(\frac{1}{q}\right)\left(\frac{1\ \mathrm{T}}{B}\right)}$$

$T = q\Delta V$

Forbidden by Maxwell



$$\nabla \times \mathbf{E} = -\frac{d\mathbf{B}}{dt}$$

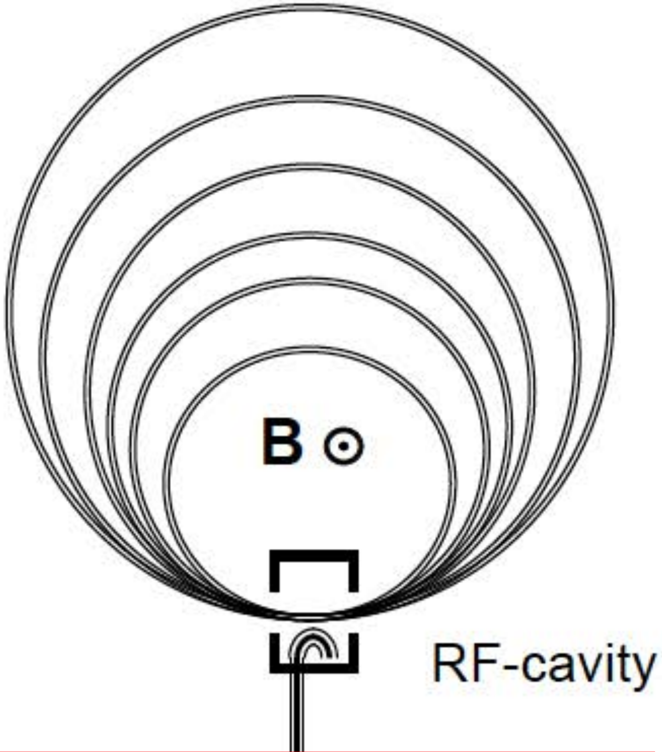
or in integral form

$$\oint_C \mathbf{E} \cdot d\mathbf{s} = -\frac{\partial}{\partial t} \int_S \mathbf{B} \cdot \mathbf{n}\, da$$

∴ There is no acceleration without time-varying magnetic flux

$$\boxed{\Delta V_T = 0}$$

B can vary in a RF cavity



$$E_z = E_0 J_0(k_r r) \cos \omega t$$

$$B_\theta = -\frac{E_0}{c} J_1(k_r r) \sin \omega t$$

Note that inside the cavity

$dB/dt \neq 0$

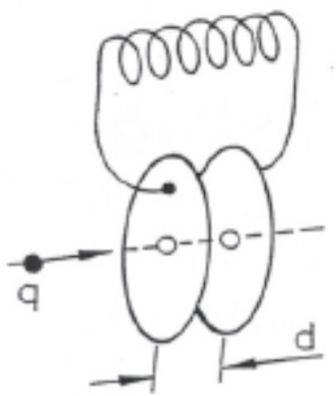
However,

Synchronism condition:

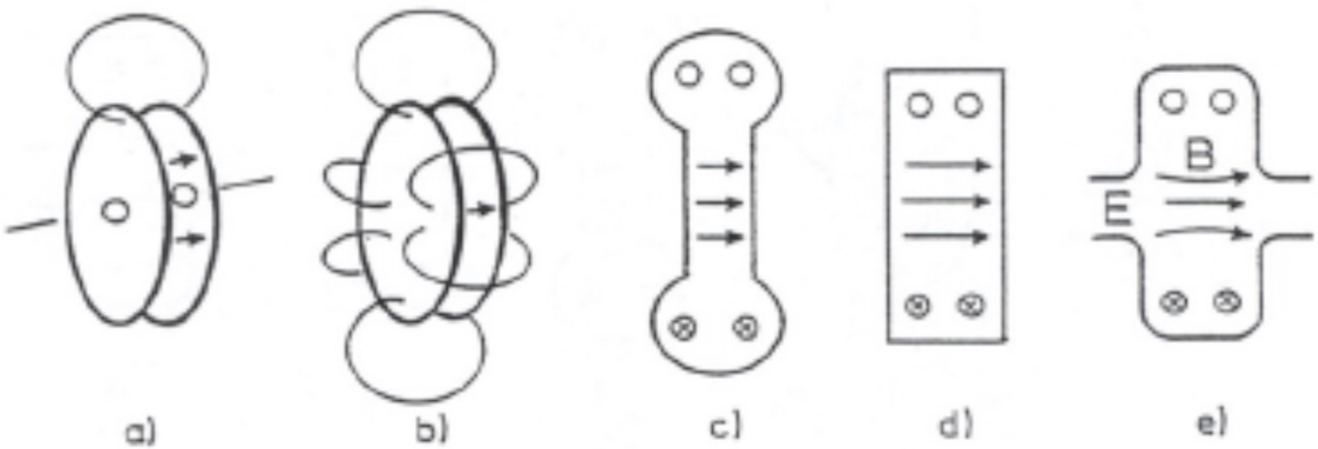
$\Delta\tau_{rev} = N/f_{rf}$

Time-oscillating electric fields imply in most cases synchronization of the incoming particle beam with the accelerating field, which has to have the desired amplitude and phase at the arrival time of particles in the accelerating region.

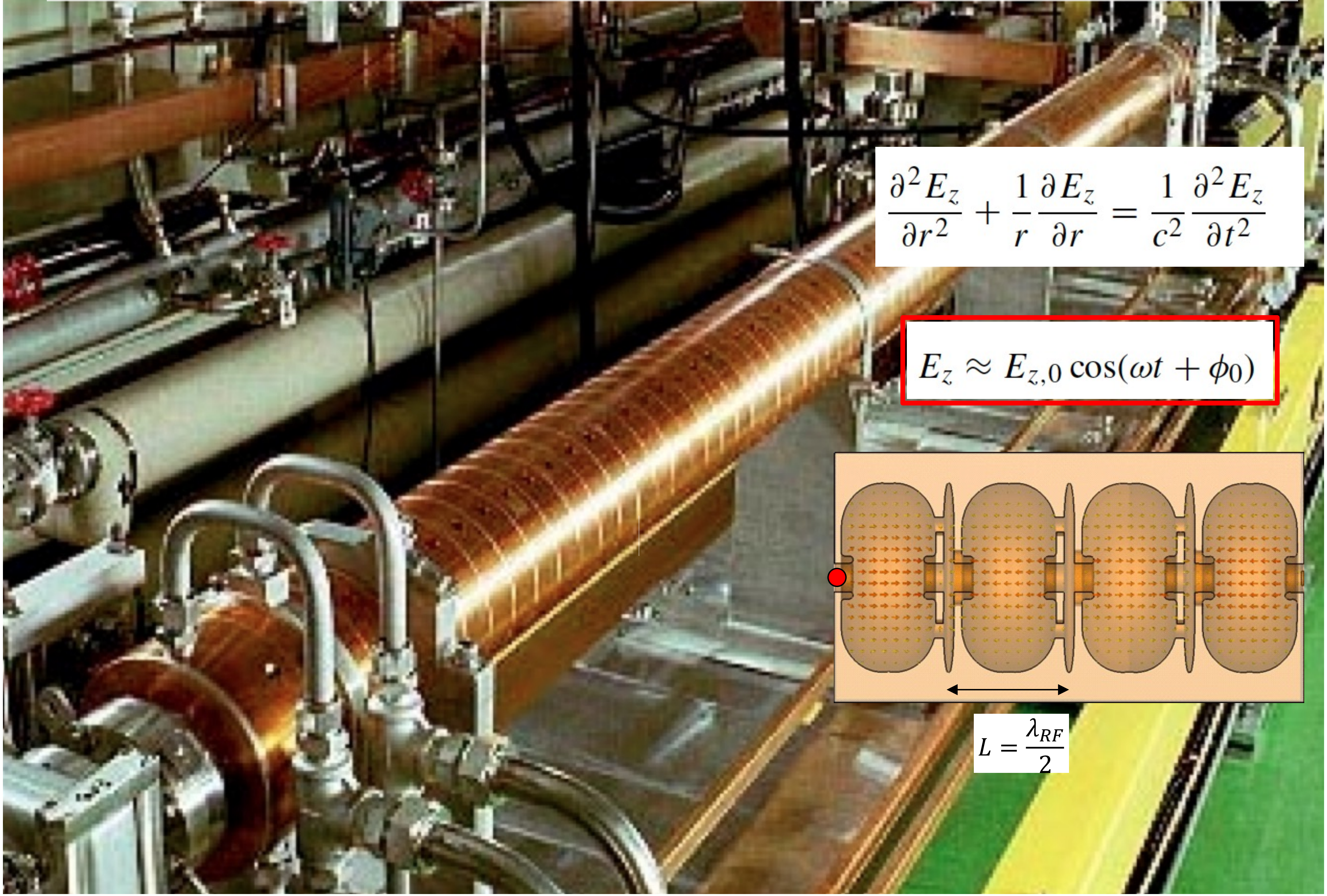
Parallel Resonant Circuit Model



- Imagine two capacitive plates with a parallel inductor.
- This creates a resonator with resonant frequency $\omega_o = \frac{1}{\sqrt{LC}}$
- If the inductor becomes many single loops of wire, this eventually becomes an accelerating cavity



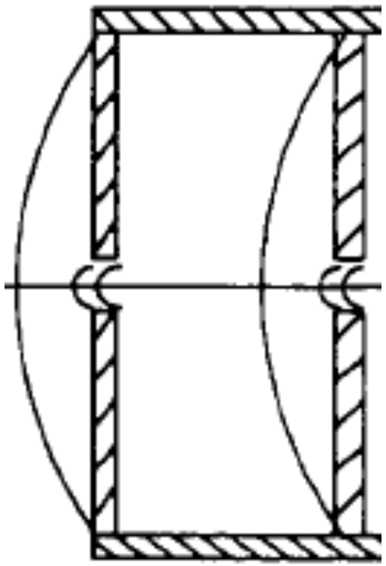
RF accelerating structures



$$\frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r}\frac{\partial E_z}{\partial r} = \frac{1}{c^2}\frac{\partial^2 E_z}{\partial t^2}$$

$$\boxed{E_z \approx E_{z,0}\cos(\omega t + \phi_0)}$$

$$L = \frac{\lambda_{RF}}{2}$$



$$E_z = E_0 J_0(k_r r) \cos \omega t$$

$$B_\theta = -\frac{E_0}{c} J_1(k_r r) \sin \omega t$$

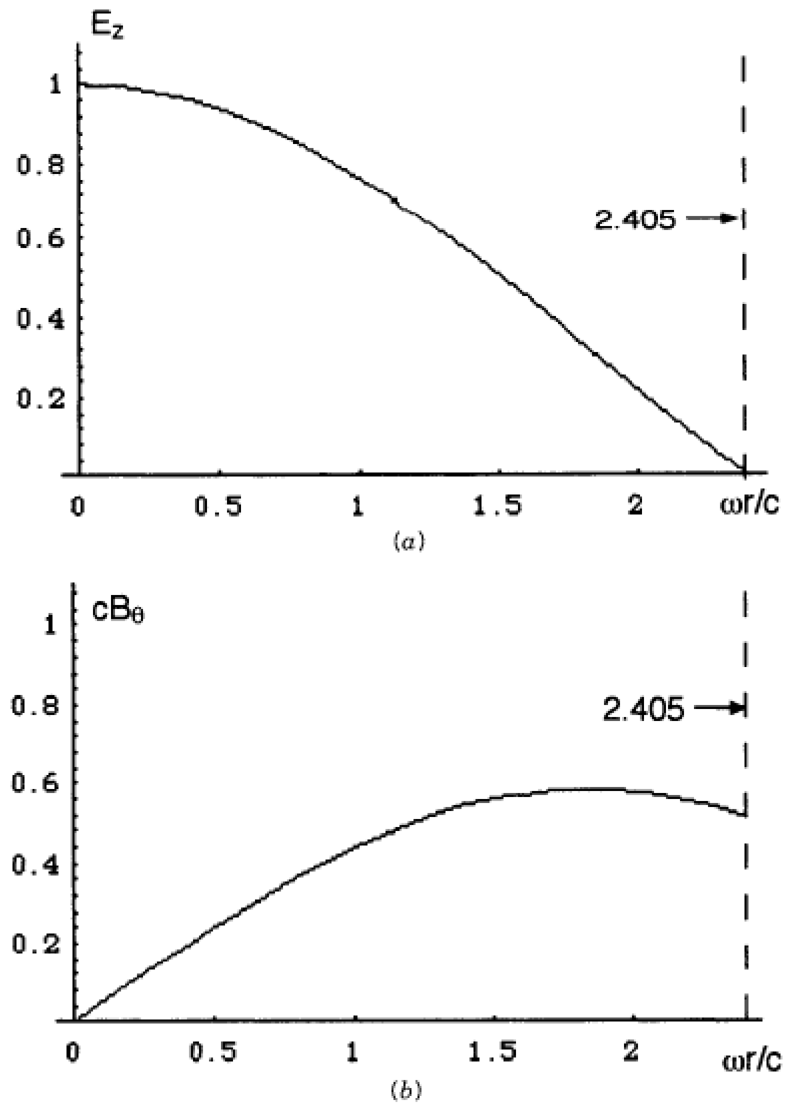
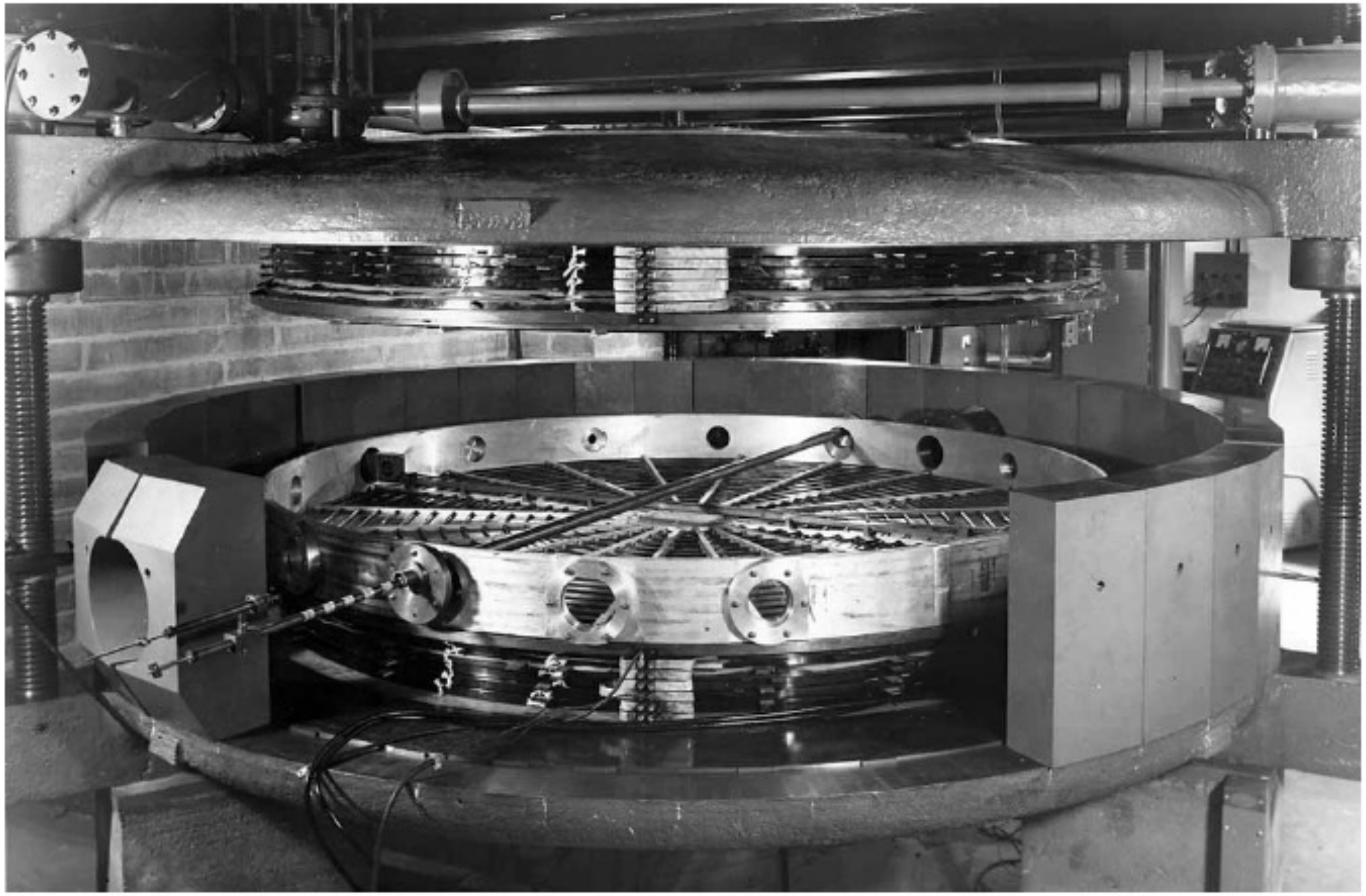


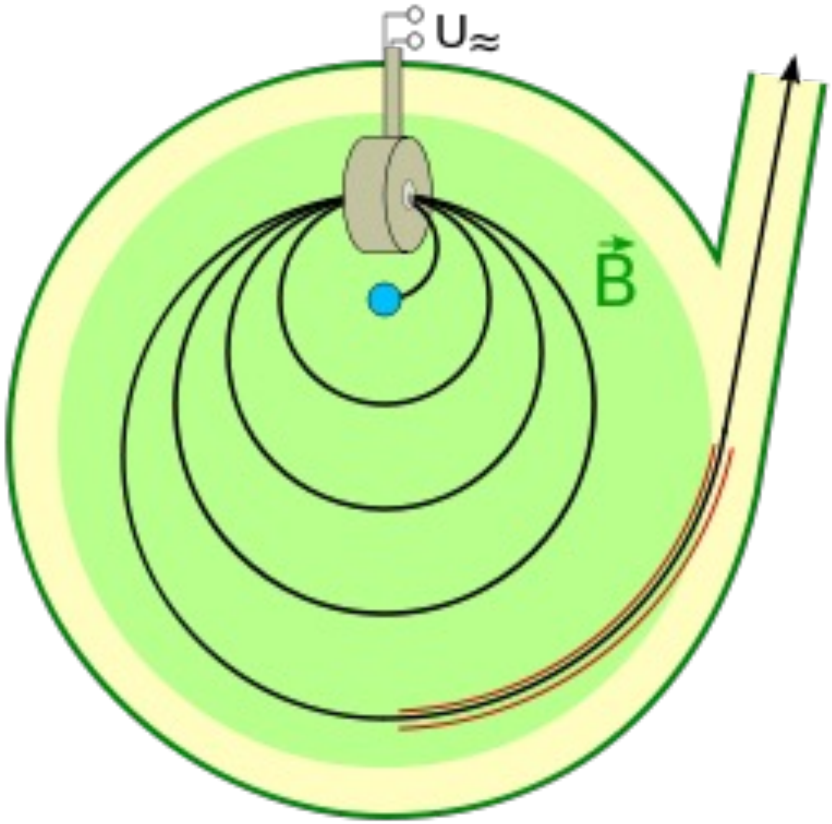
Figure 1.17 Fields for a TM_{010} mode of a cylindrical (pillbox)-cavity resonator.

Synchronization implies the production of pulsed beams, whose repetition frequency is therefore equal or a sub-multiple of the generator frequency. Each “packet” of particles is called bunch in the literature.

28 MeV Microtron at HEP Laboratory University College London



Microtron - Synchronization



$$\boxed{t_i = \frac{2\pi R_i}{v_i}}$$

$$e v_i B = m_i \frac{v_i^2}{R_i}$$

$$\boxed{R_i = \frac{v_i m_i c^2}{e c^2 B} = \frac{v_i}{e c^2 B} E_i}$$

$$\boxed{\Delta t = t_{i+1} - t_i = \frac{2\pi}{e c^2 B}\left(E_{i+1} - E_i\right) = \frac{2\pi}{e c^2 B}\Delta E}$$

Energy gain/revolution

$$\Delta t = t_{i+1} - t_i = \frac{2\pi}{ec^2 B}\left(E_{i+1} - E_i\right) = \frac{2\pi}{ec^2 B}\Delta E$$

$$\Delta t = \frac{k}{\nu_{RF}} = \frac{2\pi}{ec^2 B}\Delta E$$

$$\boxed{\Delta E = k\frac{ec^2 B}{2\pi\nu_{RF}}}$$

- In a microtron, due to the electrons' increasing momentum, the particle paths are different for each pass. The time needed for that must be an integer multiple k of the RF period. The allowed energy gain/pass must fulfill the above condition.

LORENTZ FORCE: ACCELERATION AND FOCUSING

The basic equation that describes the acceleration/bending/focusing processes is the **Lorentz Force**. Particles are **accelerated through electric** fields and are **bended and focused through magnetic** fields.

$\vec{p} = momentum$

$m = mass$

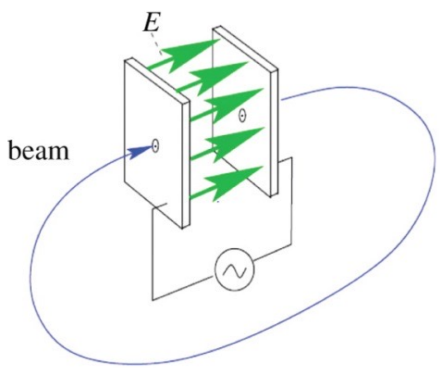
$\vec{v} = velocity$

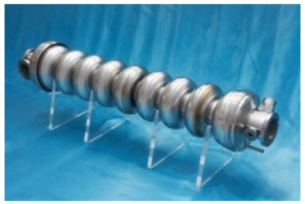
$q = charge$

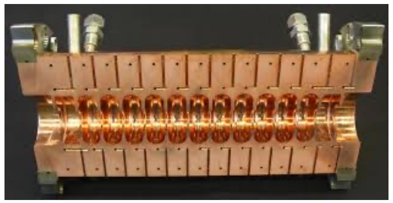
$$\frac{d\vec{p}}{dt} = q\left(\vec{E} + \vec{v} \times \vec{B}\right)$$

ACCELERATION

To accelerate, we need a force in the direction of motion



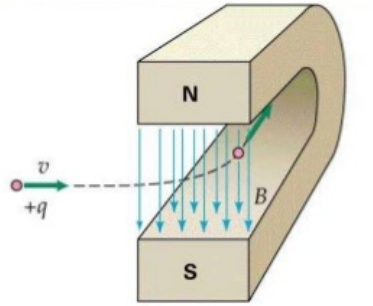


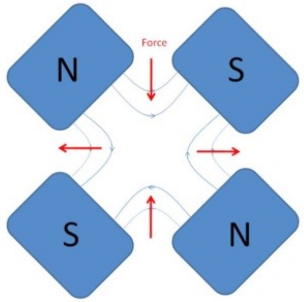


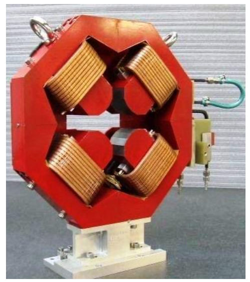
Longitudinal Dynamics

BENDING AND FOCUSING

2nd term always perpendicular to motion => no energy gain





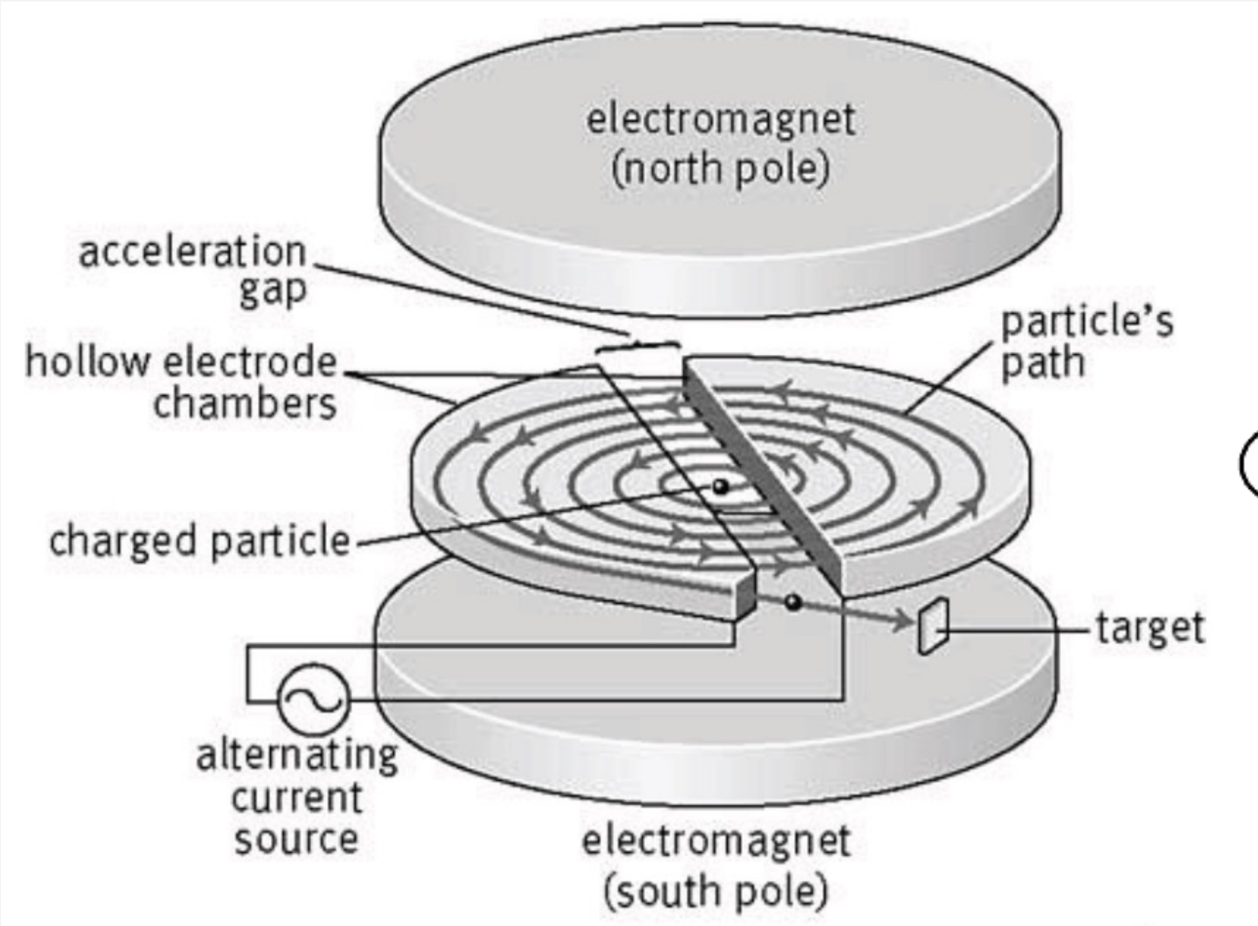


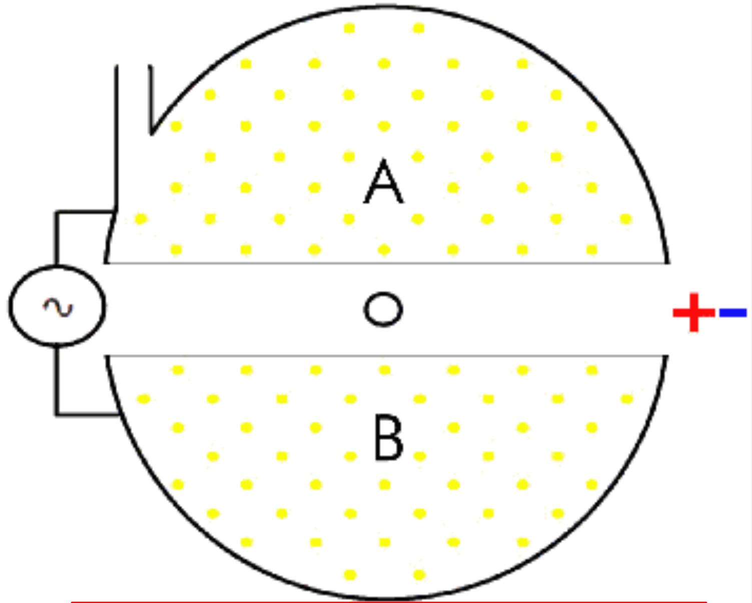
Transverse Dynamics

The Lawrence Cyclotron

FIGURE 3. ERNEST LAWRENCE, EDWIN MCMILLAN, AND LUIS ALVAREZ (left to right) admire a finished Chromatron. (Ernest O. Lawrence papers, BANC MSS 2005/200c, oversize box 3. Courtesy of the Bancroft Library, University of California, Berkeley.)

The Cyclotron concept





$$\gamma m_0 \frac{v_z^2}{R} = q v_z B_y \quad \Rightarrow \quad \begin{cases} \omega_L = \dfrac{q B_y}{\gamma m_0} \\ p_z = q B_y R \end{cases}$$

As the particle is accelerated and its energy increased, ω_L decreases for a constant B_y . In order to maintain the synchronism with the RF generator, $\omega_R = h\omega_L$, the latter has to be modulated in frequency. This process cannot be indefinite, since each RF source has a well-defined bandwidth of functionality. Within such technological limitations, the cyclotron can provide up to few hundreds of MeV kinetic energy per nucleon, and it is called synchro-cyclotron.

Magnetic Gradient to compensate for Energy increase => Vertical Defocusing

$$\gamma m_0 \frac{v_z^2}{R} = q v_z B_y \quad \Rightarrow \quad \begin{cases} \omega_L = \dfrac{q B_y}{\gamma m_0} \\ p_z = q B_y R \end{cases}$$

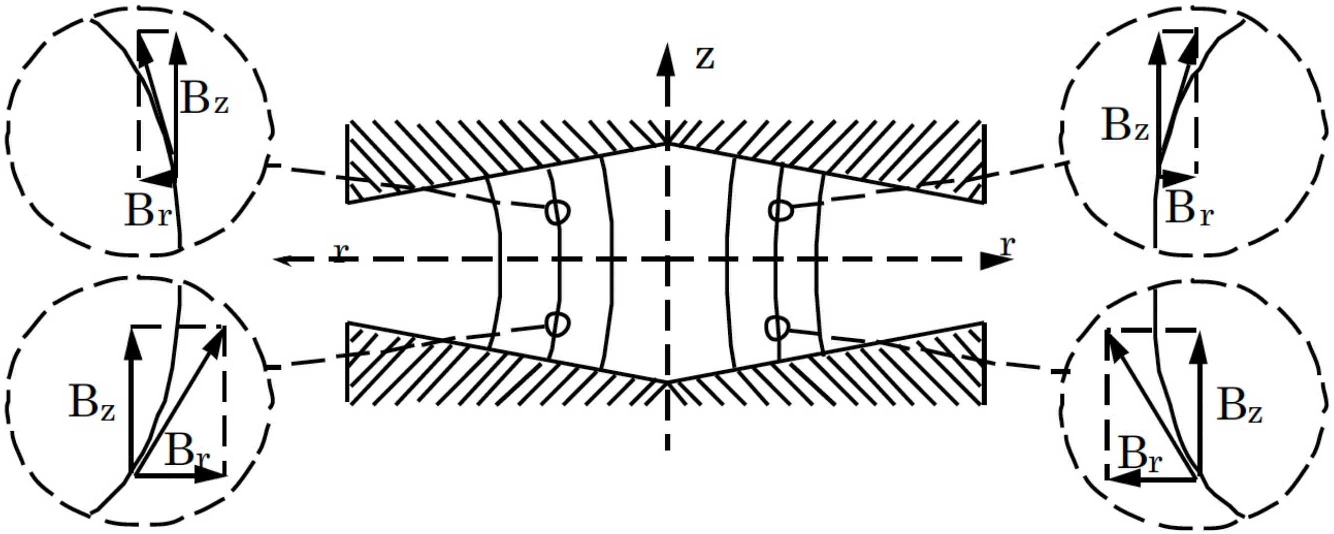


Fig. 2.5 Lateral section of the magnetic poles of a cyclotron. By virtue of the poles shaping, a charge vertically displaced with respect to the horizontal plane is subject to a horizontal component of the magnetic field. This produces a vertical defocusing Lorentz's force. (Image courtesy of S. Tazzari)

250 MeV proton cyclotron (ACCEL/Varian)

Closed He system

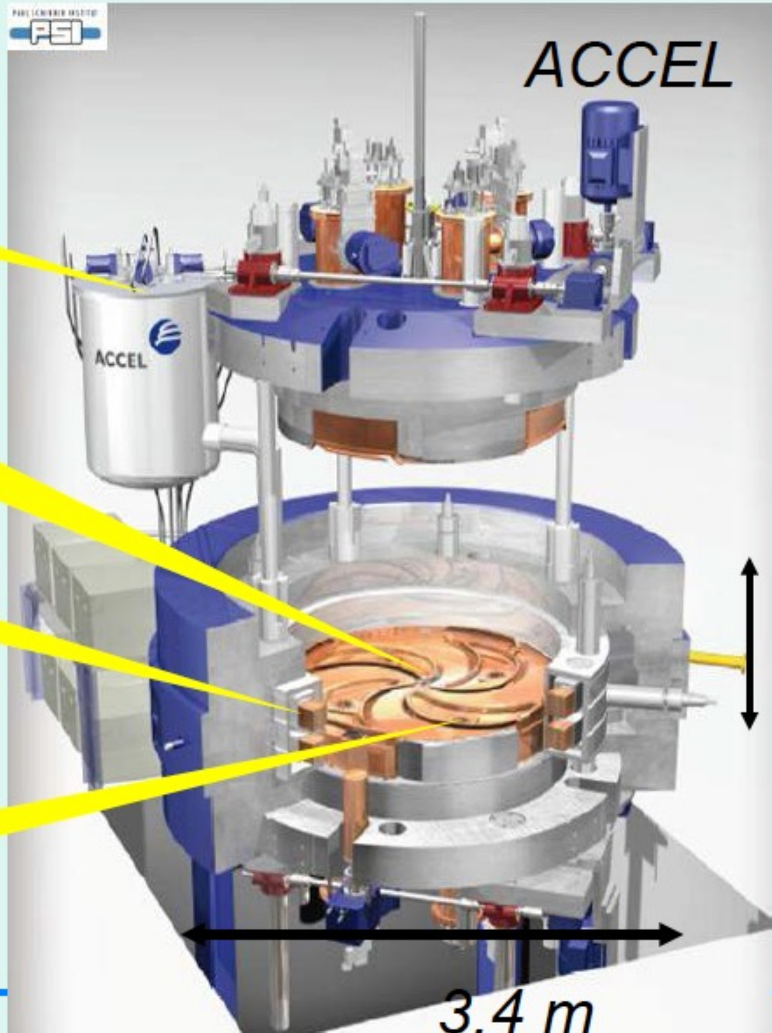
4 x 1.5 W @4K

Proton source

superconducting coils => 2.4 - 3.8 T

4 RF-cavities:

72 MHz ~80 kV



300 kW

90 tons

The Synchrotron concept

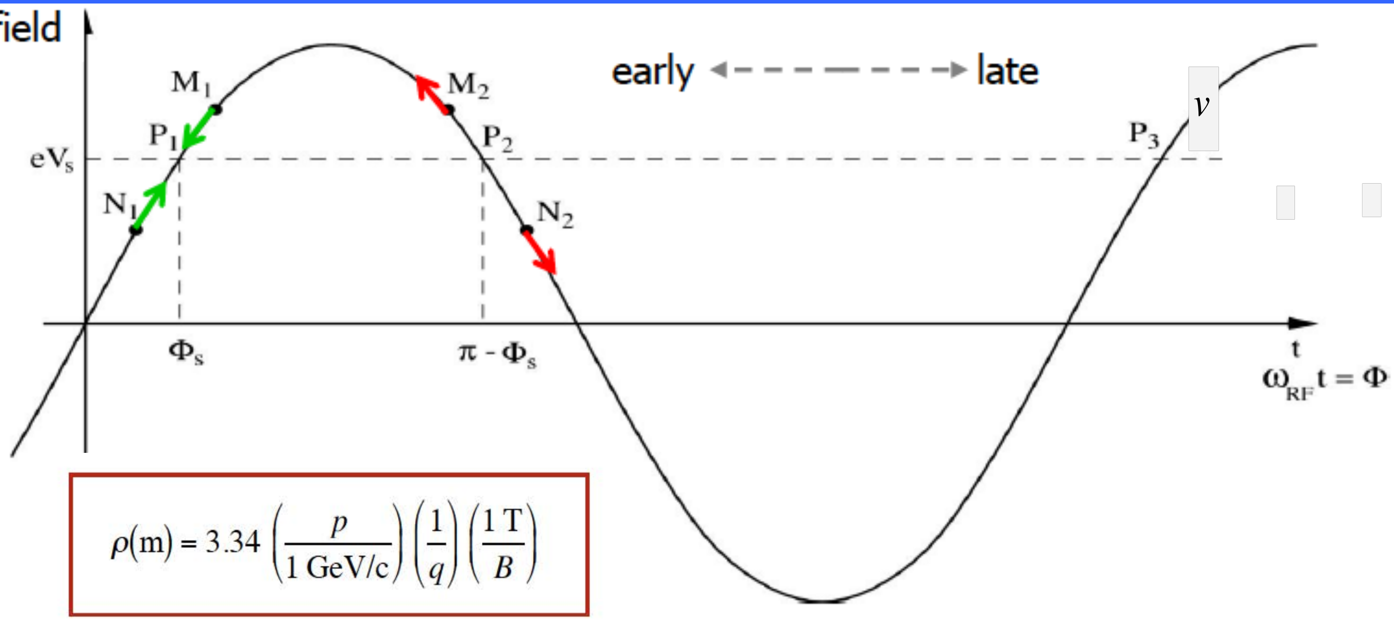
The main principle is to keep separated the bending and focusing devices (magnets of various types) from the ones that accelerates (resonant cavities).

$$\rho = \frac{mvc}{qB} = \frac{pc}{qB}$$

There is main difference from cyclotrons: the particles always ride on the same orbit. Therefore:

- **the cavities field must be synchronous with particle crossing and**
- **the bending magnet field must change in order to keep constant the radius of curvature.**

Phase stability and longitudinal focusing

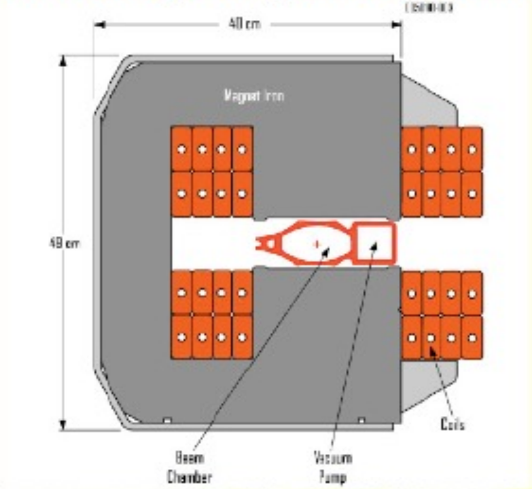


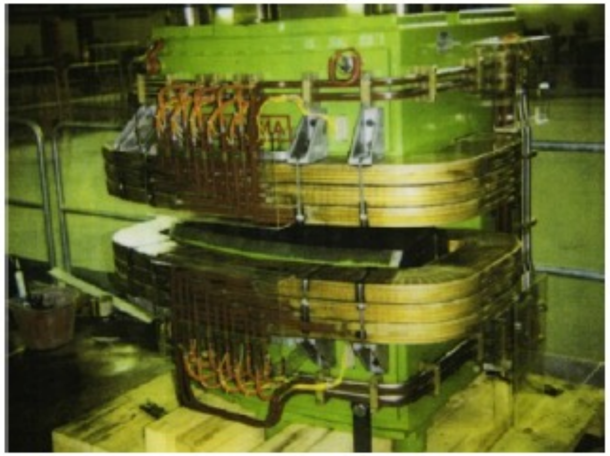
$$\rho(\mathrm{m}) = 3.34 \left(\frac{p}{1\ \mathrm{GeV/c}}\right)\left(\frac{1}{q}\right)\left(\frac{1\ \mathrm{T}}{B}\right)$$

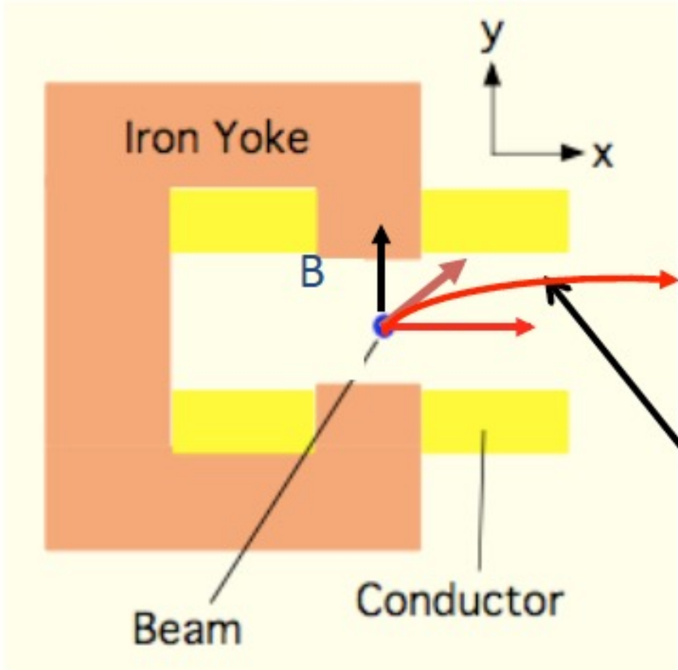
- In a certain energy range, acceleration by RF field results in early arrival of particle at next turn: for stability, this particle should undergo less acceleration
- Operating point P2 is unstable
  - Late particle N2 sees lower acceleration and gets even later
  - Early particle M2 sees higher acceleration and gets even earlier
- Operating point P1 is stable

Dipoli: deflessione

Consentono di curvare la traiettoria delle particelle. Possono essere realizzati con **magneti permanenti** o **elettromagneti** (poli ferro con avvolgimenti percorsi da corrente).







Traiettoria circolare

Raggio di curvatura

$$\rho[m] = \frac{p}{Bq} \cong \frac{W}{cqB}$$

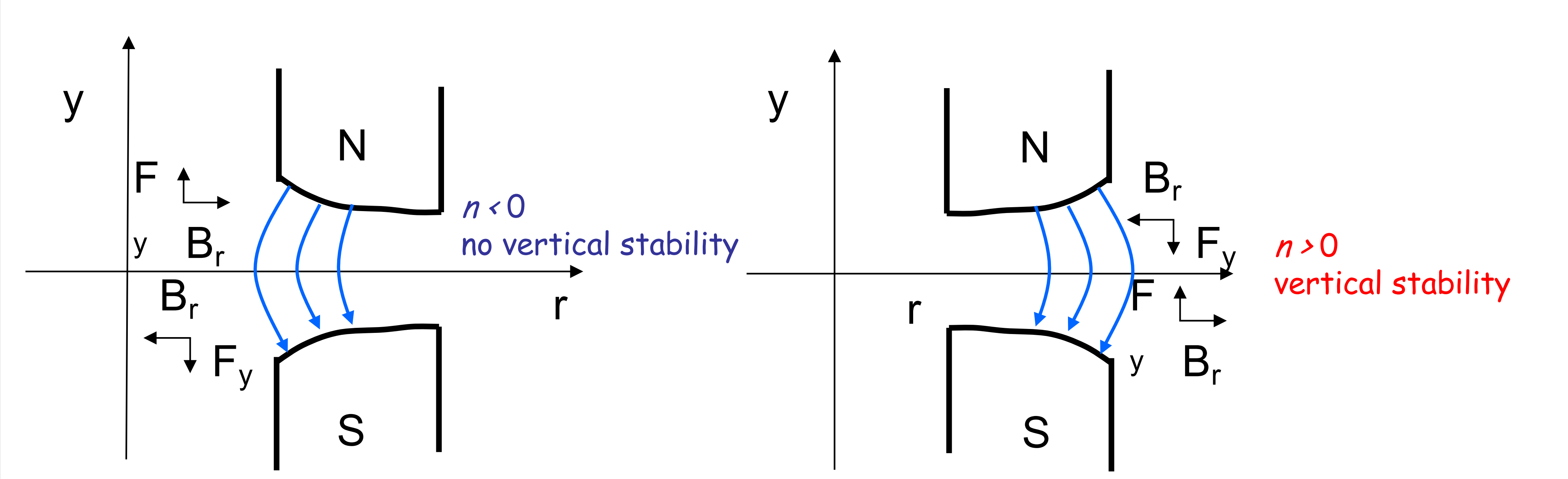
Rigidità magnetica del fascio di particelle è definita come Bρ $B\rho[Tm] = \frac{p}{q} \cong \frac{W}{cq}$

I dipoli elettromagnetici vengono usati per produrre B non oltre 1-2 T.
Per campi magnetici più intensi si ricorre a ***magneti superconduttori***

Per particelle
ultra-relativistich

Weak focusing and transverse stability

- to get vertical stability, the bending field should decrease with ρ, as in cyclotrons,
- to get horizontal stability the decrease of B with ρ should be moderate, so that, for $\rho > \rho_0$, the Lorenz force exceeds the centripetal force.

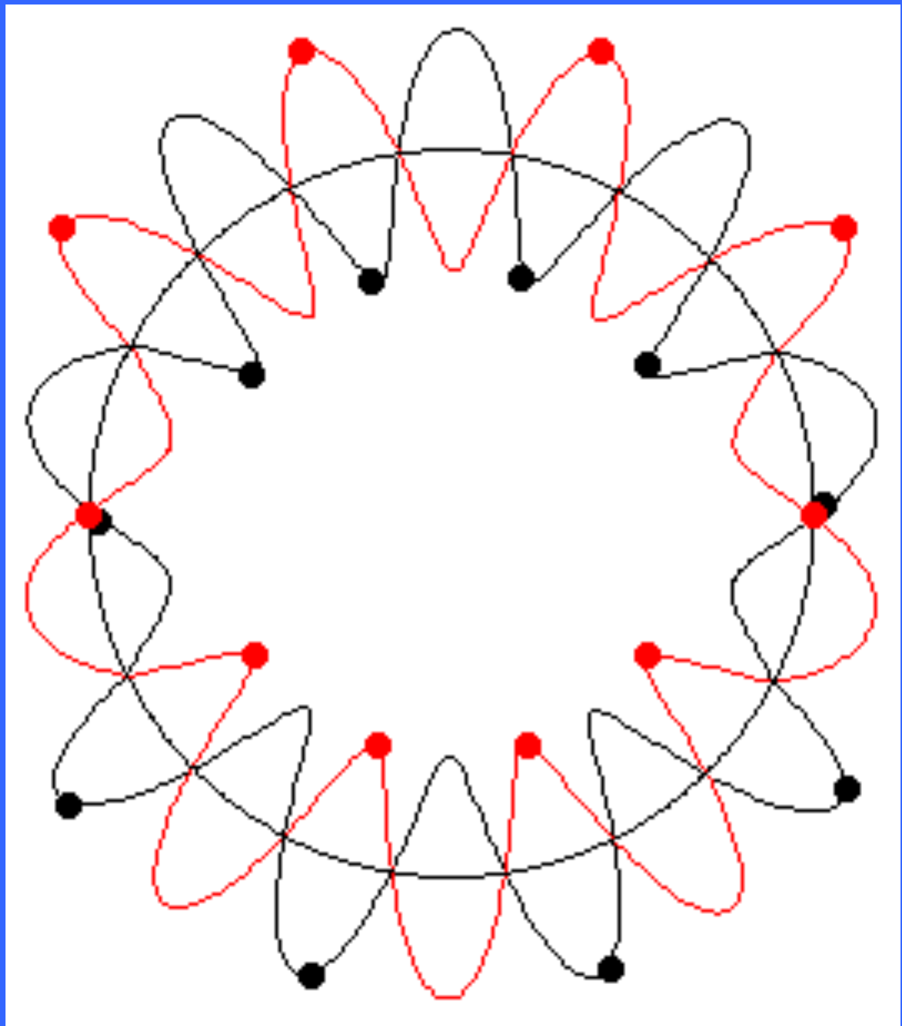


$$B_y = B_{0y} + \frac{\partial B_y}{\partial x} x = B_{0y} \left(1 + \frac{R}{B_{0y}} \frac{\partial B_y}{\partial x} \frac{x}{R} \right)$$

$$n = -\frac{R}{B_{0y}} \frac{\partial B_y}{\partial x}$$

Condizione di Stabilita': 0<n<1

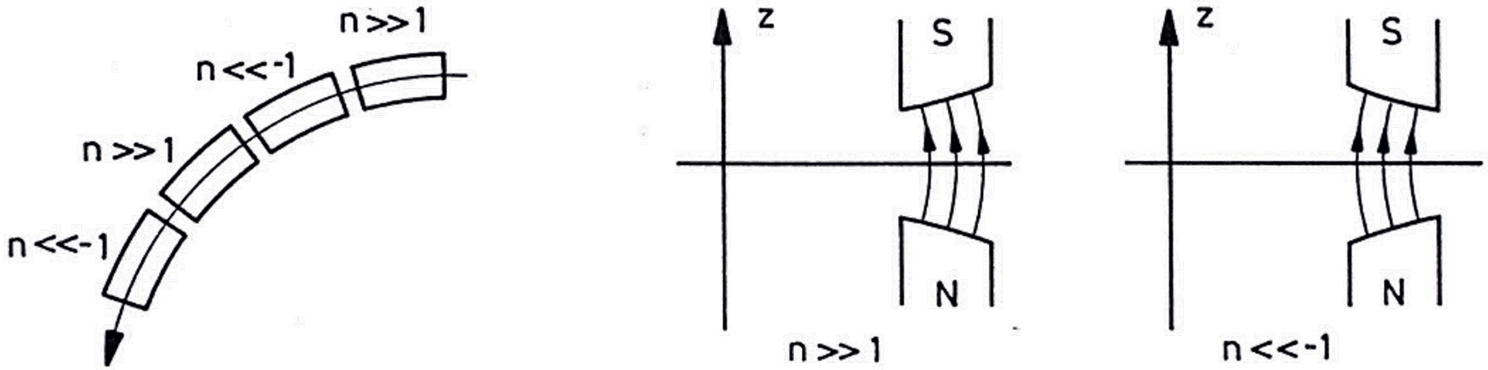
Betatron oscillations and transverse focusing

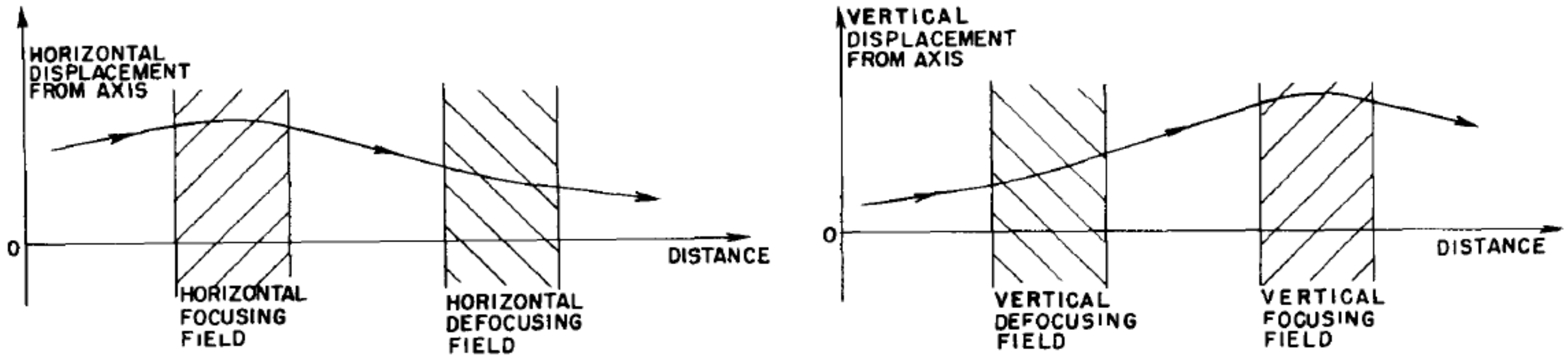


From Weak focusing To Strong Focusing

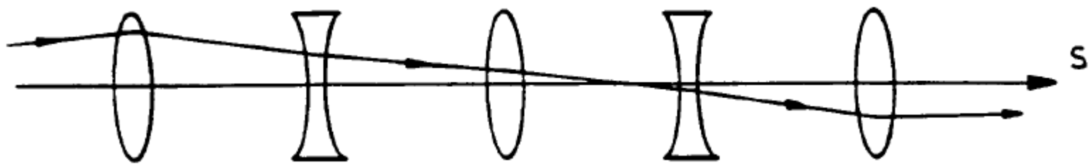
- The principle of weak focusing has one serious drawback: when the trajectory oscillation wavelength is larger than the circumference of the machine one gets large deviations from the orbit if the circumference is large.
- The magnet apertures must be very big.
- The apertures can be drastically reduced if one applies strong focusing (n much larger than 1).
- This is impossible in a machine which has a guide and focusing field independent of the azimuthal angle, since in that case the condition $0 < n < 1$ has to hold.
- It is, however, possible if we split up the machine into a series of magnetic sectors in which in alternating order the magnetic field increases strongly with increasing radius ($n << 1$) or decreases strongly with increasing radius.

Strong focusing





The overall effect is focusing!!



MAGNETIC QUADRUPOLE

Quadrupoles are used to **focalize the beam in the transverse plane**. It is a **4 poles magnet**:

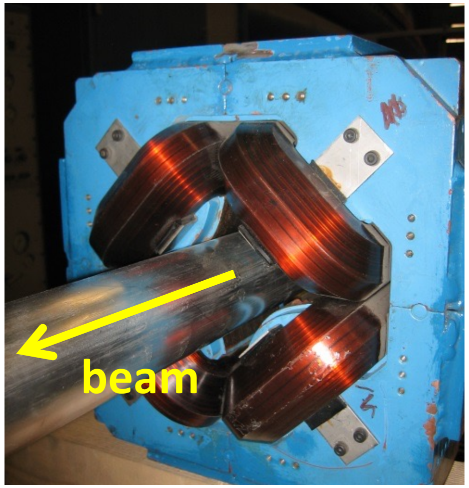
⇒**B=0 in the center of the quadrupole**

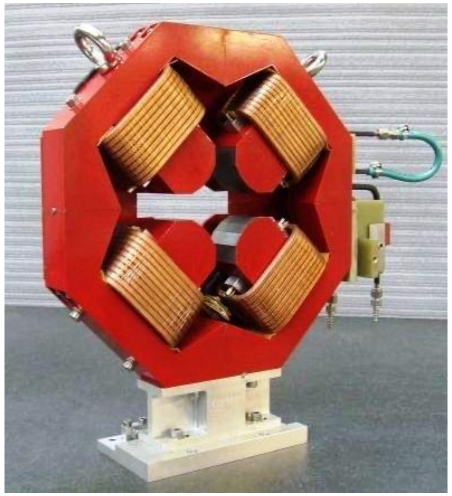
⇒The **B intensity increases linearly** with the off-axis displacement.

⇒If the quadrupole is **focusing in one plane is defocusing in the other plane**

$$\begin{cases} B_x = G \cdot y \\ B_y = G \cdot x \end{cases} \Rightarrow \begin{cases} F_y = qvG \cdot y \\ F_x = -qvG \cdot x \end{cases}$$

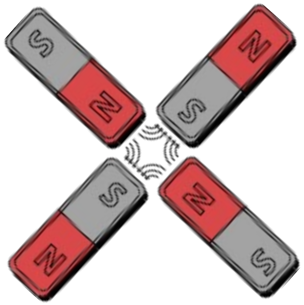
$$G = \text{quadrupole gradient} \left[\frac{T}{m} \right]$$

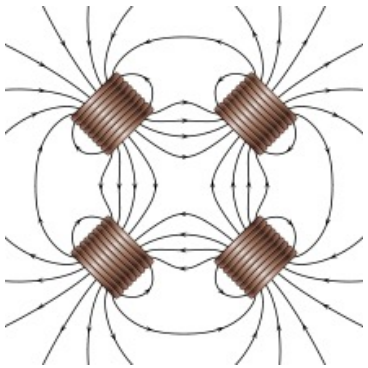


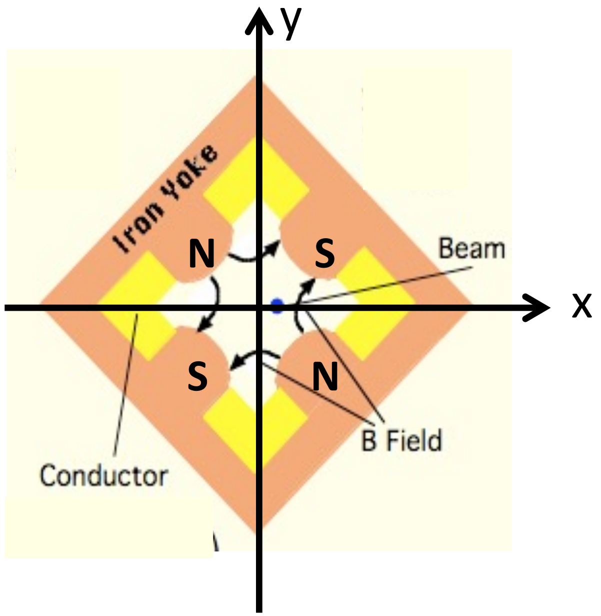


Electromagnetic quadrupoles G <50-100 T/m

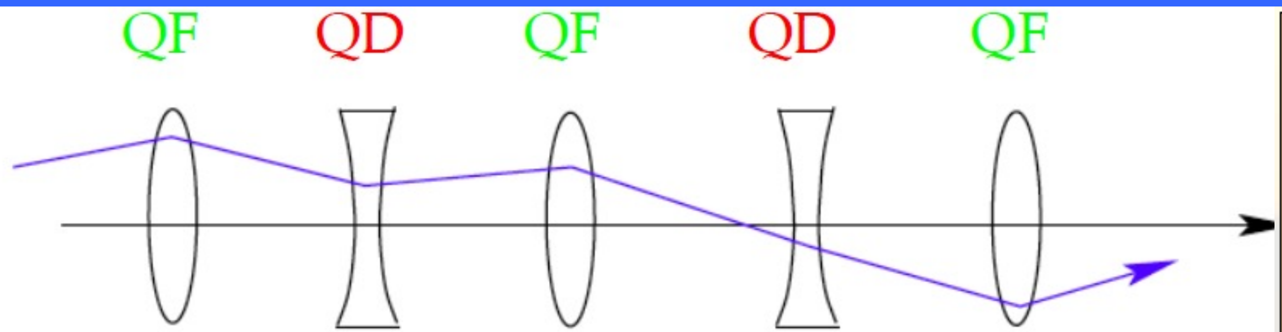
$$\frac{F_B}{F_E} = v \Rightarrow \begin{cases} F_B(1T) = F_E\left(300\frac{MV}{m}\right) @ \beta = 1 \\ F_B(1T) = F_E\left(3\frac{MV}{m}\right) @ \beta = 0.01 \end{cases}$$







Strong focusing with separated function magnets



$$\frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2} - \frac{\Delta}{f_1 f_2}$$

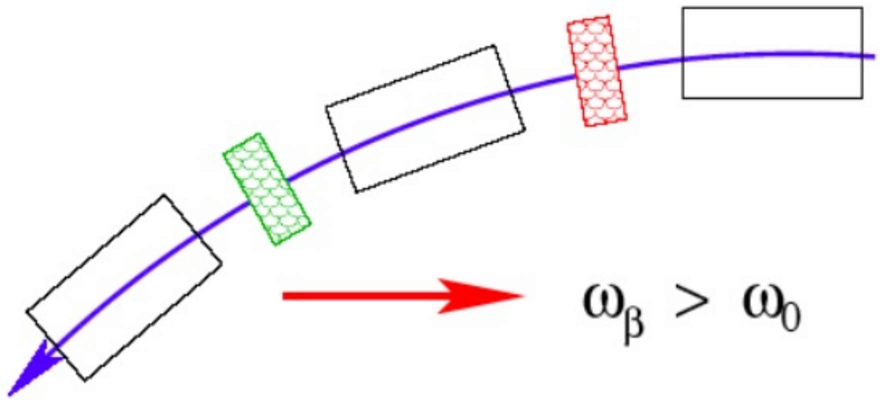
$$if \quad f_1 = -f_2$$

$$and \quad \Delta = |f_1|$$

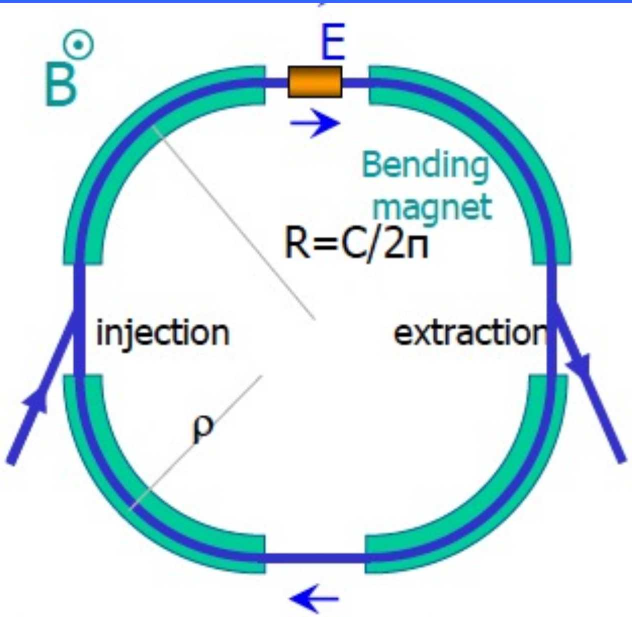
$$\frac{1}{f} = \frac{1}{|f_1|}$$

Idea: **cut the arc sections in**

***focusing* and *defocusing* elements**



3 GeV Cosmotron at BNL
weak focussing, combined function magnets



28 GeV PS at CERN
strong focussing, combined function magnets

400 GeV SPS at CERN
strong focussing, separated function magnets

Maximum energy

$$\Delta E_{\text{beam}} = eU_{\text{max}} \sin \Psi_0 - \Delta E_{\text{loss}}.$$

$$\Delta E_{\text{loss}} \propto E^4$$

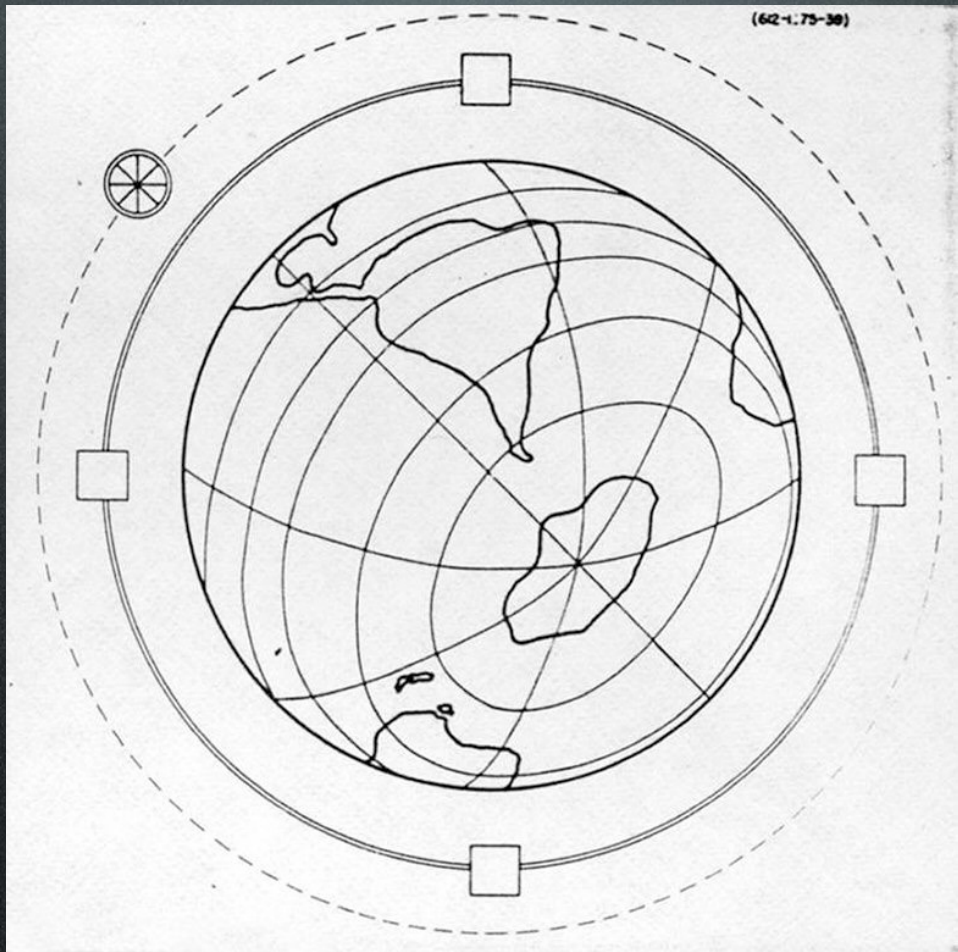
$$\boxed{P_\gamma = \frac{cC_\gamma}{2\pi} \cdot \frac{E^4}{\rho^2}}$$

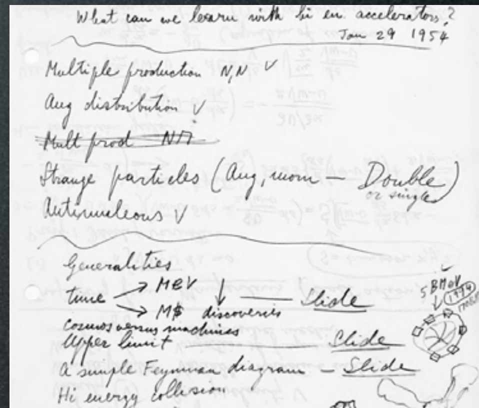
- The energy loss is very important mainly for electrons
- It is not possible to accelerate starting to E=0 MeV
- So a linac is needed in order to fill the ring
- For more stable system another rings is needed (booster)

Fermi's Globatron: ~5000 TeV Proton beam

1954 the ultimate synchrotron

B_{max} 2 Tesla
ρ 8000 km
fixed target
3 TeV c.m.
170 G$
1994

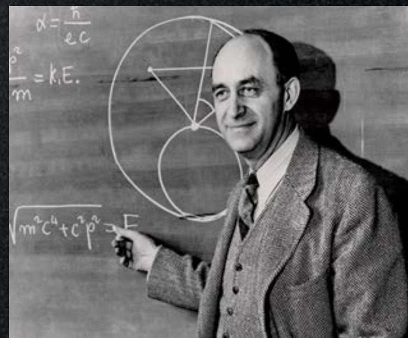



What can we learn with hi en. accelerators?
Jan 29 1954

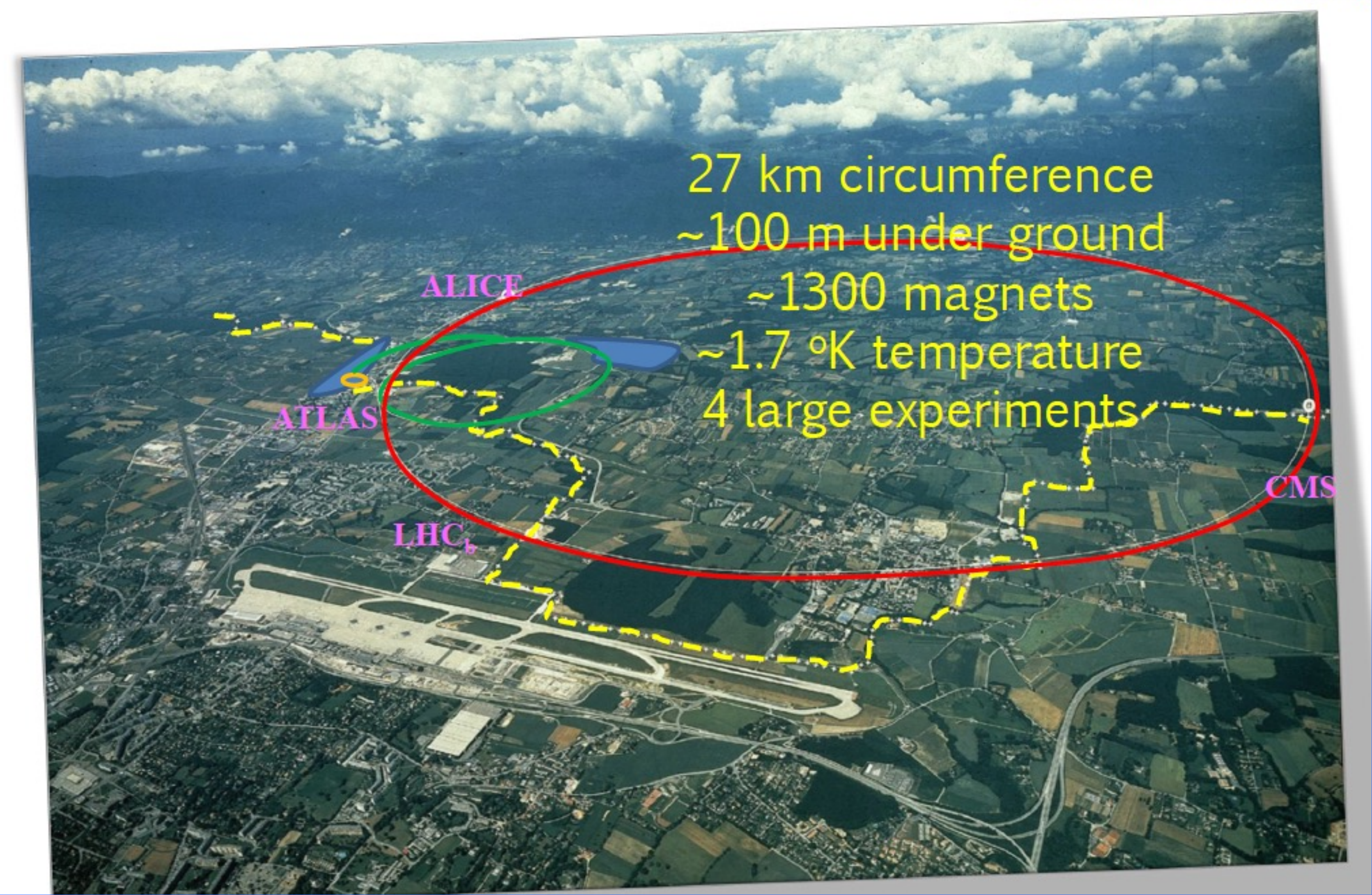
Multiple production N,N ✓
Ang distribution ✓
~~Mult prod NΠ~~
Strange particles (Ang, mom — Double or single)
Antinucleons ✓

Generalities
time → MeV ↓ — Slide
time → M$ discoveries
Cosmos versus machines
Upper limit — Slide
A simple Feynman diagram — Slide
Hi energy collision

5BMeV
1994



LHC few data


27 km circumference
~100 m under ground
~1300 magnets
~1.7 ºK temperature
4 large experiments
ALICE
ATLAS
LHCb
CMS

Hawking: the Solartron

Towards the Planck scale

Without further novel technology, we will eventually need an accelerator as large as Hawking expected.

"The Universe in a Nutshell", by Stephen William Hawking, Bantam, 2001