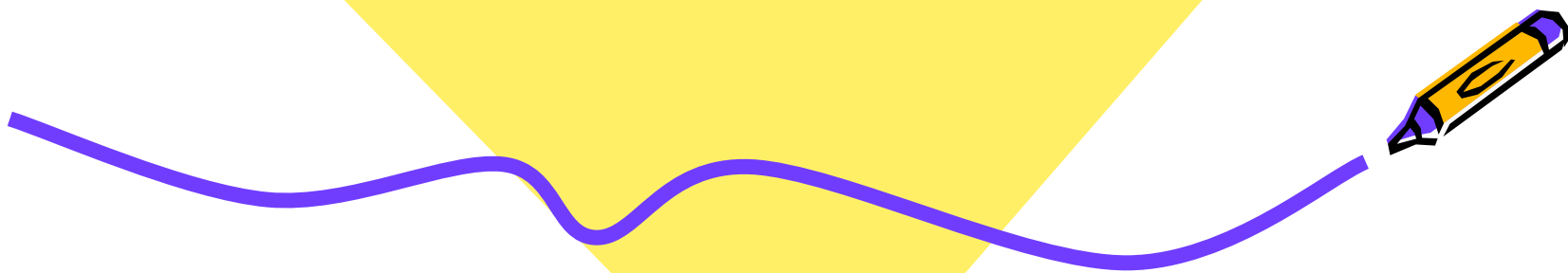




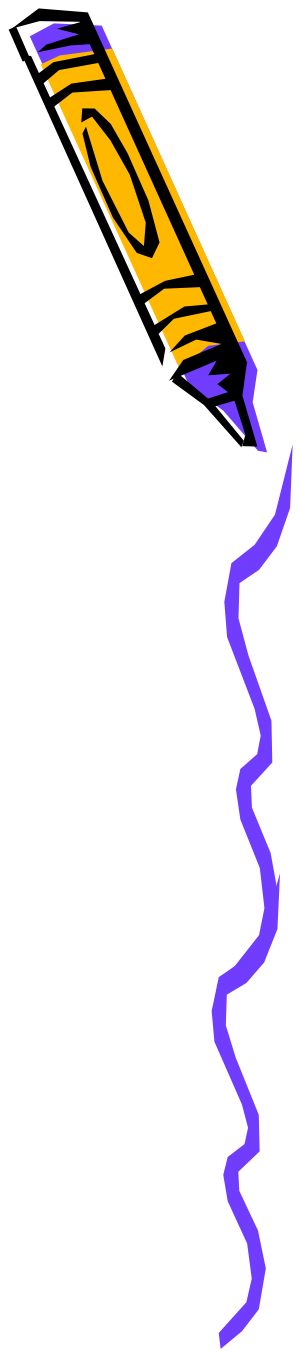
Natural Units



Relativity

Summary

Kahoot!



Relativistic kinematics/dynamics

$$\beta = \frac{v}{c} \Rightarrow v$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

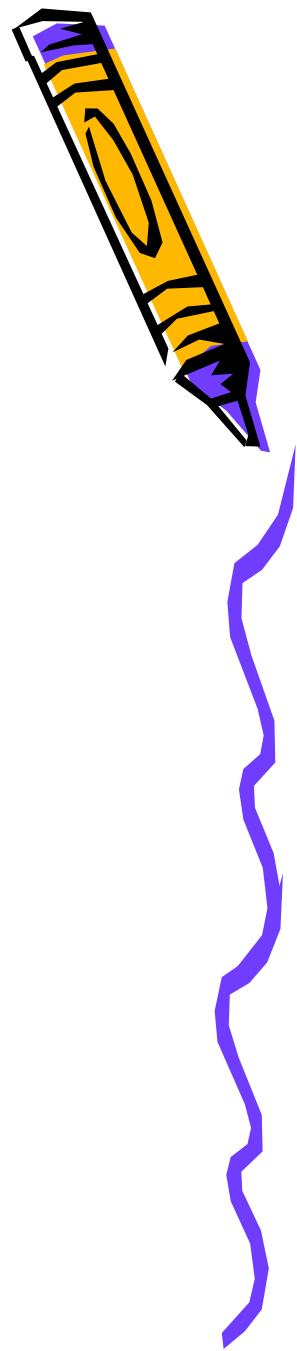
$$E = m\gamma c^2 \Rightarrow m\gamma$$

$$p = m\beta\gamma c \Rightarrow m\beta\gamma$$

$$T = E - m \Rightarrow m(\gamma - 1)$$

$$E = \sqrt{p^2 + m^2}$$

$$\beta \ll 1 \Rightarrow T \approx \frac{p^2}{2m}$$



Natural Units



- In relativistic quantum mechanics (i.e. particle physics), it is customary to express quantities assuming $\hbar=c=1 \rightarrow$ Natural Units (NU) as opposed to International System (IS)
- With this assumption relevant physics quantities can be related to powers of only one, e.g. energy
- To connect between the two systems each quantity needs to be multiplied by the powers of \hbar and c needed to restore the physics quantities (m and n uniquely determined)

$$Q[\text{IS}] = Q[\text{NU}] * \hbar^m * c^n$$

In this system $e^2/4\pi\epsilon_0 = \alpha_{em} = 1/137$
 \rightarrow Heavyside-Lorentz

Kahoot!



Dimensions of \hbar and c



	Dimensions	measurement
\hbar	Momentum*position	
	Energy*time	1.035 10 ⁻³⁴ Js 6.5 10 ⁻¹⁶ eV s 6.5 10 ⁻¹³ MeV ns
c	Position/time	3.0 10 ⁸ m/s 300 km/s 30cm/ns 3.0 10 ¹⁴ fm/ns
$\hbar c$	Energy*position	3.1 10 ⁻²⁶ Jm 200 MeV fm



My favourite approach: use

$$Q[IS]=Q[NU]*(\hbar c)^m*c^n$$

With the units in red

Natural Units: examples



1. An electron has a momentum $p=1\text{MeV}/c$, which is its momentum in IS?
 - MeV is a unit of energy $1\text{MeV}=10^6 \text{ e}[C] \text{ J}\sim 1.6 \cdot 10^{-13} \text{ J}$
 - p in the I.S. should be in kg m/s
 - to convert between the two representations one needs to multiply
 $p = 1 \text{ MeV}/c = 1.6 \cdot 10^{-13} / 3 \cdot 10^8 = 5 \cdot 10^{-22} \text{ kg m/s}$
2. Which is the e.s. energy in NU of an electron at a distance $d=0.5\text{\AA}$ from a carbon nucleus with ($Z=6$). Assume no shielding from other electrons

$$U = Ze^2 / 4\pi\epsilon_0 d = Z\alpha_{em} / d$$

To convert you need to add the correct power of \hbar and c . U is an energy (E), d a length (L), therefore the power of \hbar (m) and c (n) need to satisfy

$$E = \hbar^m c^n / L = (E T)^m (L/T)^n / L$$

$\rightarrow m=n=1 \rightarrow U = Z\hbar c \alpha_{em} / d = 6 \cdot 200 (\text{MeV}/\text{fm}) / 137 / 5 \cdot 10^4 = 1.2 \cdot 10^5 / 137 / 5 = 175 \text{ eV}$
[N.B. binding energy is half the e.s. energy]



Examples



- Find the kinetic energy of an He nucleus with $p=50\text{MeV}/u$
- Find the radius of the orbit of a $T=10\text{MeV}$ proton in $B=0.5\text{T}$ magnetic field

$P=eBR$ with $e=300\text{ MeV}/\text{TM}$

- Find the beta and beta*gamma of an electron accelerated by 2MV



Kahoot!