Natural Units

Relativity

Summary

Kahoot!

Relativistic kinematics/dynamics

$$\beta = \frac{v}{c} \Rightarrow v$$

$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$E = m\gamma c^2 \Rightarrow m\gamma$$

$$p = m\beta\gamma c \Rightarrow m\beta\gamma$$

$$T = E - m \Rightarrow m(\gamma - 1)$$

$$E = \sqrt{p^2 + m^2}$$

$$\beta << 1 \Rightarrow T \approx \frac{p^2}{2m}$$

Natural Units

- In relativistic quantum mechanics (i.e. particle physics), it is customary to express quantities assuming $\hbar=c=1$ ➔ Natural Units (NU) as opposed to International System (IS)
- With this assumption relevant physics quantities can be related to powers of only one, e.g. energy
- To connect between the two systems each quantity needs to be multiplied by the powers of $\hbar$ and c needed to restore the physics quantities (m and n uniquely determined)

$$Q[IS]=Q[NU]*\hbar^m*c^n$$

In this system $e^2/4\pi\varepsilon_0=\alpha_{em}=1/137$
➔ Heavyside-Lorentz

Kahoot!

Dimensions of ħ and c

	Dimensions	measurement
ħ	Momentum*position	
	Energy*time	1.035 10-34 Js 6.5 10^{-16} eV s 6.5 10-13 MeV ns
c	Position/time	3.0 10^{8} m/s 300 km/s 30cm/ns 3.0 10^{14} fm/ns
ħc	Energy*position	3.1 10^{-26} Jm 200 MeV fm

My favourite approach: use

$$Q[IS]=Q[NU]*(\hbar c)^{m}*c^{n}$$

With the units in red

Natural Units: examples

1. An electron has a momentum p=1MeV/c, which is its momentum in IS?
   - MeV is a unit of energy 1MeV=10^6 e[C] J~1.6 10^{-13} J
   - p in the I.S. should be in kg m/s
   - to convert between the two representations one needs to multiply

     p= 1 MeV/c=1.6 10^{-13}/3 10^8= 5 10^{-22} kg m/s
2. Which is the e.s. energy in NU of an electron at a distance d=0.5Å from a carbon nucleus with (Z=6). Assume no shielding from other electrons

$$U=Ze^2/4\pi\varepsilon_0 d=Z\alpha_{em}/d$$

To convert you need to add the correct power of ħ and c. U is an energy(E), d a length (L), therefore the power of ħ (m) and c (n) need to satisfy

$$E=\hbar^m c^n/L=(E\ T)^m(L/T)^n/L$$

$\rightarrow$m=n=1 $\rightarrow$ $U=Z\hbar c\alpha_{em}/d=6*200(MeV/fm)/137/5*10^4=1.2*10^5/137/5=175eV$

[N.B. binding energy is half the e.s. energy]

Examples

- Find the kinetic energy of an He nucleus with p=50MeV/u
- Find the radius of the orbit of a T=10MeV proton in B=0.5T magnetic field

P=eBR with e=300 MeV/TM

- Find the beta and beta*gamma of an electron accelerated by 2MV