

RELIABILITY - SOME FUNDAMENTALS

$N_0 \rightarrow$ NUMBER OF PRODUCTS JUST BORN AT TIME T_0
ALL OF THEM IDENTICAL -

$C \rightarrow$ COMPONENT

$A \rightarrow$ ROOM CONDITIONS

$t \rightarrow$ TIME (INSTANT OF LIFE OR OF OPERATING LIFE)

RELIABILITY $\rightarrow R = f(C, A, t)$

USUALLY SIMPLIFYING

$$R = f(t)$$

$N_f(t) \rightarrow$ NUMBER OF COMPONENTS FAILED AT TIME t

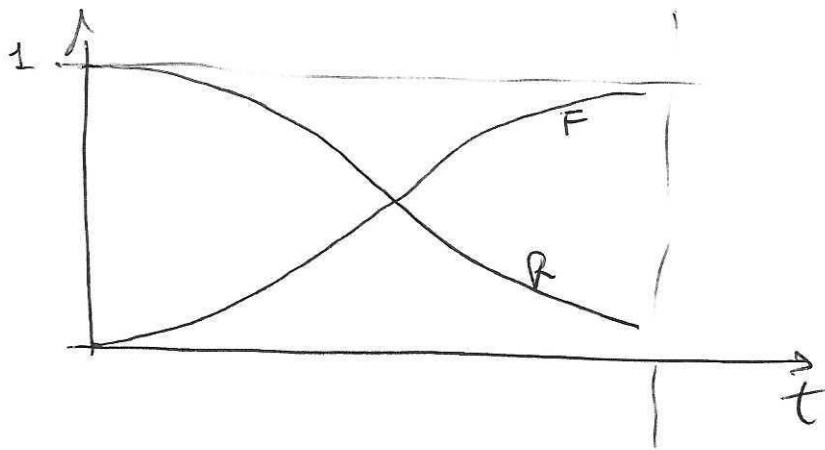
$N_f(t) \rightarrow$ NUMBER OF COMPONENTS STILL OPERATING AT TIME t

$$\Rightarrow R(t) = \frac{N_f(t)}{N_0}$$

UNRELIABILITY $\rightarrow F(t)$

$$\Rightarrow F(t) = \frac{N_f(t)}{N_0}$$

$$\Rightarrow R(t) + F(t) = 1$$



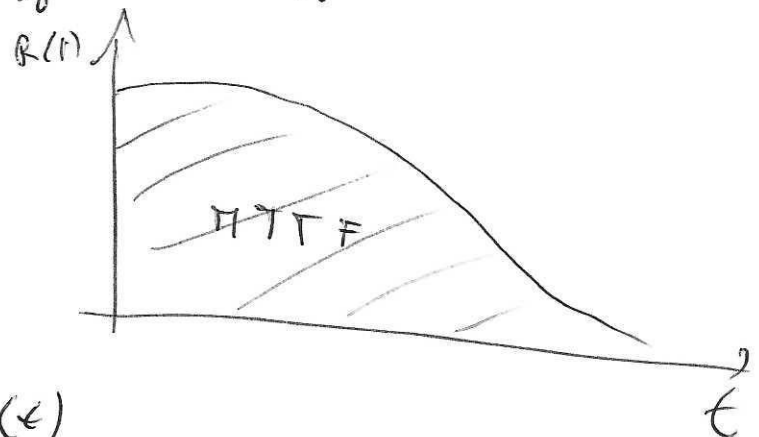
PROBABILITY DENSITY OF UNRELIABILITY $\rightarrow f(t) = \frac{dF}{dt} = \frac{d(1-R)}{dt} = -\frac{dR}{dt}$

MEAN TIME TO FAILURE $\rightarrow \text{MTTF}$

AVERAGE VALUE OF $f(t)$

$$\text{MTTF} = \int_0^{\infty} t \cdot f(t) dt = \int_0^{\infty} t \cdot \left(-\frac{dR}{dt}\right) dt =$$

$$= -\left[t \cdot \frac{dR}{dt} \right]_0^{\infty} + \int_0^{\infty} R(t) dt = \int_0^{\infty} R(t) dt$$



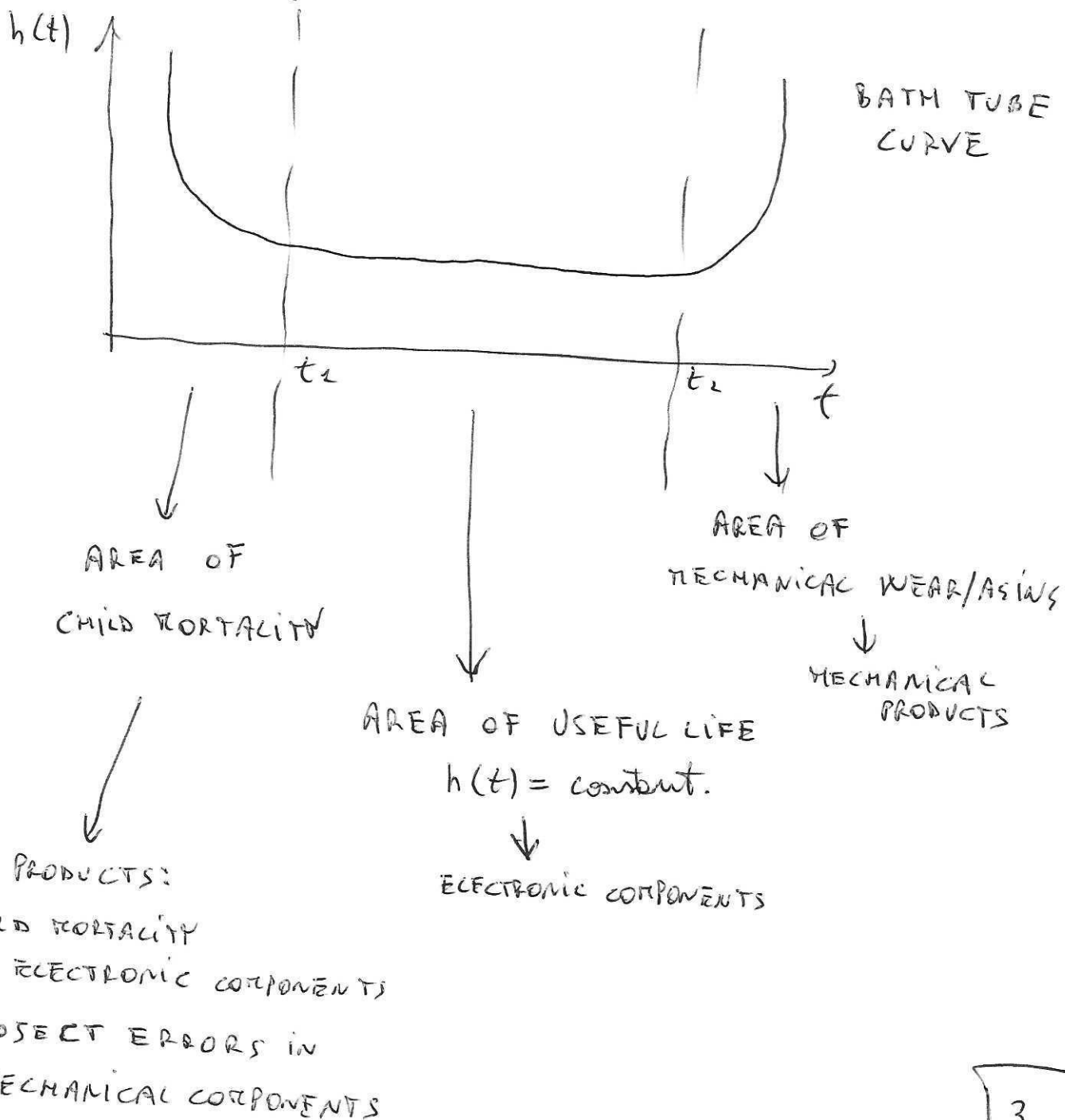
FAILURE RATE $\rightarrow h(t)$

NUMBER OF COMPONENTS FAILING AT TIME t
IN RELATION TO THE NUMBER OF COMPONENTS/PRODUCTS
STILL WORKING AT TIME t

$$h(t) = \frac{1}{N_f(t)} \cdot \frac{dN_f(t)}{dt} = \frac{1}{\underbrace{N_f(t)}_{N_0}} \cdot \underbrace{N_0}_{N_0} \cdot \frac{dN_f(t)}{dt} =$$

$$= \frac{1}{R(t)} \cdot d\left(\frac{N_f(t)}{N_0}\right)/dt = \frac{1}{R(t)} \cdot \frac{dF}{dt} = \frac{f(t)}{R(t)}$$

$$\Rightarrow f(t) = h(t) \cdot R(t)$$



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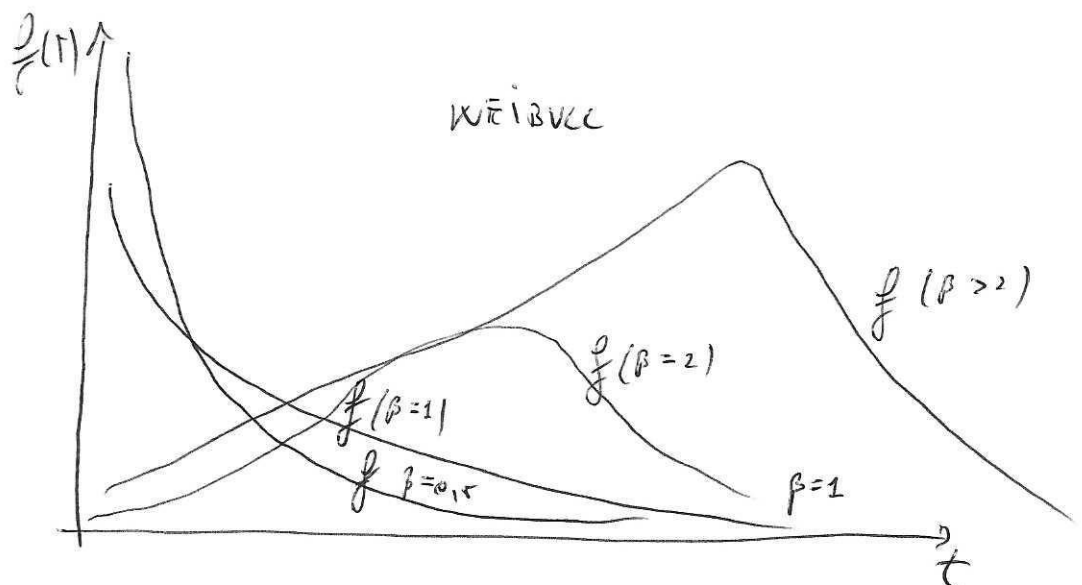
→ USUALLY EMPLOYED FOR THE THREE TYPICAL LIFE AREAS OF PRODUCTS OR THREE TYPICAL CLASSES OF PRODUCTS

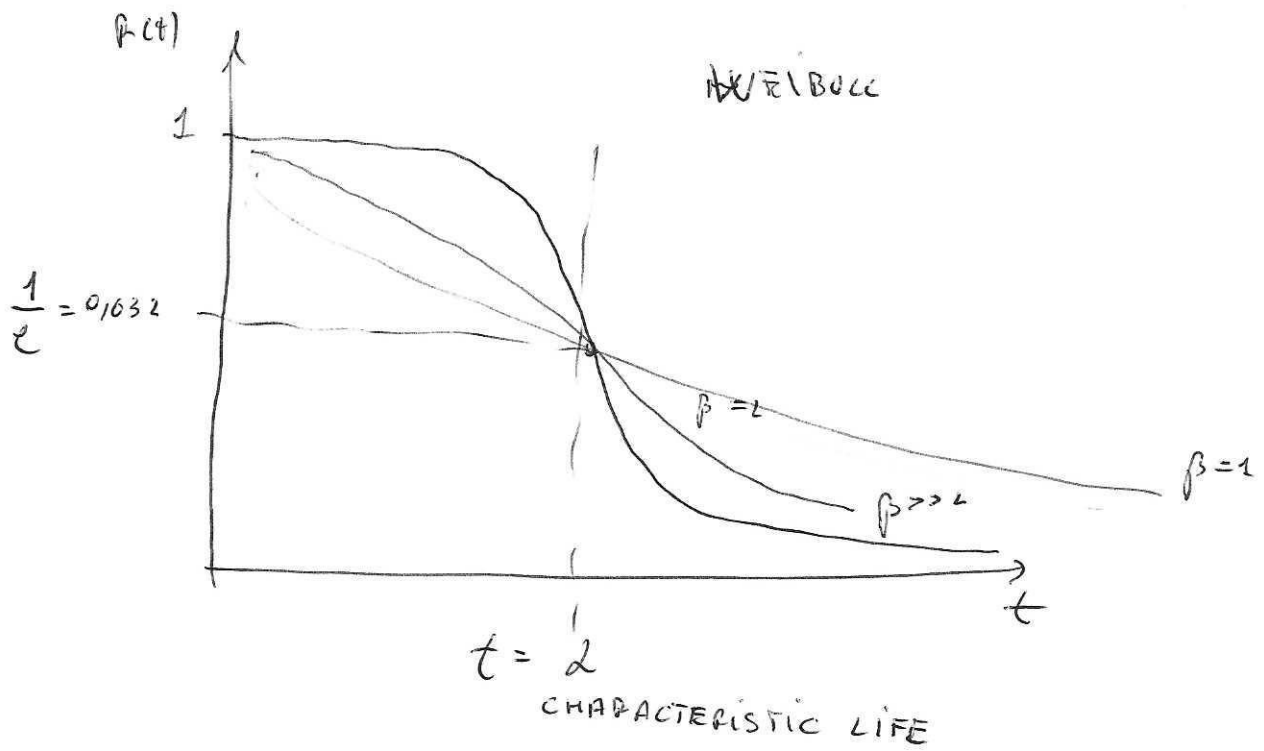
$$\Rightarrow R(t) = e^{-\left(\frac{t}{\lambda}\right)^\beta}$$

$\beta \rightarrow$ SHAPE OF THE GRAPH

$\lambda \rightarrow$ TIME OF CHARACTERISTIC LIFE

$$\begin{aligned} f(t) &= -\frac{dR}{dt} = e^{-\left(\frac{t}{\lambda}\right)^\beta} \cdot \beta \left(\frac{t}{\lambda}\right)^{\beta-1} \cdot \frac{1}{\lambda} = \\ &= \underbrace{e^{-\left(\frac{t}{\lambda}\right)^\beta}}_{R(t)} \cdot \underbrace{\frac{\beta}{\lambda} \left(\frac{t}{\lambda}\right)^{\beta-1}}_{h(t)} \end{aligned}$$





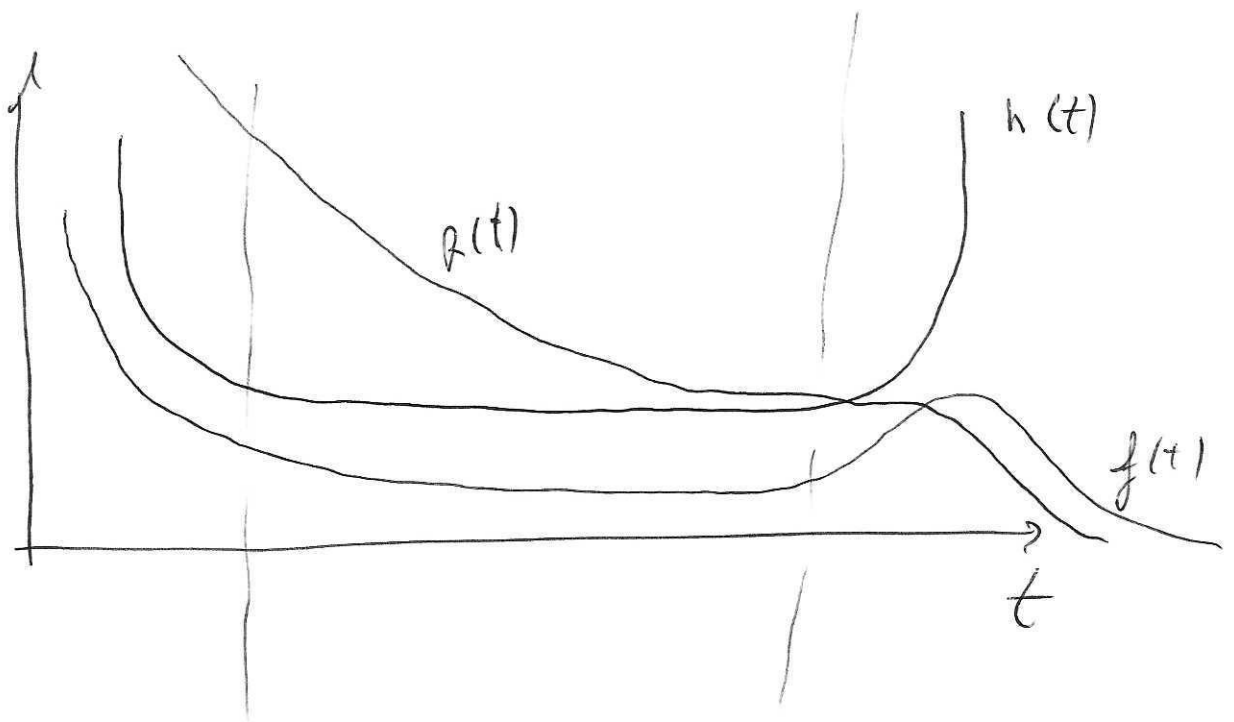
if $h(t) = \text{constant} = \lambda \Rightarrow$

AND $h(t) = \frac{\beta}{\lambda} \left(\frac{t}{\lambda} \right)^{\beta-1} \Rightarrow \beta = 1$

$\lambda = \frac{1}{\lambda}$

$\Rightarrow R(t) = e^{-\lambda t}$

$\Rightarrow \text{MTTF} = \int_0^{\infty} R(t) dt = \int_0^{\infty} e^{-\lambda t} dt = \frac{1}{\lambda}$



DUANE PLOT EXPERIMENT CUMULATIVE MEAN TIME BETWEEN FAILURES

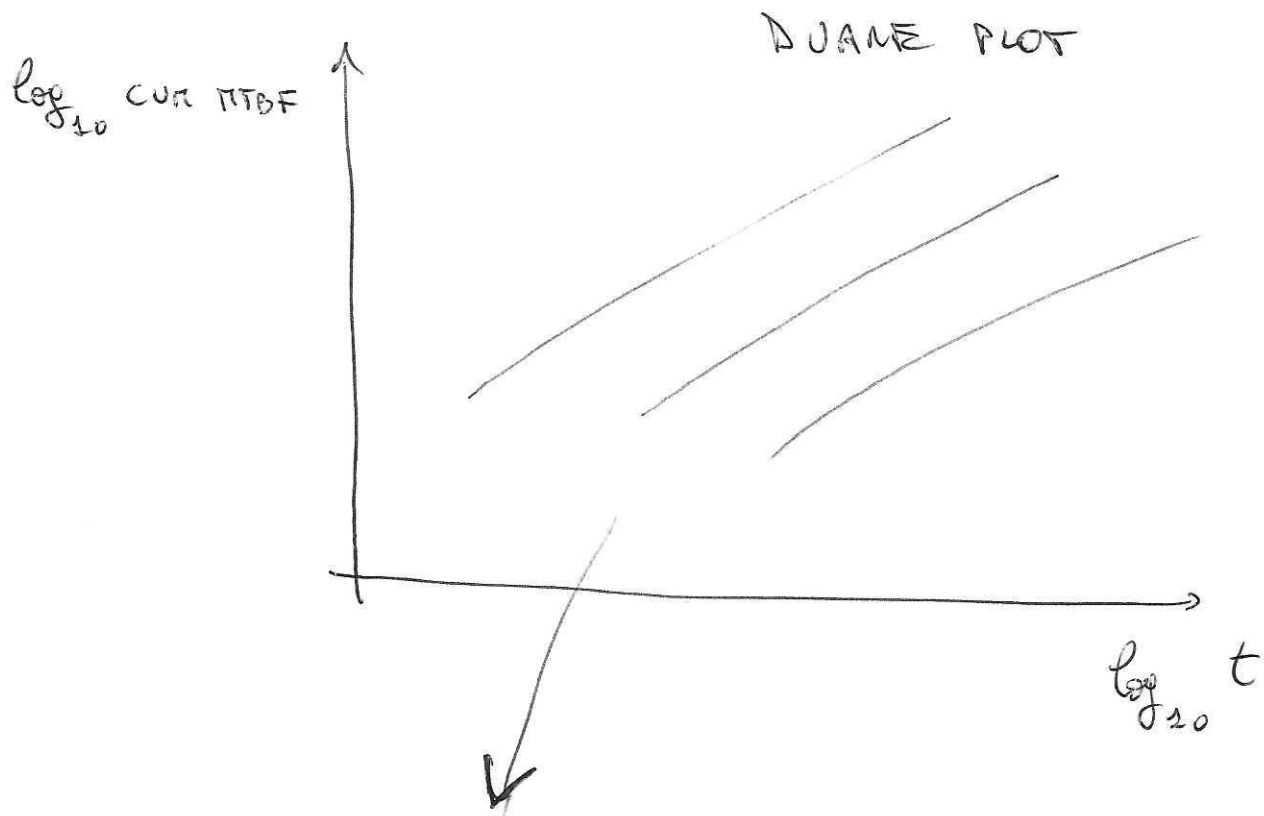
m	t	CUM MTBF
0	0	0
1	33	
2	76	
3	143	
⋮	⋮	⋮

$$\frac{33+0}{1} = 33$$

$$\frac{33+43}{2} = \frac{76}{2} = 38$$

$$\frac{33+43+67}{3} = 47,6$$

NUMERICAL EXPERIMENT



DIFFERENT PRODUCTS FOLLOW THE PLOT

DISCOVERED BY T.T. DUANE IN 1968 -