

# Safety and Maintenance for Industrial Systems

## MULTICRITERIA KNAPSACK APPROACH TO OPTIMIZATION OF INDUSTRIAL SAFETY MEASURES



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# Objective function

$$\text{Max} \sum_{i=1}^n x_i U_i$$

where  $x_i$  ( $x_1, x_2, \dots, x_n$ ) are the decision variables representing the candidate safety measures, with  $x_i = 1$  if the  $i$ -th safety measure is selected, else  $x_i = 0$ , and  $U_i$  being the utility score achieved by selecting the safety measure  $x_i$ .

$$\sum_{i=1}^n x_i C_i \leq C_{\text{MAX}}$$

where  $C_i$  is the cost of safety measure  $x_i$  and  $C_{\text{MAX}}$  is the maximum available budget.

$$U_i = \sum_{j=1}^m w_j S_j \quad \sum_{j=1}^m w_j = 1$$

$$S_j = \frac{P_j}{RP_j} RUV_j$$

Antonio C. Caputo, Pacifico M. Pelagagge, Paolo Salini,  
A multicriteria knapsack approach to economic optimization of industrial safety measures,  
Safety Science,  
Volume 51, Issue 1,  
2013,  
Pages 354-360,

# Objective function

$$S_j = \frac{P_j}{RP_j} RUV_j$$

being  $P_j$  the value of the performance measure possessed by the safety measure respect criterion  $j$  according to a predefined ranking scale,  $RP_j$  is an arbitrary reference value of that performance measure expressed in the same ranking scale, and  $RUV_j$  is the utility value corresponding to the reference value  $RP_j$  chosen for criterion  $j$ . The values of  $RUV_j$  can be assigned following Table 1 on a  $-10$  to  $+10$  scale (Alanne, 2004).

**Table 1**  
Utility values scoring scale.

Score	Utility definition
10	Huge improvement compared with situation before intervention
8	Great improvement...
6	Fair improvement...
4	Moderate improvement...
2	Slight improvement...
0	No improvement compared with situation before intervention
-2	Slight drawback...
-4	Moderate drawback...
-6	Fair drawback...
-8	Great drawback...
-10	Huge drawback compared with situation before intervention

# Objective function: effectiveness

- Effectiveness (Score  $S_1$ )
- Cost (Score  $S_2$ )
- Efficiency (Score  $S_3$ )
- Range (Score  $S_4$ )
- Applicability (Score  $S_5$ )
- Functionality (Score  $S_6$ )

*Effectiveness*, i.e. whether the SM is able to significantly reduce the risk associated to an hazard (by reducing accident probability and/or the magnitude of consequences).

**Table 2**  
Example of risk ranking parameters.

Likelihood ranking(p)	Level	Reference occurrence frequency (yr <sup>-1</sup> )	Severity ranking (M)	Level	Reference loss value
5	Frequent	$>10^{-1}$	5	Catastrophic	>10 fatalities
4	Probable	$10^{-1}-10^{-2}$	4	Critical	1 or more fatalities
3	Occasional	$10^{-2}-10^{-3}$	3	Relevant	Occasional fatality
2	Remote	$10^{-3}-10^{-4}$	2	Marginal	Major injuries
1	Improbable	$<10^{-5}$	1	Negligible	Minor injuries

Then the effectiveness of a safety measure can be assumed to be proportional to the obtainable reduction of risk scores, i.e. to the difference of risk level before and after the adoption of the safety measure.

$$\Delta R = R_{BEFORE} - R_{AFTER} \quad (8)$$

As an example, if the current risk level is characterized by  $p = 3$  and  $M = 4$ , which gives  $R_{BEFORE} = 12$ , and by adopting the candidate safety measure we obtain  $p = 2$  and  $M = 2$ , which gives  $R_{AFTER} = 4$ , we have  $\Delta R = 12 - 4 = 8$ . Thus

$$S_1 = \frac{8}{24} 10 = +3.3 \quad (10)$$

# Objective function: cost

*Cost*, i.e. the capability of the SM of being implemented with low capital and operating expenses while maintaining effectiveness and efficiency.

**Table 3**  
Annual cost of safety measures.

C	Meaning
5	Very high
4	Relevant
3	Average
2	Low
1	Negligible

As an example, let us assume that the candidate safety measure has a cost which is considered “relevant” (i.e. score 4 according to Table 3) and that a measure with a “very high cost” (score 5) would be considered as a “huge drawback” (utility = −10) then the *Cost* score would be

$$S_2 = \frac{4}{5}(-10) = -8$$

# Objective function: efficiency

*Efficiency*, i.e. whether the SM is able to affect the most critical hazards.

**Table 4**  
Efficiency ranking of safety measures.

E	Meaning
5	The SM affects mainly the most critical hazards
4	The SM affects mainly some relevant hazards
3	The SM affects mainly hazards having average criticality
2	The SM affects mainly hazards having low criticality
1	The SM affects mainly hazards having negligible criticality



# Objective function: applicability, functionality

*Applicability*, i.e. an overall judgment about the ease of implementing the safety measure, including possible disruption of productive activities, space requirements, requirements for specific know how, adaptability to existing structures and equipment.

*Functionality*, i.e. an overall judgment about the reliability, usability and acceptance by workers including any negative interaction with productive activities, possibility of expanding/upgrading/updating the safety measure.

Both the *Applicability* and the *Functionality* judgments can be given in a qualitative manner and the corresponding utility score can be assigned by referring directly to Table 1 values. Scores  $S_5$  and  $S_6$  can have both positive or negative values.

# Objective function: range

*Range*, i.e. the capability of the SM to act on a wide range of different hazards simultaneously.

**Table 5**  
Range ranking of safety measures.

RNG	Meaning
5	The SM affects virtually all hazards existing in the facility
4	The SM affects most hazards existing in the facility
3	The SM affects multiple hazards existing in the facility
2	The SM affects only a few hazards existing in the facility
1	The SM is focused on a single hazard existing in the facility



# Safety measure Utility, U

After having assigned scores  $S_1$ – $S_6$  to the safety measures and weights  $w_1$ – $w_6$  to the evaluation criteria, it is straightforward to calculate the overall utility score of the safety measure resorting to (Eq. (4)). As an example let us imagine that a candidate SM has obtained the following scores:  $S_1 = 7$ ,  $S_2 = -4$ ,  $S_3 = 2$ ,  $S_4 = 3$ ,  $S_5 = -5$ ,  $S_6 = 6$ , and that the analyst is mainly concerned with risk reduction, cost and functionality so that he assigns the following weights:  $w_1 = 0.3$ ,  $w_2 = 0.3$ ,  $w_3 = 0.05$ ,  $w_4 = 0.05$ ,  $w_5 = 0.1$ ,  $w_6 = 0.2$ . The utility of the candidate safety measure would be computed as  $U = (0.5 \times 7) + [0.3 \times (-4)] + (0.05 \times 2) + (0.05 \times 3) + [0.1 \times (-5)] + (0.2 \times 6) = 3.5 - 1.2 + 0.1 + 0.15 - 0.5 + 1.2 = 3.25$ .

# Application example

**Table 6**  
Utility values computation table.

	SM1	SM2	SM3	SM4	SM5	SM6	SM7	SM8	SM9	SM10
$\Delta R$	8	4	10	8	20	13	3	6	5	18
Effectiveness ( $S_1$ )	<b>3.33</b>	<b>1.67</b>	<b>4.17</b>	<b>3.33</b>	<b>8.33</b>	<b>5.42</b>	<b>1.25</b>	<b>2.50</b>	<b>2.08</b>	<b>7.50</b>
$C_{INV\ Max}$ (€)	24,000	14,000	10,000	12,000	32,000	26,000	14,000	2400	32,000	22,000
$C_{E\ Max}$ (€/yr)	1000	800	800	1000	2200	500	300	300	2000	1800
$C$ (€/yr)	4905.9	3078.4	2427.5	2952.9	7407.9	4731.4	2578.4	690.6	7207.9	5380.4
Cost ( $S_2$ )	<b>-5.45</b>	<b>-3.42</b>	<b>-2.70</b>	<b>-3.28</b>	<b>-8.23</b>	<b>-5.26</b>	<b>-2.86</b>	<b>-0.77</b>	<b>-8.01</b>	<b>-5.98</b>
$E$	3	3	2	1	2	2	1	3	2	1
Efficiency ( $S_3$ )	<b>6</b>	<b>6</b>	<b>4</b>	<b>2</b>	<b>4</b>	<b>4</b>	<b>2</b>	<b>6</b>	<b>4</b>	<b>2</b>
RNG	3	3	2	1	4	4	3	4	1	2
Range ( $S_4$ )	<b>4.3</b>	<b>4.3</b>	<b>2.9</b>	<b>1.4</b>	<b>5.7</b>	<b>5.7</b>	<b>4.3</b>	<b>5.7</b>	<b>1.4</b>	<b>2.9</b>
Applicability ( $S_5$ )	<b>-2</b>	<b>2</b>	<b>-4</b>	<b>-6</b>	<b>5</b>	<b>-2</b>	<b>-1</b>	<b>8</b>	<b>9</b>	<b>-5</b>
Functionality ( $S_6$ )	<b>9</b>	<b>-2</b>	<b>6</b>	<b>3</b>	<b>-3</b>	<b>4</b>	<b>-5</b>	<b>0</b>	<b>2</b>	<b>8</b>
$U$	<b>1.82</b>	<b>0.33</b>	<b>1.45</b>	<b>-0.08</b>	<b>1.74</b>	<b>1.46</b>	<b>-1.08</b>	<b>2.78</b>	<b>1.49</b>	<b>1.90</b>

**Table 7**  
Sensitivity analysis.

Budget (€/yr)	$U$	SM	$C_{tot}$	$\Delta U/\Delta C$	Budget (€/yr)	$U$	SM	$C_{tot}$	$\Delta U/\Delta C$
4000	4.24	3,8	3118	0	15,000	7.96	1,3,8,10	13,404	0
5000	4.24	3,8	3118	0	16,000	7.96	1,6,8,10	15,708	0
6000	4.61	1,8	5596	0.037	17,000	8.29	1,2,3,8,10	16,483	0.033
7000	4.68	8,10	6071	0.007	18,000	8.29	1,2,3,8,10	16,484	0
8000	5.69	3,6,8	7849.4	0.101	19,000	9.42	1,3,6,8,10	18,136	0.113
9000	6.13	3,8,10	8498.4	0.044	20,000	9.42	1,3,6,8,10	18,137	0
10,000	6.13	3,8,10	8498.4	0	21,000	9.69	1,3,5,8,10	20,812	0.027
11,000	6.5	1,8,10	10,977	0.037	22,000	9.75	1,2,3,6,8,10	21,214	0.006
12,000	7.5	1,8,10	10,978	0.1	23,000	9.75	1,2,3,6,8,10	21,214	0
13,000	7.52	1,3,6,8	12,755	0.002	24,000	10.02	1,2,3,5,8,10	23,891	0.027
14,000	7.96	1,3,8,10	13,404	0.044	25,000	10.02	1,2,3,5,8,10	23,891	0