Lezione Fermi 15

Luciano Maiani, AA 14-15

Materia Oscura 1

Sommario

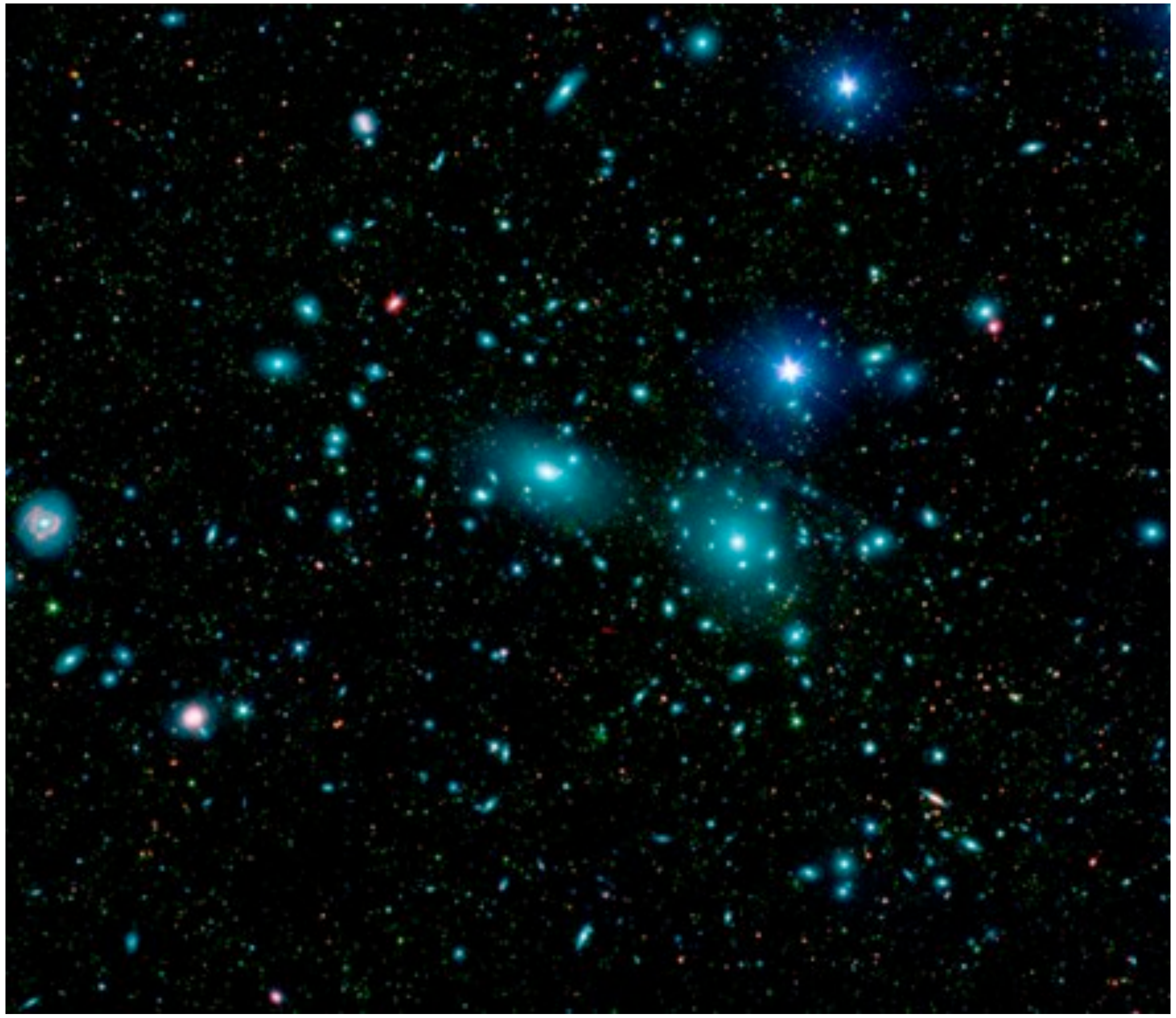
> There are more things in heaven and earth, Horatio,
> Than are dreamt of in your philosophy.
> **- *Hamlet* (1.5.167-8), Hamlet to Horatio**

VAN GOGH. STARRY NIGHT

1. Fritz Zwicky scopre la materia oscura
2. Simulazioni numeriche
3. Curve di rotazione
4. MACHOs
5. Modified Newtonian Dynamics (MOND)
6. Gravitational lensing e materia oscura
7. Bullet Galaxy

1. Fritz Zwicky scopre la Materia Oscura nel Coma Cluster

Fritz Zwicky
1898 Varna, Bulgaria
1974 Pasadena, California, USA
Residenza: USA
Cittadinanza: Svizzera



L'immagine combina dati dallo Spitzer Space Telescope con lo Sloan Digital Sky Survey, e mostra molte delle migliaia di galassie che compongono il Coma cluster.

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All'inizio degli anni '30, Zwicky mise in risalto un rapprto anomalo massa/luminosita' nell'ammasso di galassie nella costellazione Coma, partendo dall determinazione della massa con il teorema del Viriale. Zwicky lo interpreto' come dovuta alla presenza di una "materia oscura", cioe' non luminosa, in aggiunta alla usuale materia barionica che costituisce le stelle e i gas.

Fritz Swicky, una personalita’ di grande rilievo

THE FATHER OF DARK MATTER—AND MORE

Fritz Zwicky. © AIP Emilio Segrè Visual Archives, Physics Today Collection.

Zwicky's observations of supernovae in distant galaxies laid the foundation of his theoretical work. As he detected supernovae in ever-more distant galaxies, he realized that most galaxies combined in clusters. Careful measurements of the light from clusters led him to suggest the existence of dark matt

Zwicky measured the total light output of all the cluster's galaxies, which contain about a trillion stars altogether. When he compared the ratio of the total light output to the mass of the Coma Cluster with a similar ratio for the nearby Kapteyn stellar system, he found the light output per unit mass for the cluster fell short of that from a single Kapteyn star by a factor of over 100. He reasoned that the Coma Cluster must contain a large amount of matter not accounted for by the light of the stars. He called it "dark matter."

Il Teorema del Viriale applicato ai sistemi autogravitanti

- In un sistema autogravitante, l'energia totale conservata e' la somma di energia cinetica e energia potenziale: E= T + V,

$$T = \frac{1}{2}\sum_k m_k v_k^2;\ V = -\sum_k \sum_{j<k} G\frac{m_k m_j}{|r_k - r_j|} = \sum_k \sum_{j<k} V_{kj}$$

- per un sistema legato, E<0 e ci possiamo aspettare che |E| sia dell'ordine di ciascuno dei due termini dell'equazione, T e V
- Il teorema del viriale da' una forma matematica a questa considerazione, richiedendo che:

$$2 < T >_\tau = - < V >_\tau$$

  dove $<X>_\tau$ rappresenta la media temporale di X, presa su un tempo τ che sia lungo rispetto ai tempi caratteristici delle orbite del sistema.
- ***Le velocita' si misurano con l'effetto Doppler e, per galassie a distanza nota, le distanze relative si deducono dalle separazioni angolari***
- ***per un cluster molto ricco, ogni galassia del cluster puo' essere considerata come la stessa galassia vista a tempi diversi, quindi la media temporale si puo' sostituire con la media sul cluster***
- ***assumendo che le galassie abbiano masse circa eguali, possiamo ottenere il valore della massa gravitazionale dell'ammasso dal teorema del Viriale, misurare la luminosita' di ciascuna galassia ed ottenere infine il rapporto M/L***
- Zwicky ottenne dei valori di M/L fino a 100 volte $M_\odot/L_\odot$ che sarebbe il valore previsto se ci fossero solo stelle.

il Teorema del Viriale (Rudolf Clausius, 1870)

The theorem was later utilized, popularized, generalized and further developed by James Clerk Maxwell, Lord Rayleigh, Henri Poincaré, Subrahmanyan Chandrasekhar, Enrico Fermi, Paul Ledoux and Eugene Parker. Fritz Zwicky was the first to use the virial theorem to deduce the existence of unseen matter, which is now called dark matter (Wikipedia).

Definiamo:

$$\text{Momento d'inerzia}: \ I = \frac{1}{2}\sum_k m_k \mathbf{r}_k^2;$$

$$\text{e una quantita' ausiliaria}: \ G = \sum_k \mathbf{p}_k \cdot \mathbf{r}_k = \sum_k m_k \mathbf{v}_k \cdot \mathbf{r}_k$$

Il “viriale”

e troviamo:

$$\frac{1}{2}\frac{dI}{dt} = \frac{d}{dt}\frac{1}{2}\sum_k m_k \mathbf{r}_k^2 = \sum_k m_k \mathbf{v}_k \cdot \mathbf{r}_k = G$$

$$\frac{dG}{dt} = \sum_k \mathbf{p}_k \cdot \mathbf{v}_k + \sum_k \mathbf{r}_k \cdot \frac{d\mathbf{p}_k}{dt} = 2T + \sum_k \mathbf{r}_k \cdot \mathbf{F}_k$$

D’altro canto

$$\sum_k \mathbf{r}_k \cdot \mathbf{F}_k = \sum_k \sum_{j \neq k} \mathbf{r}_k \cdot \mathbf{F}_{kj} = \sum_k \sum_{j<k} (\mathbf{r}_k - \mathbf{r}_j) \cdot \mathbf{F}_{kj} \quad (\text{dato che}: \ \mathbf{F}_{kj} = -\mathbf{F}_{jk})$$

$$(\mathbf{r}_k - \mathbf{r}_j) \cdot \mathbf{F}_{kj} = -(\mathbf{r}_k - \mathbf{r}_j) \cdot \frac{\partial}{\partial \mathbf{r}_k} V_{kj} = -r_{kj} V'_{kj} = +V_{kj} \quad (r_{kj} = |\mathbf{r}_k - \mathbf{r}_j|)$$

$$\text{quindi}: \ \sum_k \mathbf{r}_k \cdot \mathbf{F}_k = \sum_k \sum_{j<k} V_{kj} = +V$$

per cui:

$$\frac{dG(t)}{dt} = 2T(t) + V(t); \ \frac{1}{\tau}\int_0^\tau \frac{dG(t)}{dt} dt = 2 < T >_\tau + < V >_\tau$$

In un sistema legato, tutte le variabili, incluso G, variano in un intervalli finito, quindi:

$$\lim_{\tau \to \infty} < \frac{dG}{dt} >_\tau = \lim_{\tau \to \infty} \frac{G(\tau) - G(0)}{\tau} = \lim_{\tau \to \infty} 2 < T >_\tau + < V >_\tau = 0$$

QED

2. Simulazioni numeriche

Zwicky's measurements took place just after astronomers had realized that galaxies are very large groups of stars. It took some time for dark matter to become the subject of active research it is today. When Zwicky first observed the Coma Cluster, tests of Einstein's theory were just starting, the first cosmological measurements were taking place, and nuclear physicists were only beginning to develop the theories that would explain the Big Bang and supernovae. Astronomers did not immediately begin to worry about "the dark matter problem."

By the early 1970s, technology, astronomy, and particle physics had advanced enough that the dark matter problem seemed more tractable. General relativity and nuclear physics had come together in the Big Bang theory of the early universe, and the detection of microwave photons from the time when the first atoms formed from free electrons and protons had put the theory on a solid footing. Larger telescopes and more precise and more sensitive light detectors made astronomical measurements quicker and better. Just as important, ***the emergence of affordable mini-computers allowed physics and astronomy departments to purchase their own high- performance computers for dedicated astronomical calculations***. Every advance set the scene for a comprehensive study of dark matter, and two very important studies of dark matter soon appeared.

Dark matter appears in galactic simulations

In 1973, Princeton University astronomers Jeremiah Ostriker and James Peebles used numerical simulation to study how galaxies evolve. Applying a technique called ***N-body simulation***, they programmed 300 mass points into their computer to represent groups of stars in a galaxy rotating about a central point. Their simulated galaxy had more mass points, or stars, toward the center and fewer toward the edge. The simulation started by computing the gravitational force between each pair of mass points from Newton's law and working out how the mass points would move in a small interval of time. By repeating this calculation many times, Ostriker and Peebles were able to track the motion of all the mass points in the galaxy over a long period of time.

Figure 5: James Peebles (left) and Jeremiah Ostriker (right) found evidence for dark matter in their computer simulations.
Source: © AIP, Physics Today Collection and Tenn Collection.

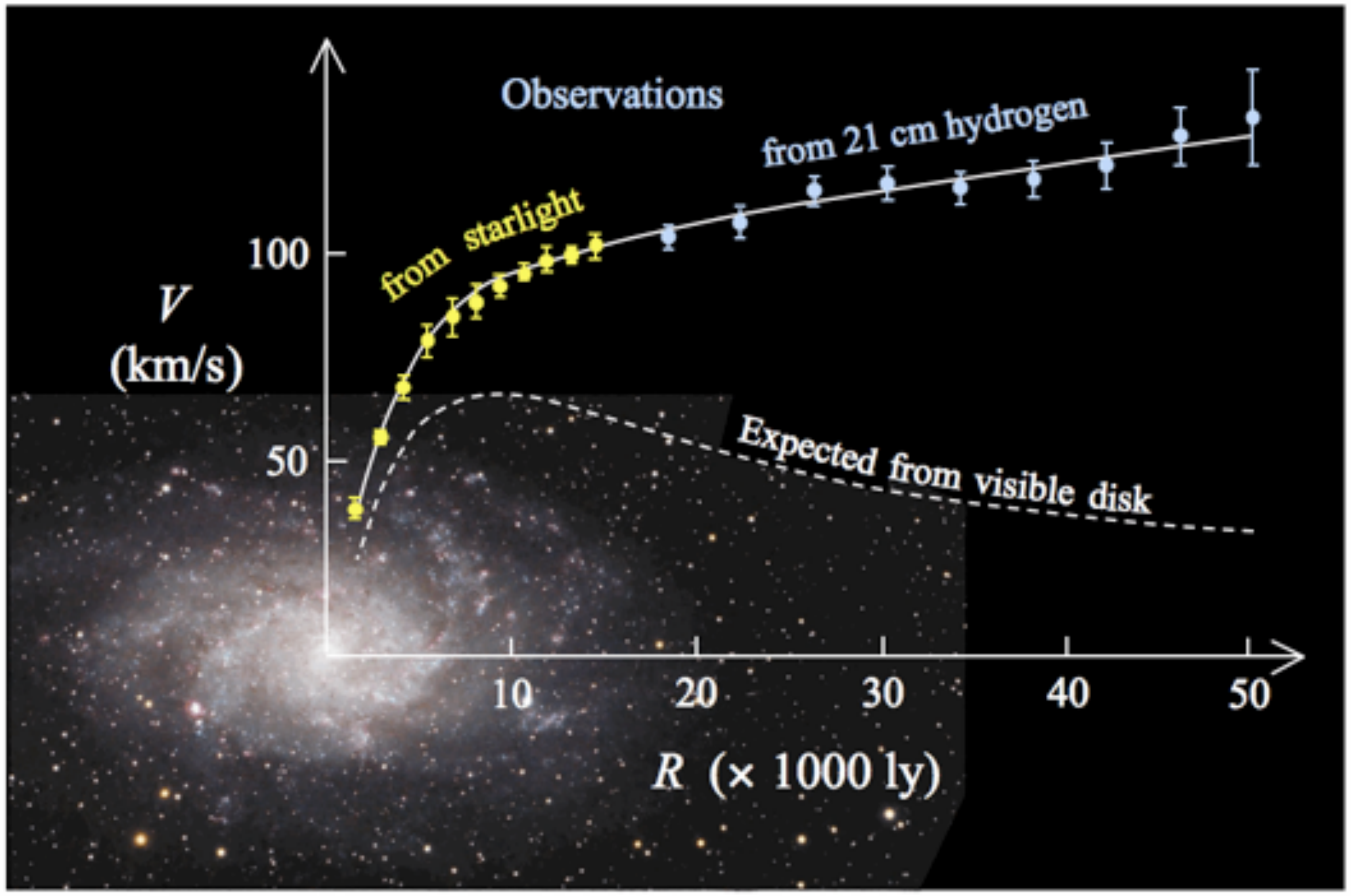
For a galaxy the size of the Milky Way (4x1020 meters), a mass point about halfway out the edge moves at about 200 kilometers per second and orbits the center in about 50 million years. Ostriker and Peebles found that in a time less than an orbital period, most of the mass points would collapse to a bar-shaped, dense concentration close to the center of the galaxy with only a few mass points at larger radii. This looked nothing like the elegant spiral or elliptical shapes we are used to seeing. ***However, if they added a static, uniform distribution of mass three to 10 times the size of the total mass of the mass points, they found a more recognizable structure would emerge.*** Ostriker and Peebles had solid numerical evidence that dark matter was necessary to form the types of galaxies we observe in our universe.

3. Curve di Rotazione

- la 3a Legge di Keplero afferma che i quadrati dei tempi di rivoluzione dei pianeti stanno tra loro come i cubi dei valori medi della distanza dal Sole
- per orbite circolari, T= 2πR/v, quindi

$$Cost = \frac{T^2}{R^3} = \frac{4\pi^2 R^2}{v^2 R^3} \rightarrow v = \frac{A}{\sqrt{R}}$$

- la previsione vale, con buona approssimazione, anche se la massa che attrae il corpo non e' puntiforme, purche' sia sfericamente simmetrica e contenuta all'interno dell'orbita
- Negli anni '70, Vera C. Rubin, studiando le curve di rotazione delle galassie scopri' che la velocita' delle nubi di gas che orbitano attorno alla galassia non decresce quando la nube e' fuori della parte luminosa, ma si mantiene circa costante fino a distanze di diverse volte il raggio della parte luminosa



E' la ***conferma della dark matter*** (ripetuta ormai in moltissimi casi), se manteniamo l'ipotesi che le leggi di Newton siano valide su scala galattica

Vera Cooper Rubin

Vera Cooper Rubin at the Lowell Observatory.
Kent Ford has his back to us. © Bob Rubin.

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Vera Rubin, with DTM image tube spectrograph attached to the Kitt Peak 84-inch telescope, 1970.

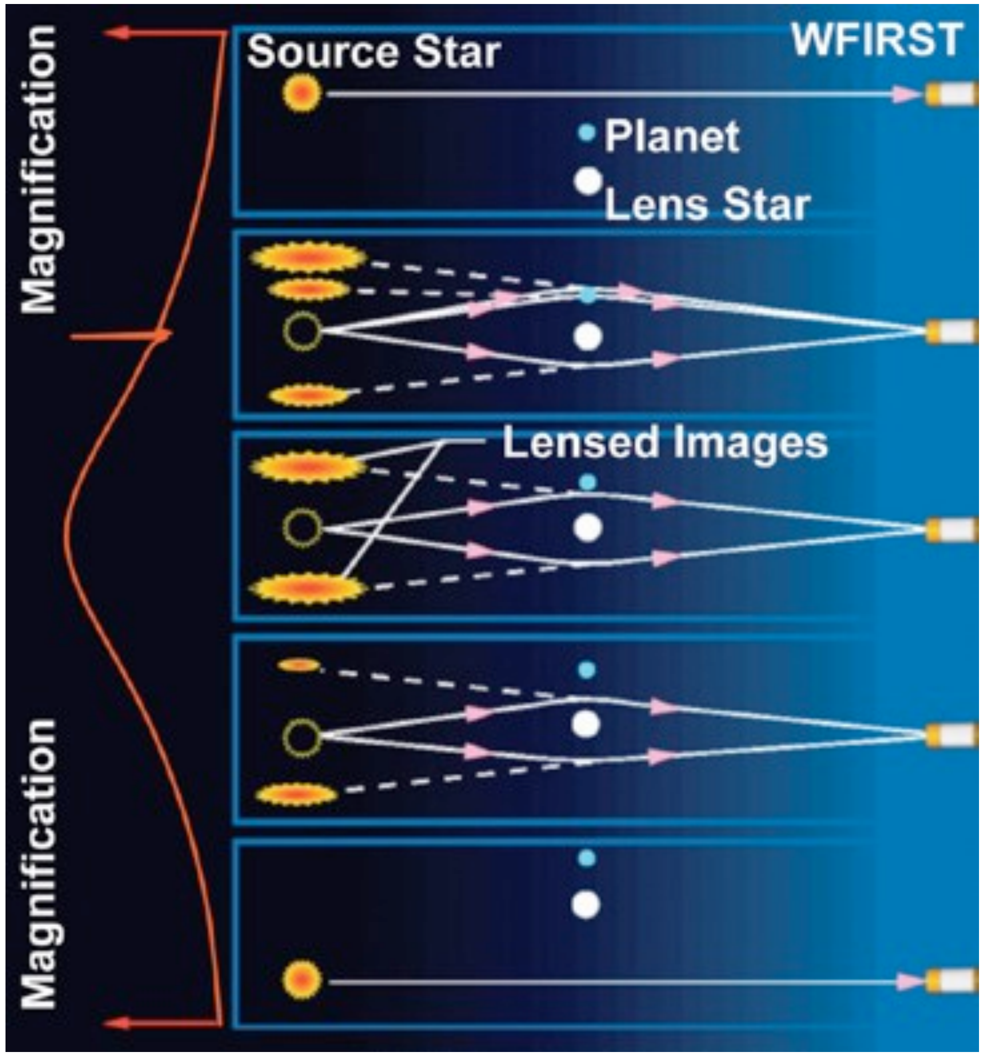
This is an image of the Andromeda galaxy (M31), the companion spiral to our own, copied from the Palomar Sky Survey. The measured optical velocities from ionized gas clouds are indicated as open and filled circles. Velocities from neutral hydrogen radio observations are shown as filled triangles. Note that the velocities remain high far beyond the optical disk.

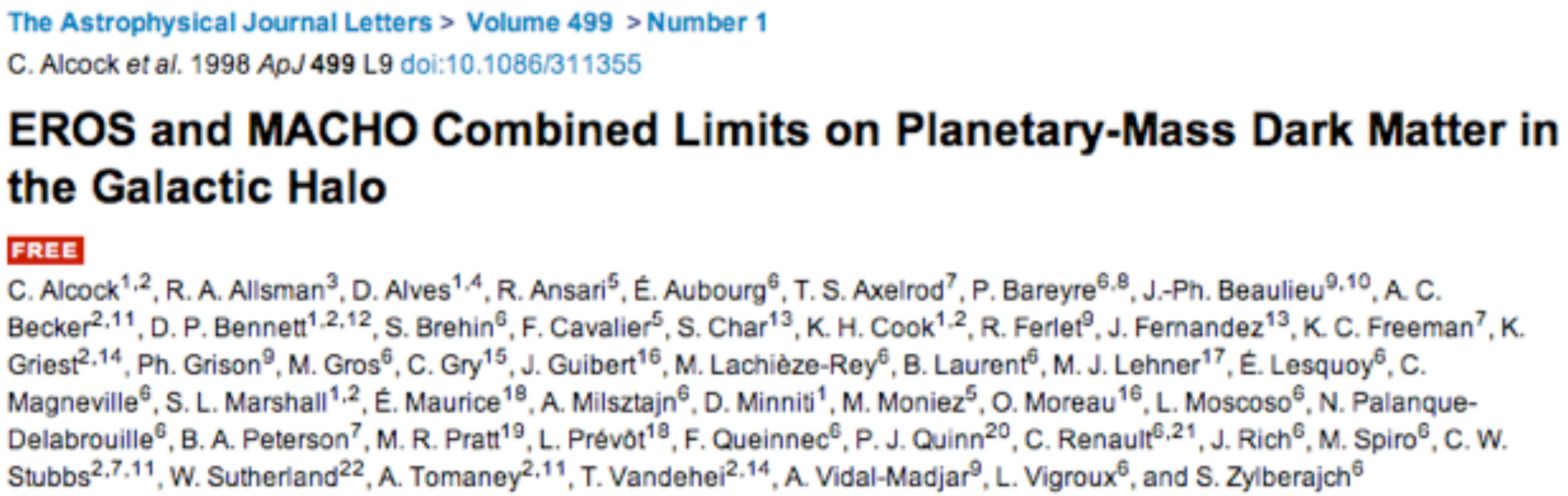
Vera Cooper Rubin faced several obstacles on her way to a career in astronomy. A high school physics teacher tried to steer her away from science. A college admissions officer suggested that she avoid majoring in astronomy. Princeton University did not grant her request for a graduate catalogue in 1948, because its graduate astronomy program did not accept women until 27 years later. Senior astronomers took a scornful view of her first paper, presented in 1950, on galactic motions independent of the classic expansion of the universe. And when she and collaborator Kent Ford expanded that research in the 1970s, they met so much dissent that they shifted to another field.

The shift proved providential. Rubin and Ford measured the rotational velocities of interstellar matter in orbit around the center of the nearby Andromeda galaxy. Their readings, confirmed by observations on other galaxies, led them to infer that the galaxies must contain dark matter. Confirmation of that fact sealed Rubin's reputation as an astronomer.

4. Massivo Compact Halo Objects: MACHOs

- Una possibile materia oscura "convenzionale" (l'unica?) e' quella dovuta ad oggetti di tipo "stelle mancate": pianeti tipo Giove che non producono energia e non sono visibili. Sono stati chiamati MACHO.
- Sono stati cercati con esperimenti di Microlensing , secondo un suggerimento di Paczynski del 1986.
- EROS (Experience de Recherche d'Objets Sombres, presso lo European Southern Observatory at La Silla, Chile, CCD camera) e MACHO (telescopio da 1.27 m, Mount Stromlo Observatory, Australia) hanno cercato fluttuazioni di luce nelle stelle della Nube di Magellano, con risultati negativi

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EROS and MACHO Combined Limits on Planetary-Mass Dark Matter in the Galactic Halo

FREE

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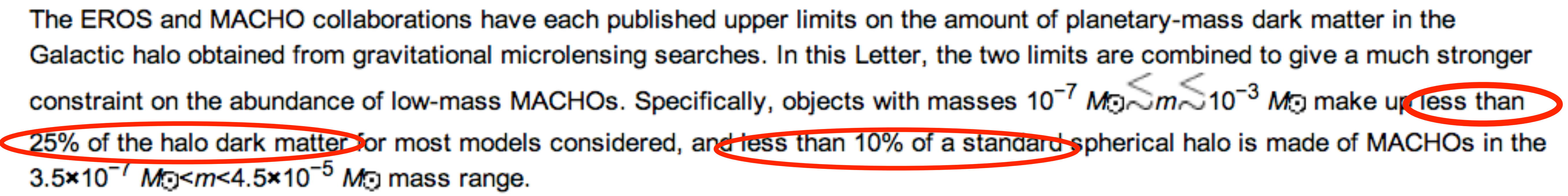
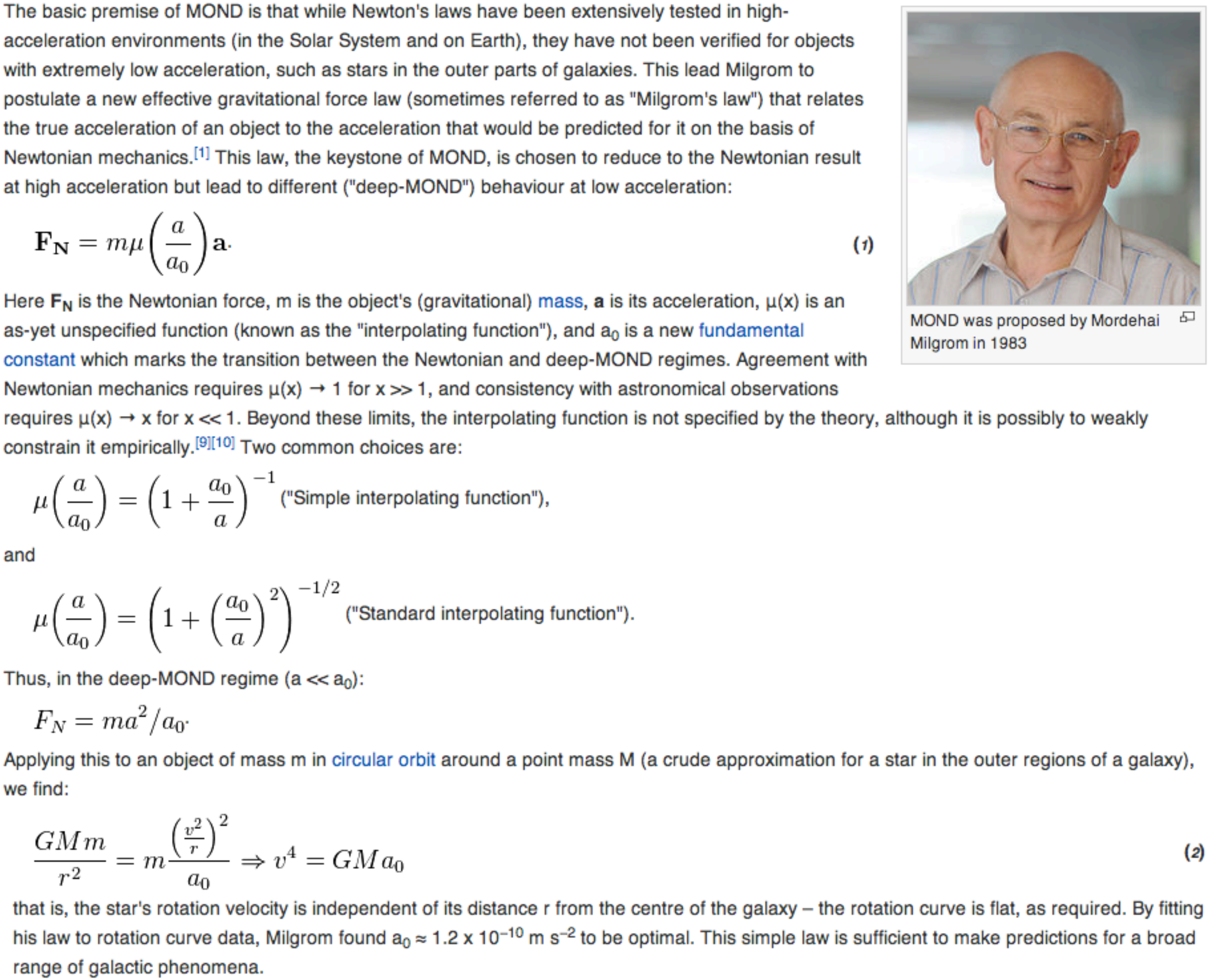

The EROS and MACHO collaborations have each published upper limits on the amount of planetary-mass dark matter in the Galactic halo obtained from gravitational microlensing searches. In this Letter, the two limits are combined to give a much stronger constraint on the abundance of low-mass MACHOs. Specifically, objects with masses $10^{-7}\ M_\odot \lesssim m \lesssim 10^{-3}\ M_\odot$ make up less than 25% of the halo dark matter for most models considered, and less than 10% of a standard spherical halo is made of MACHOs in the $3.5\times10^{-7}\ M_\odot < m < 4.5\times10^{-5}\ M_\odot$ mass range.

5. Modified Newtonian Dynamics

- E se la terza Legge di Keplero non valesse su scala galattica?
- tentativi di modifica delle leggi del moto o della legge di gravita' sono stati proposti, in modo da riprodurre i successi del moto planetario
- nessuno di questi tentativi riesce ad integrarsi o a modificare in modo accettabile la Teoria della Relativita' Generale ed e' dubbio che una di queste MOND riesca a tener conto di tutti i fenomeni che la RG prevede, a partire dalla deflessione della luce (prossima Sezione), ai buchi neri, all'effetto Einstein etc.
- Di fatto, i seguaci di MOND sottolineano soprattutto l'inadeguatezza delle varie teorie sulla Materia Oscura a rendere conto degli effetti a livello di dinamica galattica di questa idea, in assenza di un'indicazione precisa sulla natura della Materia Oscura stessa



The basic premise of MOND is that while Newton's laws have been extensively tested in high-acceleration environments (in the Solar System and on Earth), they have not been verified for objects with extremely low acceleration, such as stars in the outer parts of galaxies. This lead Milgrom to postulate a new effective gravitational force law (sometimes referred to as "Milgrom's law") that relates the true acceleration of an object to the acceleration that would be predicted for it on the basis of Newtonian mechanics.[1] This law, the keystone of MOND, is chosen to reduce to the Newtonian result at high acceleration but lead to different ("deep-MOND") behaviour at low acceleration:

$$\mathbf{F_N} = m\mu\left(\frac{a}{a_0}\right)\mathbf{a}. \qquad (1)$$

MOND was proposed by Mordehai Milgrom in 1983

Here $\mathbf{F_N}$ is the Newtonian force, m is the object's (gravitational) mass, **a** is its acceleration, $\mu(x)$ is an as-yet unspecified function (known as the "interpolating function"), and a_0 is a new fundamental constant which marks the transition between the Newtonian and deep-MOND regimes. Agreement with Newtonian mechanics requires $\mu(x) \rightarrow 1$ for $x >> 1$, and consistency with astronomical observations requires $\mu(x) \rightarrow x$ for $x << 1$. Beyond these limits, the interpolating function is not specified by the theory, although it is possibly to weakly constrain it empirically.[9][10] Two common choices are:

$$\mu\left(\frac{a}{a_0}\right) = \left(1 + \frac{a_0}{a}\right)^{-1}$$ ("Simple interpolating function"),

and

$$\mu\left(\frac{a}{a_0}\right) = \left(1 + \left(\frac{a_0}{a}\right)^2\right)^{-1/2}$$ ("Standard interpolating function").

Thus, in the deep-MOND regime ($a << a_0$):

$$F_N = ma^2/a_0.$$

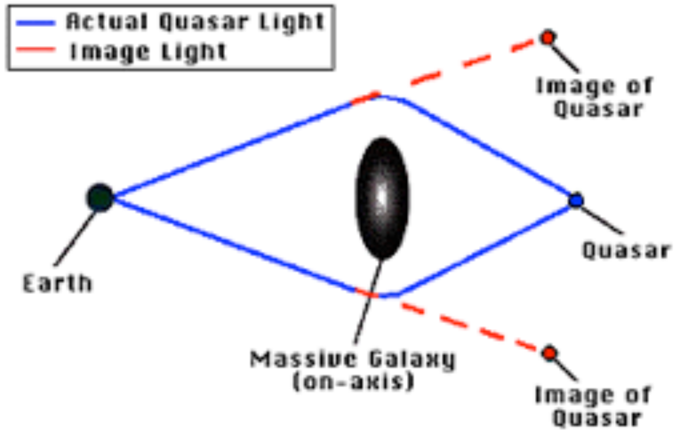
Applying this to an object of mass m in circular orbit around a point mass M (a crude approximation for a star in the outer regions of a galaxy), we find:

$$\frac{GMm}{r^2} = m\frac{\left(\frac{v^2}{r}\right)^2}{a_0} \Rightarrow v^4 = GMa_0 \qquad (2)$$

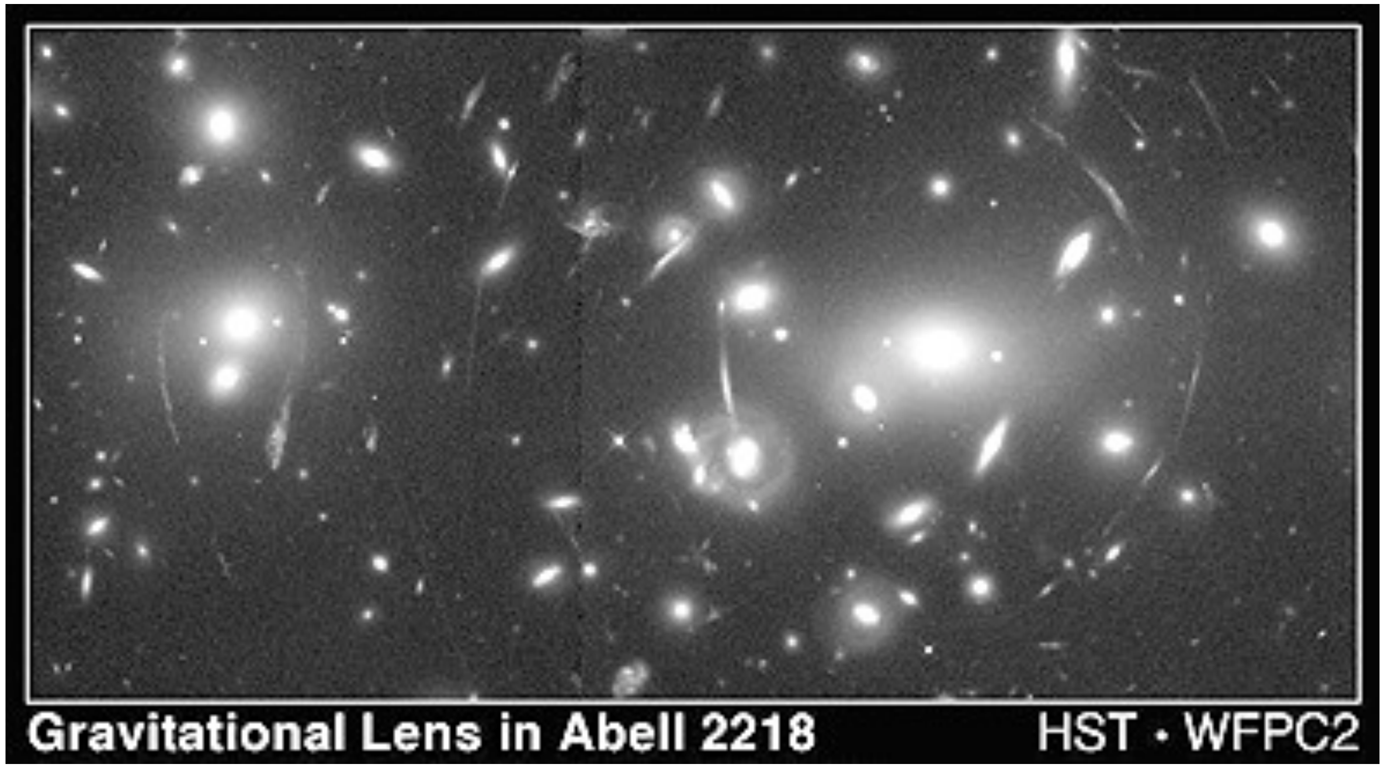
that is, the star's rotation velocity is independent of its distance r from the centre of the galaxy – the rotation curve is flat, as required. By fitting his law to rotation curve data, Milgrom found $a_0 \approx 1.2 \times 10^{-10}$ m s^{-2} to be optimal. This simple law is sufficient to make predictions for a broad range of galactic phenomena.

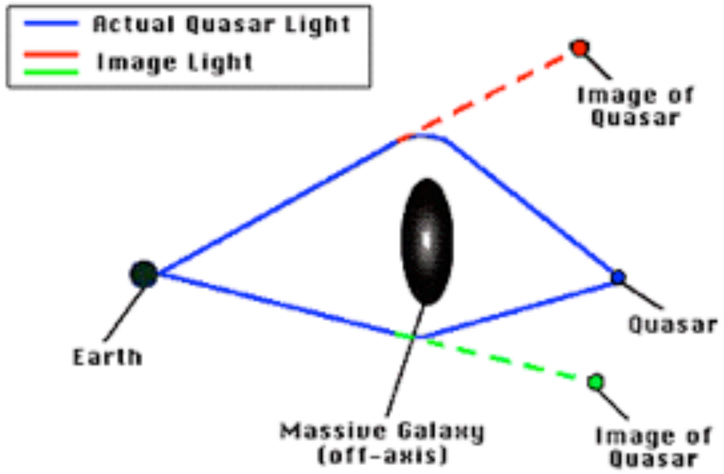
6. Gravitational lensing e materia oscura

- La luce che passa vicino ad un oggetto massivo come una galassia, si curva e ci da immagini multiple degli oggetti che stanno “dietro” la galassia in primo piano
- a seconda delle posizioni relative, si posssono avere immagini multiple o archi luminosi



Now, if the massive galaxy is off-center (as might be expected) with respect to the line between the quasar and the Earth, then the two light paths would be different distances around the galaxy. This makes the twin images be formed at different distances away from the actual quasar.





- Da queste immagini, possiamo misurare la massa della galassia e mettere alla prova l’esistenza di un alone oscuro, indipendentemente dalle leggi di Keplero

lente gravitazionale...alla lettera

- Le immagini (a), (b) e (c) sono tre immagini dello stesso oggetto, una galassia primordiale molto piu' lontana
- (c) da' un'immagine "ingrandita" rispetto all'immagine diretta e mostra una galassia di soli 850 anni-luce e 40.000.000 di soli, formata 500 Milioni di anni dopo il Big Bang

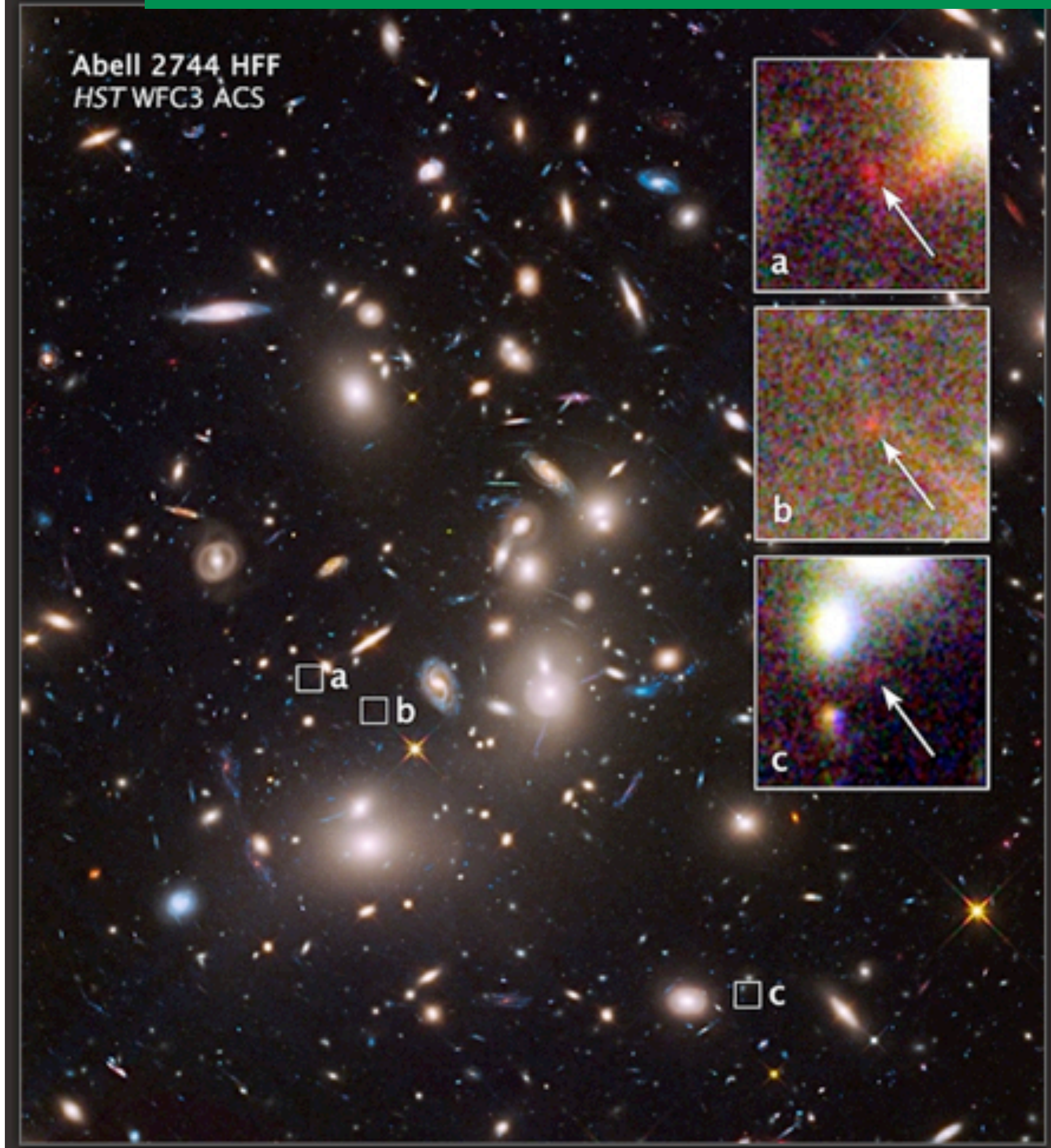
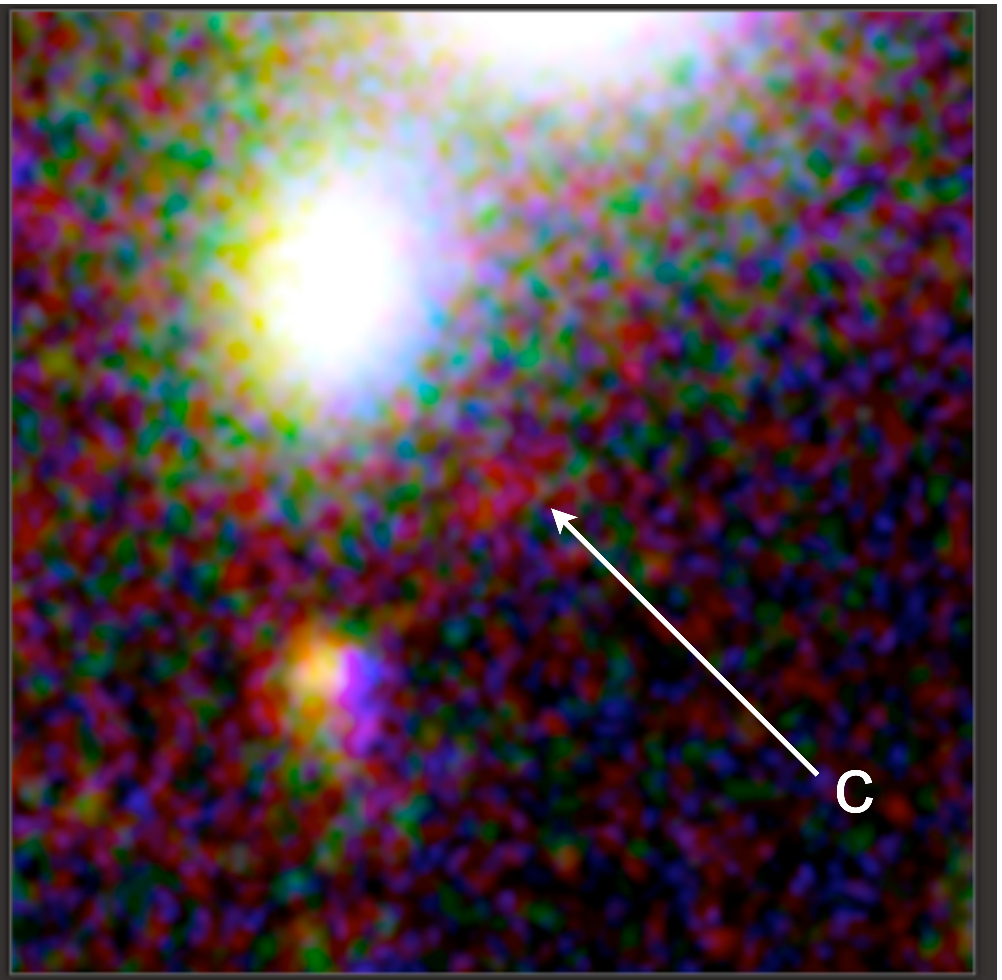


Image Credit: NASA, J. Lotz, (STScI)

Because the light was bent by the gigantic galaxy cluster Abell 2744, or Pandora's Cluster, three images of the distant galaxy where actually captured in the image above. The gravitational lens magnifies the image over 10 times what would normally be captured by Hubble's imaging sensors.

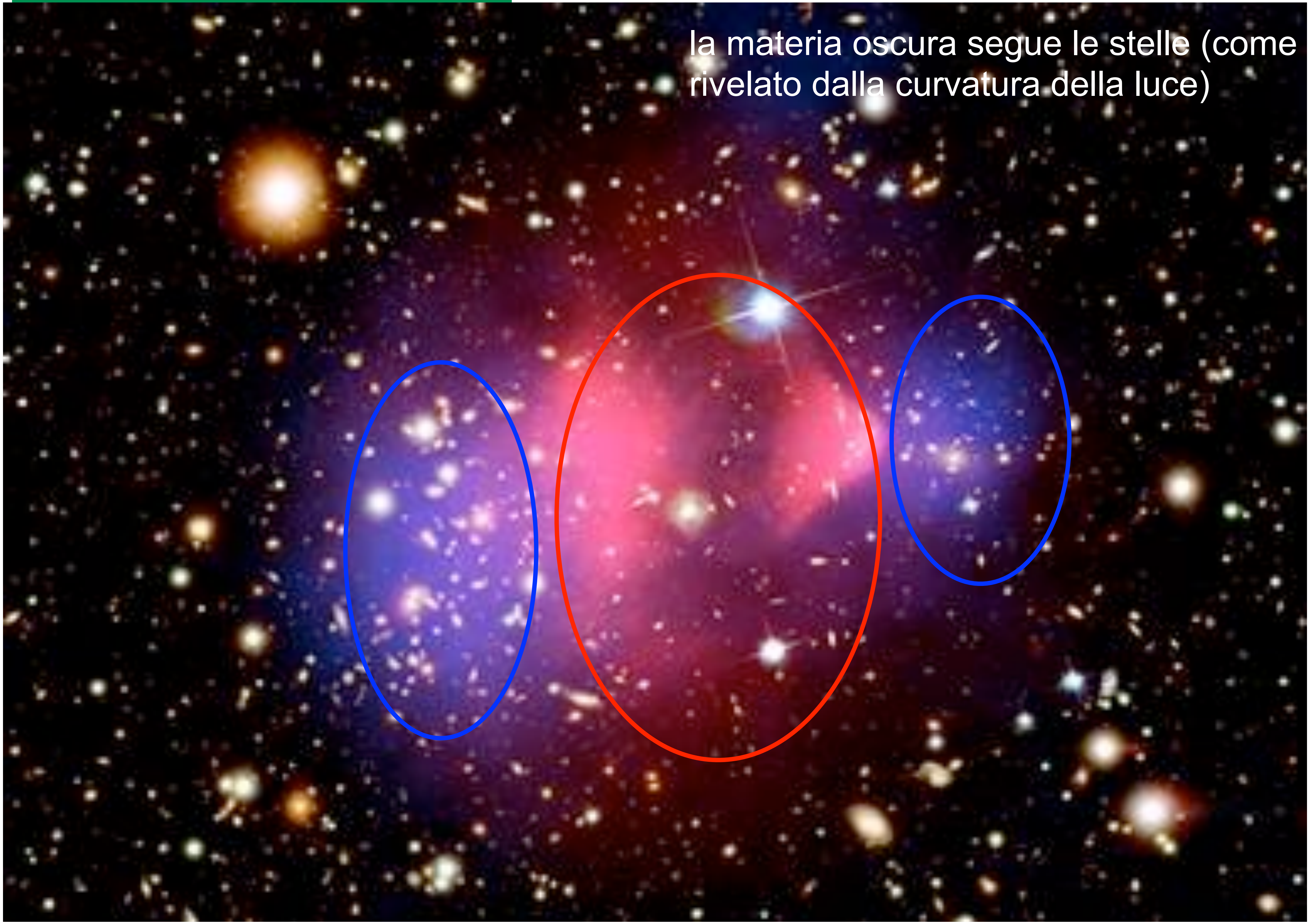


Spectrographic analysis shows that the galaxy is only 850 light-years across, about 500 times smaller than the Milky Way galaxy's 100,000 light year size. Researchers also estimate that it only has a mass of 40 million suns, compared to the 100,000,000,000 we have in our galaxy. The galaxy is so far back that it formed about 500 million years after the big bang.

7. The Bullet Galaxy

I gas si bloccano

Le stelle passano



la materia oscura segue le stelle (come rivelato dalla curvatura della luce)