Clarida, Gali & Gertler (1999)

demand shock

$$\begin{cases} x_t = -\varphi[i_t - E_t\pi_{t+1}] + E_t x_{t+1} + g_t & \text{(IS)} \\ \pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t & \text{(AS)} \end{cases}$$

cost push shock

$$\begin{cases} g_t = \mu g_{t-1} + \tilde{g}_t & \mu \in [0,1] \\ u_t = \rho u_{t-1} + \tilde{u}_t & \rho \in [0,1] \end{cases} \quad \left.\begin{matrix} \tilde{g}_t \\ \tilde{u}_t \end{matrix}\right\} \text{i.i.d } 0 \begin{cases} \sigma_g^2 \\ \sigma_u^2 \end{cases}$$

[$x_t = y_t - z_t$ output gap
π_t inflation rate between $t-1$ and t]

Consumption Euler Equation

↳ log linearized (IS)
φ interest rate elasticity in the IS, i.e. the intertemporal elasticity of substitution between consumption today & consumption tomorrow. [real interest rate is the "price" of the consumption today]

Iterating forward

$$x_t = E_t\left[\sum_{j=0}^{\infty} -\varphi(\underbrace{i_{t+j} - \pi_{t+1+j}}) + g_{t+j}\right]$$

[effectiviness depends on current & future policies. (expected)]

short vs. long run rate

Phillips curve (AS) } calvo adjustment
adjusting cost (quadratic)

↓

similar Taylor/Fischer

└ ≠ explicit optimization problem

Calvo (each period a fraction 1/X sets prices for X>1 periods)

↓

[ad hoc]

$(1-\theta)$ probability to adjust

price fixed for an average of $(1-\theta)^{-1}$ periods. $\theta = 0{,}75$ [Q] prices change once a year.

λ is decreasing in θ → the longer prices are fixed the less sensitive is inflation to changes in the output gap.

Crucial difference

$E_t \pi_{t+1}$ instead of $E_{t-1}\pi_t$

iterating

$$\pi_t = E_t\left[\sum_{j=0}^{\infty} \beta^j (\lambda x_{t+j} + u_{t+j})\right]$$

future marginal costs

Δ exess demand

cost push

shock on the monopoly mark-up.

- Stochastic wedge MRS = W/P (tax)
- MRS ≠ W/P sticky nominal wages } ≠
- Cost channel
- Exchange rate mov. (imp. pass through)

Central Bank

$$\max -\frac{1}{2} E_t\left[\sum_{j=0}^{\infty} \beta^j \left(\overset{\text{weight}}{a}\, x_{t+j}^2 + \pi_{t+j}^2\right)\right] \quad (CB)$$

(quadratic approximation of the expected discounted utility of the representative house hold) → β discount factor representative household / a is a function of the deep parameters (pref./tech)

$\boxed{\text{Discretion :}}$ S1

max CB s.t. IS/AS wrt $\{x_t, \pi_t, i_t\}$ each period.

Markow Perfect Equilibria

- (step 1) $x_t\ \pi_t$ max CB s.t. AS taking expectations as given. ⇒ $x_t^*\ \pi_t^*$
- (step 2) given $x_t^*\ \pi_t^*$ → choose i_t^* from IS

Step 1

$$\max_{\{\pi_t\, x_t\}} -\frac{1}{2}\left[a x_t^2 + \pi_t^2\right] + F_t \quad \text{s.t. } \pi_t = \lambda x_t + f_t$$

where
$$\begin{cases} F_t = -\frac{1}{2} E_t\left[\sum_{j=1}^{\infty} \beta^j (a x_{t+j}^2 + \pi_{t+j}^2)\right] \\ f_t = \beta E_t \pi_{t+1} + u_t \end{cases}$$

i.e.

$$\max_{\{x_t\}} -\frac{1}{2}\left[a x_t^2 + (\overbrace{\lambda x_t + f_t}^{\pi_t})^2\right]$$

see AS

$$\beta E_t \pi_{t+1} + u_t + \lambda x_t = \pi_t$$

foc

$$-a x_t - \lambda(\lambda x_t + f_t) = 0$$

thus $\boxed{x_t = -\frac{\lambda}{a}\pi_t}$

lean against the wind

$\pi = 0$ do noting

$\pi > 0$ reduce x of λ/a (↓ employment) by ↑ real interest rate

combining with the AS

$$\lambda x_t + \beta E_t \pi_{t+1} + u_t = \pi_t$$

$$\pi_t = -\frac{\lambda^2}{a}\pi_t + \beta E_t \pi_{t+1} + u_t$$

$$\pi_t = \frac{a(u_t + \beta E_t \pi_{t+1})}{\lambda^2 + a}$$

notice $E_t \pi_{t+1} = E_t\left[\frac{a(u_{t+1} + \beta E_{t+1}\pi_{t+2})}{\lambda^2 + a}\right]$

$$E_{t+1}\pi_{t+2} = E_{t+1}\left[\frac{a(u_{t+2} + \beta E_{t+2}\pi_{t+3})}{\lambda^2 + a}\right]$$

⇓

$$E_t \pi_{t+1} = E_t\left[\frac{a(u_{t+1} + \frac{\beta a}{\lambda^2 + a}(u_{t+2} + \beta E_{t+2}\pi_{t+3})}{\lambda^2 + a}\right]$$

$$\pi_t = \frac{a}{\lambda^2 + a}u_t + \left(\frac{a}{\lambda^2 + a}\right)^2 \beta E_t u_{t+1} + \left(\frac{a}{\lambda^2 + a}\right)^3 \beta^2 E_t u_{t+2} + \dots$$

$$+\left(\frac{a\beta}{\lambda^2+a}\right)^3 E_t \pi_{t+3}$$

law of iterated expectations
$E_t(E_{t+1}) = E_t$

assuming $\lim_{T\to\infty} \left(\frac{a\beta}{\lambda^2+a}\right)^T E_t \pi_{t+T} = 0$

$$\pi_t = \frac{a}{\lambda^2+a} E_t \left[\sum_{j=0}^{\infty} \left(\frac{a\beta}{\lambda^2+a}\right)^j u_{t+j} \right]$$

$u_t = \rho u_{t-1} + \tilde{u}_t$ & $E_t[\tilde{u}_{t+1}] = 0$

$$\boxed{\pi_t = \frac{a}{\lambda^2+a} \sum_{j=0}^{\infty} \left(\frac{a\beta\rho}{\lambda^2+a}\right)^j u_t}$$

[↑ as a ↑]

$$\pi_t = \frac{\frac{a}{\lambda^2+a} u_t}{1-\frac{a\beta\rho}{\lambda^2+a}} = \frac{a}{\lambda^2+a(1-\beta\rho)} u_t$$

$$\begin{cases} \pi_t^* = a q\, u_t \\ x_t^* = -\lambda q\, u_t \end{cases} \qquad a\uparrow \begin{cases} aq\uparrow \Rightarrow \pi_t \uparrow \\ \lambda q \downarrow \Rightarrow x_t \uparrow \end{cases}$$

optimal values for x & π under discretion

Step 2

$$\pi_t^* = \vartheta q u_t ; \quad x_t^* = -\lambda q u_t$$

$$x_t^* = -\varphi\left[i_t - E_t \pi_{t+1}^*\right] + E_t x_{t+1}^* + g_t$$

$$-\lambda q u_t = -\varphi\left[i_t - \vartheta q \underbrace{E_t u_{t+1}}_{E_t(\rho u_t + \tilde{u}_{t+1}) = \rho u_t}\right] + E_t(-\lambda q u_{t+1})$$

$$-\lambda q u_t = -\varphi i_t + \varphi \vartheta q \rho u_t - \lambda q \rho u_t + g_t$$

$$i_t^* = \underbrace{\left[1 + \frac{\lambda(1-\rho)}{\varphi \vartheta \rho}\right]}_{\gamma_\pi > 1} \vartheta q \rho u_t + \frac{1}{\varphi} g_t$$

$$E_t \pi_{t+1} = \vartheta q E_t u_{t+1} = \overbrace{\vartheta q \rho u_t}$$

$$i_t^* = \left(\gamma_\pi \vartheta q \rho u_t \Rightarrow\right) \gamma_\pi E_t \pi_{t+1} + \frac{1}{\varphi} g_t$$

Policy rule

Summarizing

$$\pi_t^* = \vartheta q u_t$$

$$x_t^* = -\lambda q u_t$$

$$i_t^* = \gamma_\pi E_t \pi_{t+1} + \frac{1}{\varphi} g_t$$

R1 if $\exists \sigma_\mu^2 \Rightarrow$ short run $\boxed{\text{trade off}}$ between ? inflation & output variabilities

$$var(x_t) = x_t^2 = \left(\frac{\lambda}{\lambda^2 + a(1-\beta\rho)}\right)^2 \sigma_\mu^2 = \sigma_a^2$$

$$var(\pi_t) = \pi_t^2 = \left(\frac{a}{\lambda^2 + a(1-\beta\rho)}\right)^2 \sigma_\mu^2 = \sigma_\pi^2$$

$\downarrow$

$$\left(\frac{1}{\frac{\lambda^2}{a} + 1 - \beta\rho}\right)$$

$\uparrow a$

$\sigma_x^2 = E[x - E(x)]^2$

$$a \to 0 \Rightarrow \begin{cases} \sigma_x = \frac{\sigma_\mu}{\lambda} \\ \sigma_\pi = 0 \end{cases}$$

a inverse of Rogoff conservativeness

$$a \to +\infty \Rightarrow \begin{cases} \sigma_x = 0 \\ \sigma_\pi = \frac{\sigma_u}{1-\beta\rho} \end{cases}$$

R2 $\boxed{\text{Inflation targeting}}$ $\rightarrow$ inflation converge to the target over time.

$$\lim_{j\to\infty} E_t \pi_{t+j} = \lim_{j\to\infty} aq\rho^j u_t = 0$$

$\hookrightarrow aq E_t u_{t+j}$

$\rho E_t u_{t+j-1}$

$\rho E_t u_{t+j-2}$

$\vdots$

R3 $\boxed{\text{Taylor policy}}$ if $E_t \pi_{t+1} \uparrow$ then $i_t \uparrow$ such as $i_t - E_t(\pi_{t+1}) \uparrow$

in fact $\gamma_\pi > 1$

R4 Demand shock stabilization (as a free lunch) 8

$$i_t = \gamma_\pi E_t \pi_{t+1} + \frac{1}{\varphi} g_t$$

$$x_t = -\varphi i_t + \varphi E_t \pi_{t+1} + E_t x_{t+1} + \underbrace{g_t}_{+5}$$

$+5/\varphi \xrightarrow{-5} +5$

off set

US policy in the late 1990? $\longleftrightarrow$ no unemployment & costs, no inflation

Commitment (S2)

A Barro-Gordon's inflationary bias

$$\max -\frac{1}{2} E_t \left[\sum_{j=0}^{\infty} \beta^j (a[x_{t+j} - k]^2 + \pi_{t+j}^2) \right]$$

$k>0$

discretion

$$x_t^k = -\frac{\lambda}{a} \pi_t^k + k$$

IS

$$-\frac{\lambda}{a}\pi_t^k + \cancel{k} = -\varphi[i_t - E_t \pi_{t+1}^k] + E_t\left[-\frac{\lambda}{a} \pi_{t+1}^k + \cancel{k}\right] + g_t \rightarrow$$

No effects on x (anticipation)

But:

$$x_t^k = -\frac{\lambda}{a}\pi_t^k + k \qquad x_t^* = -\frac{\lambda}{a}\pi_t^*$$

$$x_t^k = x_t^*$$

$$-\frac{\lambda}{a}\pi_t^* = -\frac{\lambda}{a}\pi_t^k + k$$

R5 $\boxed{\text{inflation bias}}$

$$\begin{cases} \pi_t^k = -\frac{a}{\lambda}\left(-\frac{\lambda}{a}\pi_t^* - k\right) = \pi_t^* + \frac{a}{\lambda}k \\ x_t^k = x_t^* \end{cases}$$

[High inflation 1960-80
Pre-Volcker Era]

Paul Volcker

$\begin{cases} a = 0 \text{ Rogoff} \\ k = 0 \text{ contract} \end{cases}$

solutions

B* Intertemporal stabilization bias

$k = 0$ + simple rule (non-inertial op. rule)
suboptimal

$\max_{\{x_{t+j};\, \pi_{t+j}\}_{j=0}^{\infty}}$ CB s.t AS (Two steps)

$\neq$ discretion $\Rightarrow$ expectations are not considered as given.

$\omega^* = \lambda q$ special case

$\omega > 0$
feedback

policy rule $\boxed{x_t^c = -\omega u_t}$

$$\pi_t^c = \frac{1}{1-\beta\rho} u_t + \frac{\lambda}{1-\beta\rho} x_t^c$$

$$\downarrow$$

$$-\frac{\lambda}{\alpha(1-\beta\rho)} \pi_t^c$$

$$\boxed{\begin{aligned} \pi_t^c &= \frac{\alpha(1-\beta\rho)}{\alpha(1-\beta\rho)^2+\lambda^2} u_t = \frac{\alpha_c}{\lambda^2+\alpha^c(1-\beta\rho)} u_t \\ x_t^c &= -\frac{\lambda}{\lambda^2+\alpha^c(1-\beta\rho)} u_t \end{aligned}}$$

optimal policy $\begin{cases} \pi_t^c = \alpha^c q^c u_t & < \pi_t^* \\ x_t^c = -\lambda q^c u_t & < x_t^* \end{cases}$ commitment closer to zero; farer to zero

$$q^c = \frac{1}{\lambda^2+\alpha^c(1-\beta\rho)} \qquad \begin{cases} \alpha^c < \alpha \\ q^c > q \end{cases}$$

R6

Commitment $\boxed{\pi_t^c \to 0 \text{ but } x_t^c \leftarrow 0}$ wrt discretion. (as on higher α in discretion)

R7 commitment improves the short run trade off between π/x: By the AS

C: $\pi_t^c = \frac{\lambda}{1-\beta\rho} x_t^c + \frac{1}{1-\beta\rho} u_t$

$>$

D: $\pi_t = \lambda x_t + \beta E_t \pi_{t+1} + u_t$

a change in x_t^c has a bigger effect on π_t^c than in the discretionary regime.

Commitment (global optimum)

$\max_{\{x_{t+j}, \pi_{t+j}\}_{j=0}^{\infty}}$ CB s.t AS (step 1)

Lagrangian

$$\max -\frac{1}{2} E_t \Big[\sum_{j=0}^{\infty} \beta^j \Big\{ \underbrace{a x_{t+j}^2 + \pi_{t+j}^2}_{\text{inst ot.}} + \overset{\text{multiplier}}{\phi_{t+j}} \cdot \big(\underbrace{\pi_{t+j} - \lambda x_{t+j} - \beta \pi_{t+1+j} - u_{t+j}}_{\text{constraint}} \big) \Big\} \Big]$$

$\frac{1}{2}\beta^j \phi_{t+j}$ Lagrangian multiplier associated with the constraint at $t+j$

1 Loo thus (relevant) [dynamic inconsistency] REMARK ON

$$-\frac{1}{2}\beta^{j-1}\Big\{(a x_{t+j-1}^2 + \pi_{t+j-1}^2) + \phi_{t+j-1}$$
$$(\pi_{t+j-1} - \lambda x_{t+j-1} - \beta \pi_{t+j} - u_{t+j-1})\Big\}$$
$$-\frac{1}{2}\beta^{j}\Big\{(a x_{t+j}^2 + \pi_{t+j}^2) + \phi_{t+j}(\pi_{t+j} - \lambda x_{t+j}$$
$$-\beta \pi_{t+1+j} - u_{t+j})\Big\}$$

FO 1

$$-\frac{\beta^j}{2}\cdot\{2a x_{t+j} - \lambda \phi_{t+j}\} = 0$$

FOC 2

$$-\frac{1}{2}\beta^{j-1}\{\phi_{t+j-1}(-\beta)\} - \frac{1}{2}\beta^j\{2\pi_{t+j} + \phi_{t+j}\} = 0$$

$$x_t^c = -\omega u_t$$

$$\pi_t^c = \lambda x_t^c + \beta E_t \pi_{t+1}^c + u_t$$ Iterating forward

$$= E_t\left[\sum_{j=0}^{\infty} \beta^j \left(\underbrace{\underbrace{\lambda x_{t+j}}_{-\omega\lambda u_{t+j}} + u_{t+j}}_{(1-\omega\lambda)\, u_{t+j}}\right)\right]$$

$$= (1-\omega\lambda) E_t \sum_{j=0}^{\infty} \beta^j u_{t+j}$$ $\begin{cases} \sum_0^\infty a^j = \frac{1}{1-a} \\ a<1 \end{cases}$

$$= (1-\omega\lambda) E_t \sum_{j=0}^{\infty} \beta^j \rho^j u_t$$

$$= \frac{1-\omega\lambda}{1-\beta\rho} u_t$$ static constraint

$$\boxed{\pi_t^c = \frac{1}{1-\beta\rho} u_t + \frac{\lambda}{1-\beta\rho} x_t^c}$$ (AS^c)

$$\max -\frac{1}{2} E_t\left[\sum_{j=0}^{\infty} \beta^j \left(a (x_{t+j}^c)^2 + (\pi^2_{t+1})^2\right]\right.$$ s.t.

$$= \max\left[-\frac{1}{2} a (x_t^c)^2 + (\pi_t^c)^2\right] + \left[-\frac{1}{2} a \left(\underbrace{x_{t+1}^c}_{\to \frac{x_t^c}{u_t} u_{t+1}}\right)^2 + \left(\underbrace{\pi_{t+1}^c}_{\to \frac{\pi_t^c}{u_t} \cdot u_{t+1}}\right)^2\right]\beta + \left[-\frac{1}{2} a \left(\underbrace{x_{t+2}^c}_{\to \frac{x_t^c}{u_t} u_{t+2}}\right)^2 + \left(\underbrace{\pi_{t+2}^c}_{\to \frac{\pi_t^c}{u_t} \cdot u_{t+2} \dots}\right)^2\right]\beta^2 + \dots$$ i.e

$$\max\left[-\frac{1}{2} a (x_t^c)^2 + (\pi_t^c)^2\right] \cdot \underbrace{E_t\left[\sum_{j=0}^{\infty} \beta^j \left(\frac{u_{t+j}}{u_t}\right)^2\right]}_{C_t}$$

Notice that (linearity) [show as exercise]

$$\pi_t^c = \frac{1-\lambda\omega}{1-\beta\rho} u_t = q\, u_t \Rightarrow q = \frac{\pi_t^c}{u_t}$$

$$\pi_{t+1}^c = q\, u_{t+1} = \frac{\pi_t^c}{u_t} u_{t+1}$$

...

$$\pi_{t+j}^c = q\, u_{t+j} = \frac{\pi_t^c}{u_t} \cdot u_{t+j}$$

in a similar manner $(x_t^c = -\omega u_t)$

$$x_{t+j}^c = \frac{x_t^c}{u_t} \cdot u_{t+j}$$

Plug in the constraint the problem becomes

$$\max -\frac{1}{2}\left[a (x_t^c)^2 + \left(\frac{\lambda}{1-\beta\rho} x_t^c + \frac{1}{1-\beta\rho} u_t\right)^2\right] L_t$$

foc
$$-\left[a x_t^c + \frac{\lambda}{1-\beta\rho}\underbrace{\left[\frac{\lambda}{1-\beta\rho} x_t^c + \frac{1}{1-\beta\rho} u_t\right]}_{\pi_t^c}\right] L_t = 0$$

$$\boxed{x_t^c = -\frac{\lambda}{a^c} \pi_t^c} \longrightarrow \left\{\begin{array}{c}\text{Remark lean against the wind} \\ x_t = -\lambda/a\, \pi_t\end{array}\right\}$$

$a^c = a(1-\beta\rho) < a$

By using the constraint we find

x_t^c & π_t^c optimal as function of u_t

simplifying

$$\left.\vartheta x_{t+j} - \frac{\lambda}{2}\phi_{t+j} = 0 \quad \forall j \geq 0 \right\} x$$

$$\left.\begin{array}{l} \pi_{t+j} + \frac{1}{2}\phi_{t+j} - \frac{1}{2}\phi_{t+j-1} = 0 \quad \forall j \geq 1 \\ \pi_t + \frac{1}{2}\phi_t = 0 \end{array}\right\} \pi$$

time inconsistency
[the promise becomes sub-optimal $t \to t+1$]

$$\phi_{t+j} = 2\frac{\vartheta}{\lambda} x_{t+j} \quad \forall j \geq 0$$

$$\left.\begin{array}{l} x_{t+j} - x_{t+j-1} = -\frac{\lambda}{\vartheta}\pi_{t+j} \quad \forall j \geq 1 \\ x_t = -\frac{\lambda}{\vartheta}\pi_t \end{array}\right\} \pi$$

with IS (step 2): $\varphi[i_t - E_t \pi_{t+1}] = E_t x_{t+1} - x_t + g_t$

$$\varphi[i_t - E\pi_{t+1}] = -\frac{\lambda}{\vartheta}E_t\pi_{t+1} + g_t$$

$$\varphi i_t = \varphi E_t \pi_{t+1} - \frac{\lambda}{\vartheta}E_t\pi_{t+1} + g_t$$

$$\boxed{i_t = \underbrace{\left[1 - \frac{\lambda}{\varphi\vartheta}\right]}_{<1} E_t \pi_{t+1} + \frac{1}{\varphi}g_t}$$

R7

the CB adjusts demand only partially in response to an increase in expected inflation.

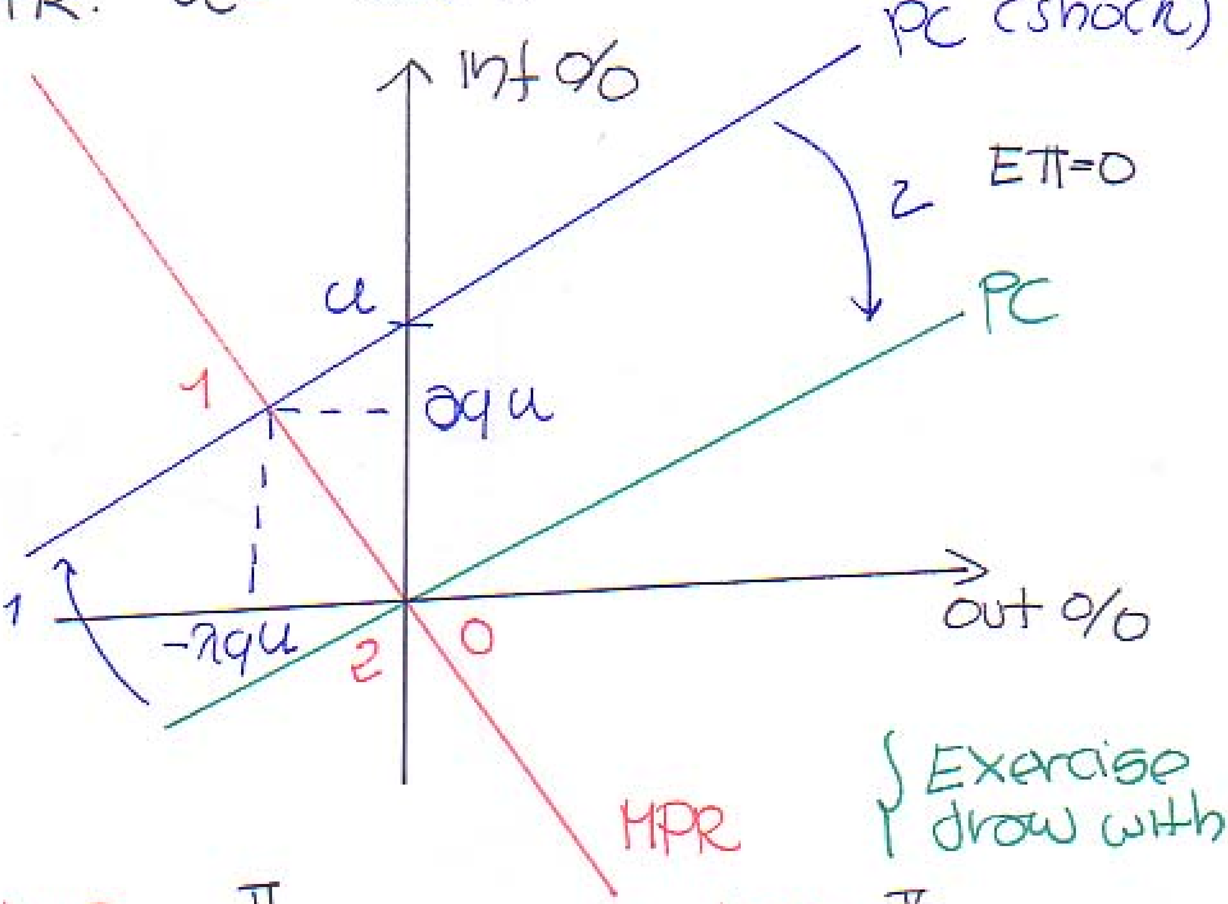
instability
$\uparrow E\pi \to \uparrow i$ such that $\downarrow(i - E\pi)$ ($\neq$ discretion)

Example (discretion)

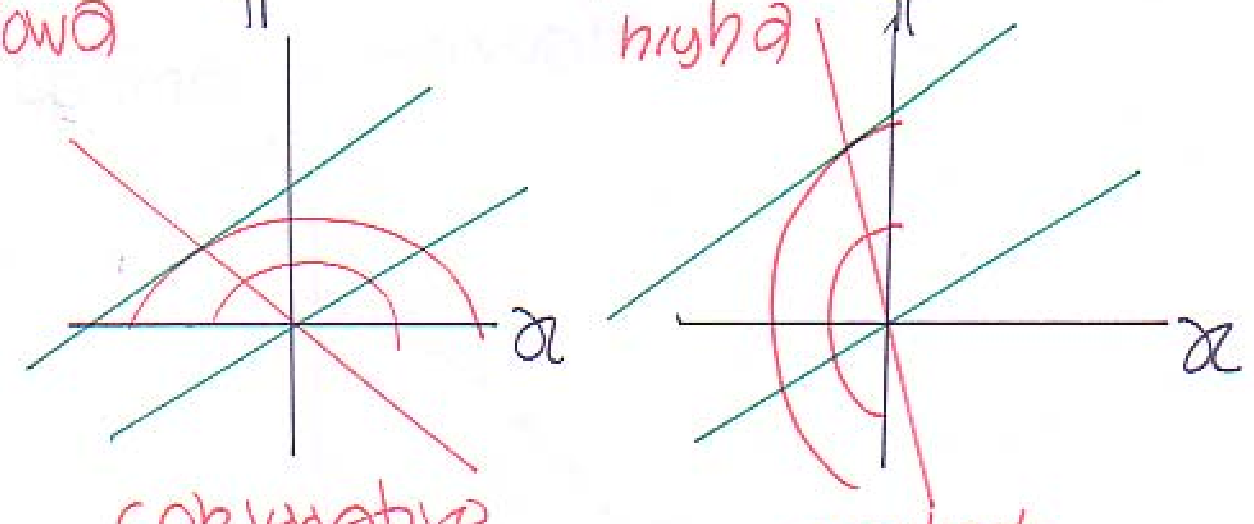
AS: $\pi = \lambda x + \beta E\pi + u$

u: u iid $(\rho = 0)$ $\quad q = 1/(\lambda^2 + a(1-\beta))$

MPR: $x = -\lambda/a\,\pi$



{Exercise draw with $k>0$}



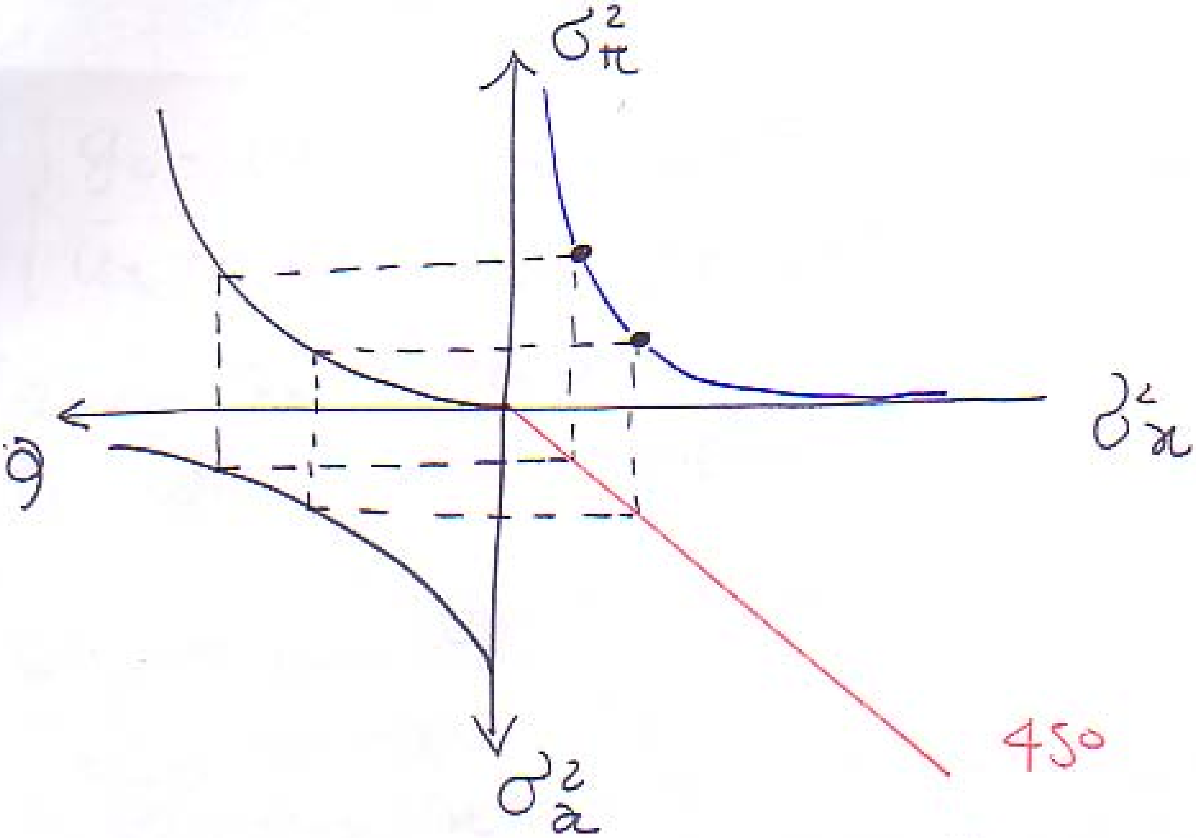
conservative (commitment)*

populist

Volatility tradeoff

$\pi = a q u \quad x = -\lambda q u \quad q = (\lambda^2 + (1-\beta) a)^{-1}$

$\sigma_\pi^2 = a q^2 \sigma_u^2 \quad \sigma_x^2 = \lambda^2 q^2 \sigma_u^2$



optimal conservativeness (a)