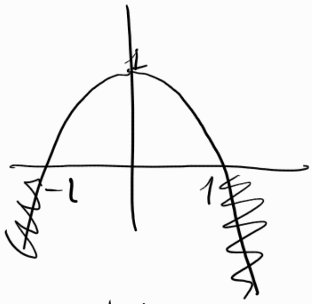
Disegnare, senza fare derivate, un grafico approssimato di

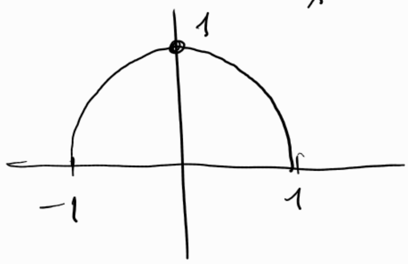
$$f(x) = \frac{1}{1+\sqrt{1-x^2}}$$

Dominio $f = [-1, 1]$ $\qquad$ f pari (dipende solo da x^2).

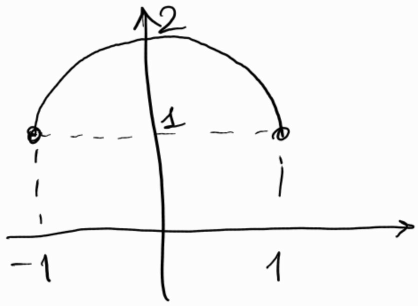
$$y = 1 - x^2$$



$$y = \sqrt{1-x^2}$$
$$\Downarrow$$
$$y^2 = 1 - x^2$$



$$y = 1 + \sqrt{1-x^2}$$



$$y = \frac{1}{1+\sqrt{1-x^2}}$$

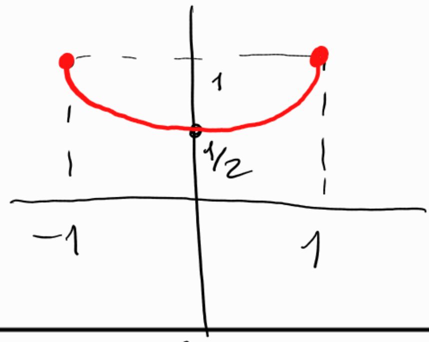
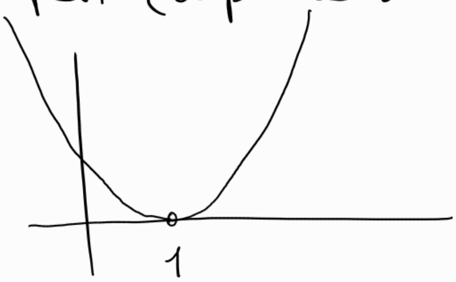


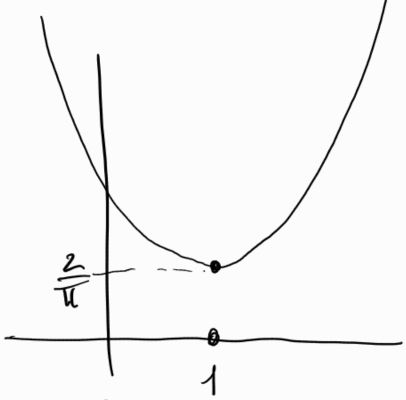
Grafico di $f(x) = \cos\left(\frac{1}{(|x|-1)^2 + \frac{2}{\pi}}\right)$

Dominio: $\mathbb{R}$ $\qquad$ f pari (dipende solo da $|x|$)

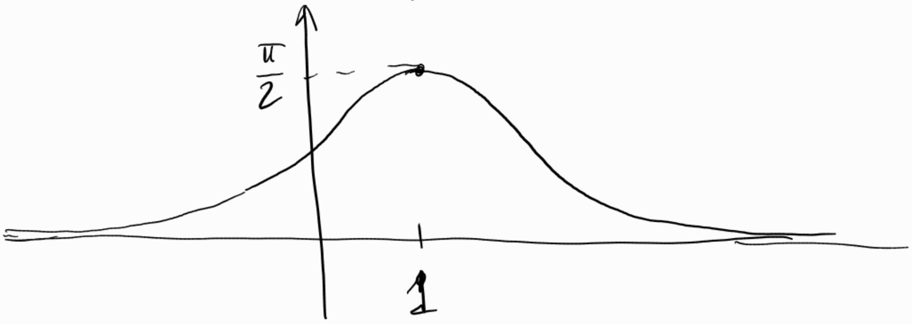
$$y = (x-1)^2$$



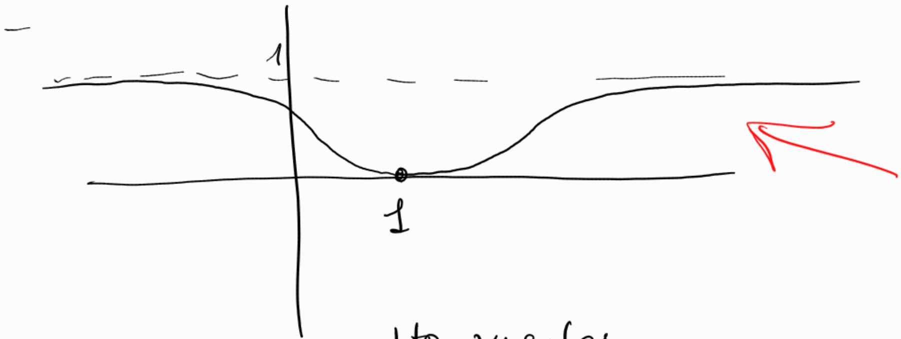
$$y = (x-1)^2 + \frac{2}{\pi}$$



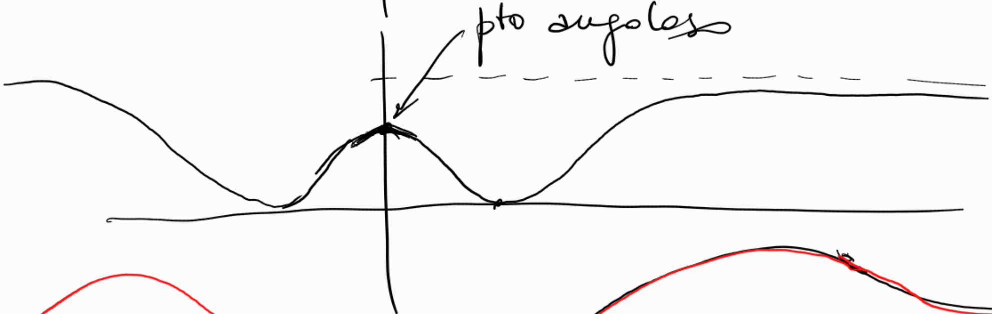
$$y = \frac{1}{(x-1)^2 + \frac{2}{\pi}}$$



$$y = \cos\left(\frac{1}{(x-1)^2 + \frac{2}{\pi}}\right)$$

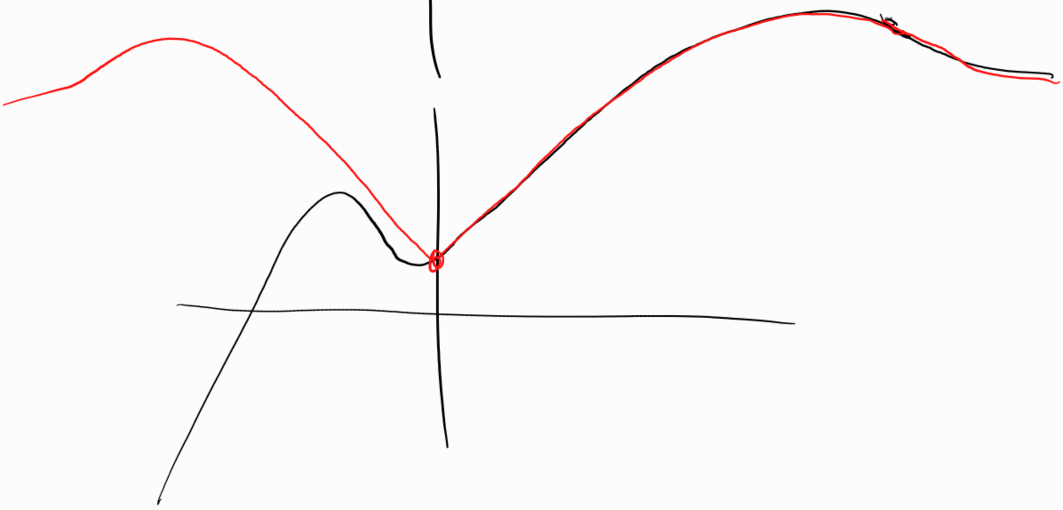


$$y = \cos\left(\frac{1}{(|x|-1)^2 + \frac{2}{\pi}}\right)$$

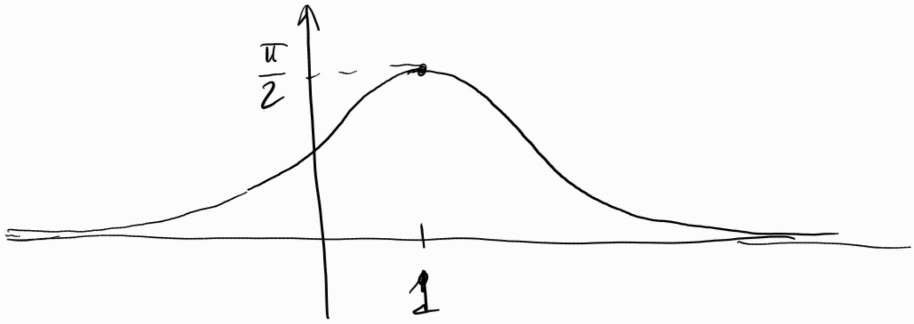


$$y = f(x)$$

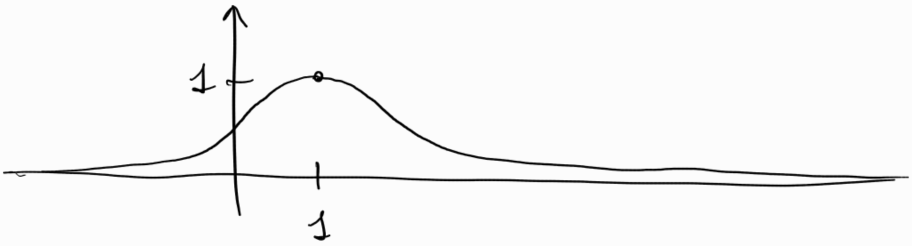
$$y = f(|x|)$$



$$y = \frac{1}{(x-1)^2 + \frac{2}{\pi}}$$



$$y = \operatorname{sen}\left(\frac{1}{(x-1)^2 + \frac{2}{\pi}}\right)$$



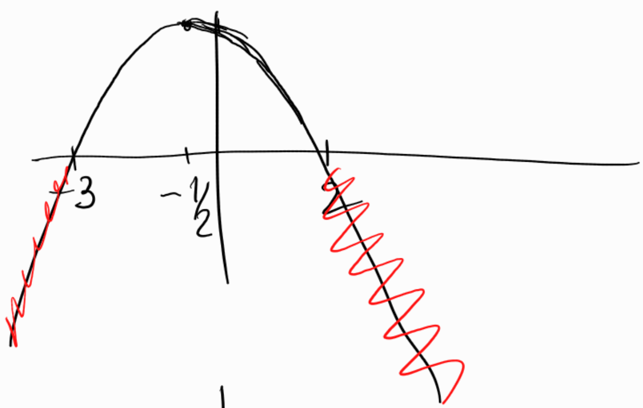
Trovare un intervallo in cui $f(x) = \sqrt{-x^2 - x + 6}$ è invertibile, e scrivere la funzione inversa.

Dominio di $f = \{x : -x^2 - x + 6 \geq 0\} = [-3, 2]$

$$x^2 + x - 6 \leq 0 . \qquad x_{1,2} = -3, 2$$

$$-3 \leq x \leq 2$$

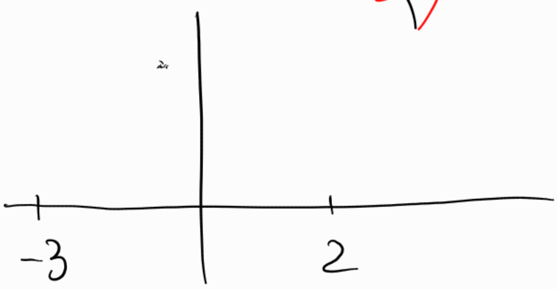
$$y = -x^2 - x + 6$$



$$y = \sqrt{-x^2 - x + 6}$$

$$\Downarrow$$

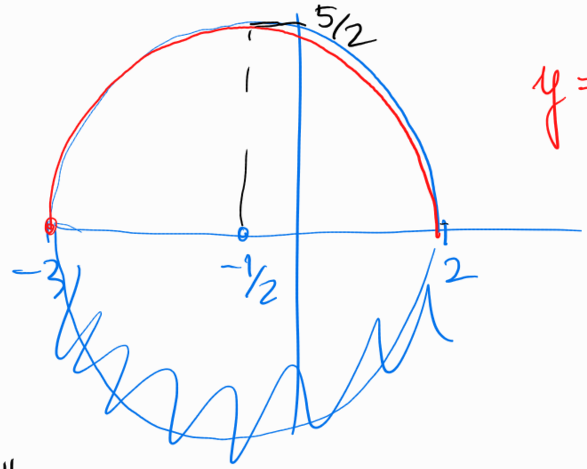
$$y^2 = -x^2 - x + 6$$



$$x^2 + x + \frac{1}{4} + y^2 = 6 + \frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 + y^2 = \frac{25}{4}$$

circonferenza con centro $\left(-\frac{1}{2}, 0\right)$ e raggio $\frac{5}{2}$



$y = \sqrt{-x^2-x+6} = f(x)$

Come intervallo "di invertibilità" posso scegliere $[-\frac{1}{2}, 2]$ oppure $[-3, -\frac{1}{2}]$ oppure un sottointervallo di uno di questi due.

Scegliamo $[-\frac{1}{2}, 2]$

$f\big|_{[-\frac{1}{2},2]} : [-\frac{1}{2}, 2] \longrightarrow [0, \frac{5}{2}]$. è biiettiva?

È iniettiva? sì perché strett. decrescente.

È suriettiva? sì perché è continua in $[-\frac{1}{2}, 2]$ quindi assume tutti i valori compresi tra

$\min f = 0$ $\max f = \frac{5}{2}$

$\exists$ f^{-1} (in realtà $\left(f\big|_{[-\frac{1}{2},2]}\right)^{-1}$

$$f^{-1} : [0, \tfrac{5}{2}] \longrightarrow [-\tfrac{1}{2}, 2].$$

$y \in [0,\frac{5}{2}] \longmapsto$ l'unico $x \in [-\frac{1}{2},2]$ t.c. $f(x) = y$

$\Updownarrow$

$\Updownarrow$ $\sqrt{-x^2-x+6} = y$

$$-x^2 - x + 6 = y^2$$

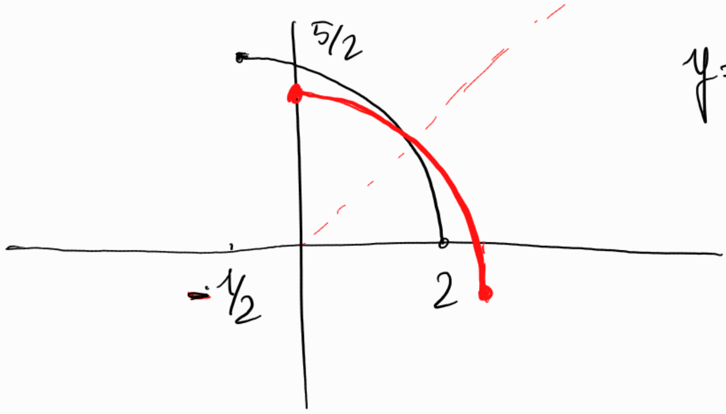
$$x^2 + x - 6 + y^2 = 0$$

$$x = \frac{-1 \pm \sqrt{1 + 4(6-y^2)}}{2} = \frac{-1 \pm \sqrt{25-4y^2}}{2}$$

scelgo + perché
ho scelto $x \in \left[-\frac{1}{2}, 2\right]$.

$$f^{-1}(y) = \cancel{x} = \frac{-1 + \sqrt{25-4y^2}}{2}$$

$$f^{-1}(x) = \frac{-1 + \sqrt{25-4x^2}}{2} \qquad \forall x \in \left[0, \frac{5}{2}\right]$$



$y = f(x)$ $\quad$ $y = f^{-1}(x)$