Serie

Studiare la convergenza delle seguenti serie:

1 $\sum_{n=1}^{+\infty}\left(\left(1+\frac{3}{n}\right)^{\frac{1}{n}}-1\right)$

2 $\sum_{n=1}^{+\infty}\left(\left(1+\frac{3}{n}\right)^{n}-1\right)$

3 $\sum_{n=1}^{+\infty} n^3\pi^{-\arctan(1/n)}$

4 $\sum_{n=1}^{+\infty}\frac{(-1)^n(2n-1)}{n^2-35n+250-\cos n}$.

5 $\sum_{n=1}^{+\infty}\frac{n!}{(2n)!}$

6 $\sum_{n=1}^{+\infty} 2^{-\ln n}$

7 $\sum_{n=2}^{+\infty}\left(\frac{(-1)^n}{n\ln n}+\sin\left(n\pi+\frac{1}{n}\right)\right)$

8 $\sum_{n=1}^{+\infty}(-1)^n\frac{n^2+\cos n}{n^3+1}$

9 $\sum_{n=1}^{+\infty}(\sqrt{n^2+n}-n)(n\tan\frac{1}{n}-1)$

10 $\sum_{n=0}^{+\infty}(-1)^n\frac{7n+24}{n^2+7n+12}$

11 $\sum_{n=1}^{+\infty}\frac{(-1)^n}{n+(-1)^n(n+\sqrt{n})}$

12 $\sum_{n=1}^{+\infty}\left(e-\left(1+\frac{1}{n}\right)^n\right)$

13 $\sum_{n=0}^{+\infty}\frac{n^n}{2^{n^3}}$

14 $\sum_{n=1}^{+\infty}\frac{e^{3n}n!}{\sqrt{n^n}}$

15 $\sum_{n=0}^{+\infty}\frac{n}{5^n+(3\cos n)^n}$

16 $\sum_{n=1}^{+\infty} e^{-n}(1+\frac{3}{n})^{n^2}$

17 $\sum_{n=0}^{+\infty}(\sqrt{n^{\frac{4}{3}}+1}-\sqrt{n^{\frac{4}{3}}+2})$

18 $\sum_{n=0}^{+\infty}(-1)^n(\sqrt{n+1}-\sqrt{n+2})$.

19 $\sum_{n=0}^{+\infty} n^2(e^{\frac{1}{n}}-1)^4$

20 $\sum_{n=0}^{+\infty} n^3(e^{\frac{1}{n^3}}-1)^{\frac{4}{3}}$

21 $\sum_{n=0}^{+\infty}\frac{\log(1+\frac{3}{n^{\frac{1}{5}}})}{n^{\frac{3}{5}}}$

22 $\sum_{n=0}^{+\infty}\frac{|\cos(\frac{1}{n^2\sqrt{\log(n+1)}})-1|}{\sin\frac{1}{n\log(n+1)}}$

23 $\sum_{n=0}^{+\infty}\frac{3+\log(2+n^5)}{n^2+\sqrt{n}}$

24 $\sum_{n=0}^{+\infty}\frac{\log(1+e^{n^2})-n^{\frac{1}{3}}}{(1+n^8)^{\frac{1}{3}}(\log n)^2+\sqrt{1+n^2}}$

25 $\sum_{n=1}^{+\infty}\left(\sqrt[3]{1+\frac{3}{n^2}}-1\right)\sqrt{n}$

26 $\sum_{n=1}^{+\infty}(-1)^n\,\mathrm{sen}\,\frac{1}{\sqrt{n^2+5}}$

27 $\sum_{n=1}^{+\infty}\left(\mathrm{tg}\,\frac{2}{n}-\frac{1}{n}\right)$

28 $\sum_{n=1}^{+\infty}(-1)^n\left(\mathrm{tg}\,\frac{2}{n}-\frac{1}{n}\right)$

29 $\sum_{n=1}^{+\infty}(-1)^n\frac{2n}{n^2+4\ln n}$

30 $\sum_{n=1}^{+\infty}\tan\left(\left|\frac{1}{n+1}-\sin\frac{1}{n}\right|^{\frac{2}{3}}\right)$

Al variare del parametro reale indicato, studiare il carattere delle seguenti serie:

31 (∗) $\sum_{n=1}^{+\infty}(-1)^{n\left[\cos\frac{1}{n^{3/2}}+\frac{1}{2n^{\alpha}}\right]}\frac{1}{2n+\sqrt{n}}$, dove $[s]=\max\{n\in\mathbf{Z}:n\le s\}$ $(\alpha>0)$.

32 $\sum_{n=1}^{+\infty}\left(\tan\frac{1}{n}-\frac{1}{n+1}\right)\left(\sqrt[3]{8n^3+1}-2n\right)^{\alpha}$

33 $\displaystyle\sum_{n=1}^{+\infty} \frac{\left[\log\left(\alpha+\frac{1}{n}\right)\right]^n}{\alpha^2+n^2}$

34 $\displaystyle\sum_{n=1}^{+\infty} \log\left(1+\frac{3}{n}\right)\; x^n$

35 $\displaystyle\sum_{n=1}^{+\infty} \log\left(1+\frac{3}{n}\right)\; \frac{(x-1)^n}{5^n+n^3}$

36 $\displaystyle\sum_{n=1}^{+\infty} \left(\sqrt[3]{8n^3+3n}-2n\right)^\alpha$

37 $\displaystyle\sum_{n=1}^{+\infty} \frac{(n!)^2x^{2n}}{(2n)!}$

38 $\displaystyle\sum_{n=1}^{+\infty} \left(\operatorname{tg}\frac{2}{n}-\frac{\alpha}{n}\right)$

39 $\displaystyle\sum_{n=1}^{+\infty} (-1)^n\,\frac{(2\alpha+5)^n}{5n^2-n+4}$

40 $\displaystyle\sum_{n=1}^{+\infty} \frac{\ln(3+n^3)-3\ln n}{(n+2)^\alpha}$

41 $\displaystyle\sum_{n=1}^{+\infty} \left(\operatorname{ch}\frac{x}{n}-\cos\frac{2}{n}\right)$

42 $\displaystyle\sum_{n=1}^{+\infty} (\ln x)^n(3^n+n)$

43 $\displaystyle\sum_{n=1}^{+\infty} \frac{(\sqrt{3}\operatorname{tg}\alpha)^n}{3n^2-3\sqrt{n}+2}$

44 $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2+\alpha}{2n^3+n^\alpha}$

45 $\displaystyle\sum_{n=1}^{+\infty} \frac{e^{\alpha n}}{5n^2+2n^{3/2}+1}$

46 $\displaystyle\sum_{n=1}^{+\infty} \frac{n+\operatorname{arctg} n}{5n^2+2n^\alpha+1}$

47 $\displaystyle\sum_{n=1}^{+\infty} \frac{(3^n+n^3)\,(2-\ln x)^n}{n^5}$

48 (∗) $\displaystyle\sum_{n=1}^{\infty} n\left(\sin\frac{x^n}{n}+\arctan\frac{n}{x^n}-\frac{\pi}{2}\right)$

49 $\displaystyle\sum_{n=1}^{+\infty} \left(\tan\frac{x}{n}-\tan\ln\left(1+\frac{x}{n}\right)\right)$

50 $\displaystyle\sum_{n=1}^{+\infty} \frac{\left|x+\frac{1}{4}\right|^{n^2}}{n^{2x+1}}$

51 $\displaystyle\sum_{n=1}^{+\infty} \frac{nx}{x^{\frac{3}{2}}n^3+\sqrt{n+1}}$

52 $\displaystyle\sum_{n=0}^{+\infty} \frac{nx\sin^4 x}{1+n^3|x|^3}\,.$

53 $\displaystyle\sum_{n=1}^{\infty} n\left(\sqrt[3]{n^{3\alpha}+n^\alpha}-n^\alpha\right)$

54 $\displaystyle\sum_{n=1}^{+\infty} n^{10}\sin(n^2)\left[\left(\frac{7^n+3^n}{5^n+2^n}\right)^{\frac{1}{n}}-x\right]$

55 $\displaystyle\sum_{n=1}^{+\infty} \frac{n^2+n+1}{n^x+n^2+1}$

56 $\displaystyle\sum_{n=1}^{+\infty} \left(\cosh\frac{n^2+n}{n^3+3}-1\right)^x$

57 (*) $\displaystyle\sum_{n=1}^{+\infty} \left\{\left(\frac{\sqrt{x^2+2}}{x^2}\right)^n+\left(\frac{\sqrt{2n^2+1}}{n^2}\right)^{x+\frac{1}{n}}\right\}$

58 $\displaystyle\sum_{n=1}^{+\infty} \frac{\ln(1+n^{2x})}{(n+\sin n)^{x^2}}$

59 $\displaystyle\sum_{n=1}^{+\infty} \frac{1+\sin(n\pi+\frac{\pi}{2}x)}{(n+1)^x}, \quad x\in[0,2]$

60 $\displaystyle\sum_{n=1}^{+\infty} \frac{2^n+|x|^n}{n^2+1}$

61 $\displaystyle\sum_{n=1}^{+\infty} \ln\left(1+\frac{|x|^n}{n}\right)$

62 $\displaystyle\sum_{n=3}^{+\infty} \frac{n^2}{(x+\frac{1}{n})^{\ln n}}$

63 $\displaystyle\sum_{n=3}^{+\infty} \frac{\ln n}{n}x^{(\ln n)^2}$

64 $\displaystyle\sum_{n=1}^{+\infty} \frac{1}{n!}|n-x^2+3x|^{nx^3}$

65 $\displaystyle\sum_{n=2}^{+\infty} \frac{n}{(x-\frac{1}{n})^{\ln n}}$

66 $\displaystyle\sum_{n=2}^{+\infty} (n-\sqrt[3]{n^3-2n})^x$

67 $\sum_{n=3}^{+\infty} \frac{x^{2^n}}{1+x^{2^{n+1}}}$

68 $\sum_{n=1}^{+\infty} \frac{n^{x^2}}{(n+x)^{\sqrt{13x}}}$

69 $\sum_{n=1}^{+\infty} \frac{|\arctan x|^{3n}}{\sqrt{n(n+1)}}$

70 $\sum_{n=1}^{+\infty} (\sqrt{n^3+(x^2+2)n^2+1} - \sqrt{n^3+3xn^2-4})$

71 $\sum_{n=1}^{+\infty} \frac{n+10}{3^n(n+5)}(x^2-1)^{3n}$

72 $\sum_{n=1}^{+\infty} \frac{(-1)^n}{n^2+3}\left(\frac{\operatorname{sen}\alpha}{\sqrt{3}+\operatorname{sen}\alpha}\right)^n$

73 $\sum_{n=1}^{+\infty} \frac{e^{n+\beta\ln n}+5}{e^{n+2}+3n^2}$

74 Al variare del parametro reale α, trovare l'ordine di infinitesimo, per $x \to 0^+$, della funzione

$$f_\alpha(x) = \ln(1-\alpha\, x \operatorname{sen} x) - e^{-x^2} + 1\,;$$

dire inoltre per quali α converge la serie $\sum_{n=1}^{+\infty}(n + 5\ln^2 n)\, f_\alpha(1/n)$.

75 (*) Sia $f(x)$ un polinomio di terzo grado, in cui il coefficiente del termine di grado massimo è positivo. Dire per quali valori del parametro x la serie $\sum_{n=1}^{+\infty} \frac{(-1)^n}{f(n^{\sin x}\ln n)+3n}$. converge semplicemente e per quali valori converge assolutamente.

76 Studiare il carattere della serie

$$\sum_{n=1}^{+\infty} \frac{(3n+2)\,e^n}{e^{2n}+\ln n}\,,$$

e, al variare di $x \in \mathbb{R}$, della serie

$$\sum_{n=1}^{+\infty} \frac{(x^2-4)^n}{(2e)^n+3}\,.$$

77 Studiare il carattere delle serie

$$\sum_{n=1}^{+\infty} \operatorname{tg}\frac{n}{n^2+6}\,, \qquad \sum_{n=1}^{+\infty} (-1)^n \operatorname{tg}\frac{n}{n^2+6}\,,$$

$$\sum_{n=1}^{+\infty} \left(\operatorname{tg}\frac{n}{n^2+6} - \frac{n}{n^2+6}\right)^\alpha$$

(al variare di $\alpha > 0$).

1 Risposte ad alcuni esercizi

1: converge (confronto asintotico con $\frac{3}{n^2}$);
2: diverge positivamente (il termine non è infinitesimo);
3: diverge positivamente (il termine non è infinitesimo);
4: converge (criterio di Leibniz);
5: converge (criterio del rapporto);
6: diverge positivamente (in realtà è una serie armonica generalizzata);
11: diverge positivamente (si noti che è una serie a segno costante);
12: diverge positivamente;
13: convergente;
14: divergente;
15: convergente;
16: divergente;
17: divergente;
18: convergente;
19: convergente;
20: divergente;
21: divergente;
22: convergente;
23: convergente;
24: divergente;
76: prova d'esame del 2.4.2007: soluzione disponibile sulla pagina web del corso;
??: $x > 0$, $\frac{1+x}{2}$;
??: la serie converge per ogni $x \in \mathbb{R}$, la somma vale $\frac{e^{x+1}-1}{x+1}$ se $x \neq -1$, 1 se $x = -1$;
??: $\operatorname{ch}\left(\sqrt{|\cos x|}\right)$;
??: convergenza in $\left[1+\frac{1}{e}, 1+e\right]$, la somma vale $\frac{\operatorname{arctg}(\log(x-1))}{\log(x-1)}$ se $x \neq 2$, 1 se $x = 2$;