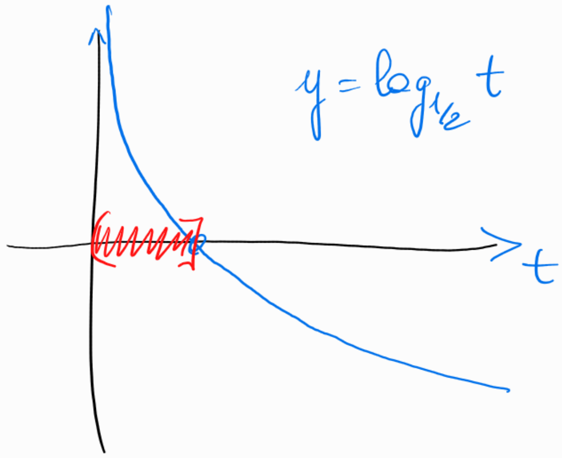
Trovare il dominio di $f(x) = \sqrt{\log_{1/2}\left(\log_2\left(2\sin^2 x - \cos x\right)\right)}$

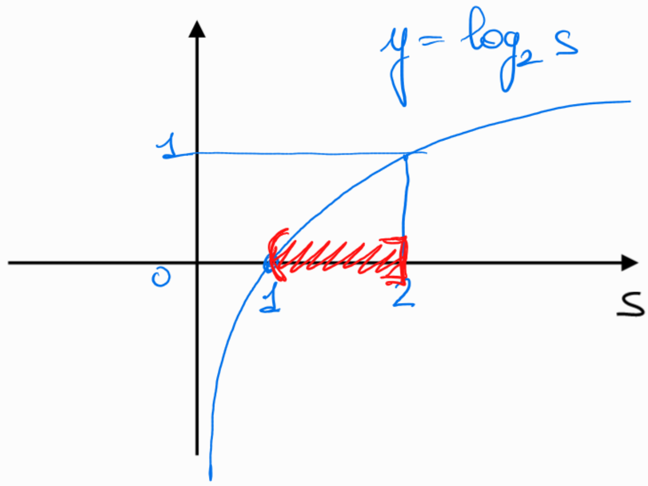
$$\log_{1/2}\big(\underbrace{\log_2\left(2\sin^2 x - \cos x\right)}_{t}\big) \geq 0$$



$$\log_{1/2} t \geq 0 \iff 0 < t \leq 1$$

$$0 < \log_2\big(\underbrace{2\sin^2 x - \cos x}_{s}\big) \leq 1$$

$$0 < \log_2 s \leq 1 \iff 1 < s \leq 2$$



$$\boxed{1 \overset{(A)}{<} 2\sin^2 x - \cos x \overset{(B)}{\leq} 2}$$

Risolvo (A)

$$2\sin^2 x - \cos x > 1$$

$$2 - 2\cos^2 x - \cos x > 1$$

$$2\cos^2 x + \underbrace{\cos x}_{t} - 1 < 0$$

$$2t^2 + t - 1 < 0$$

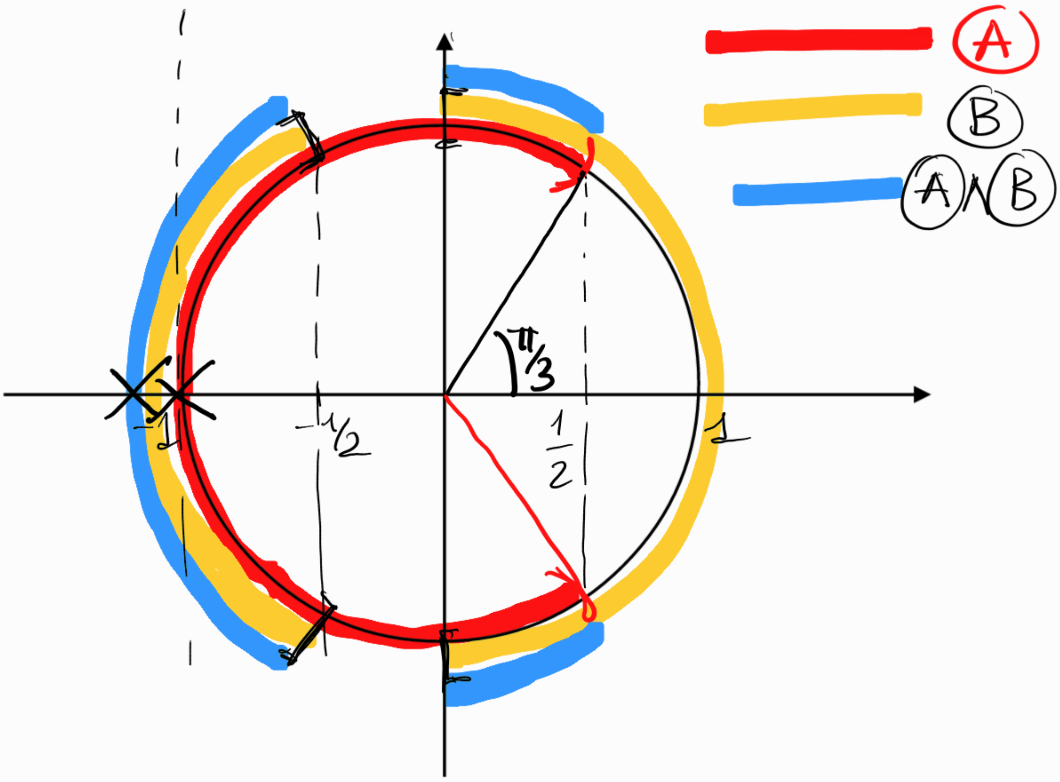
$$t_{1,2} = \frac{-1 \pm \sqrt{1+8}}{4} = \frac{-1 \pm 3}{4} = \begin{cases} -1 \\ \frac{1}{2} \end{cases}$$

$$\Updownarrow$$

$$-1 < t < \frac{1}{2}$$

(A) $\iff -1 < \cos x < \frac{1}{2}$

(A) $\iff \frac{\pi}{3} + 2k\pi < x < \frac{5}{3}\pi + 2k\pi, \quad x \neq \pi + 2k\pi$



(B) $\quad 2\sin^2 x - \cos x \le 2$

$$\cancel{2} - 2\cos^2 x - \cos x \le \cancel{2}$$

$$2\cos^2 x + \cos x \ge 0$$

$$\cos x\,(2\cos x + 1) \ge 0$$

(B) $\iff \left(\cos x \le -\frac{1}{2}\right) \vee \left(\cos x \ge 0\right)$

(B) $\iff \left(-\frac{\pi}{2} + 2k\pi \le x \le \frac{\pi}{2} + 2k\pi\right) \vee \left(\frac{2}{3}\pi + 2k\pi \le x \le \frac{4}{3}\pi + 2k\pi\right)$

$k \in \mathbb{Z}$

Dominio di f:

$$\left(\frac{\pi}{3} + 2k\pi < x \le \frac{\pi}{2} + 2k\pi\right) \vee \left(\frac{2}{3}\pi + 2k\pi \le x \le \frac{4}{3}\pi + 2k\pi\right) \vee$$

$$\vee \left(\frac{3}{2}\pi + 2k\pi \le x < \frac{5\pi}{3} + 2k\pi\right)$$

$x \ne \pi + 2k\pi$

Risolvere la diseq^{ne}

$$2\sin x - 1 < 2\cos x - tg\,x$$

$tg\,x = \dfrac{\sin x}{\cos x}$

$x \neq \dfrac{\pi}{2} + k\pi$

$k \in \mathbb{Z}$

$$2\sin x - 1 < 2\cos x - \frac{\sin x}{\cos x}$$

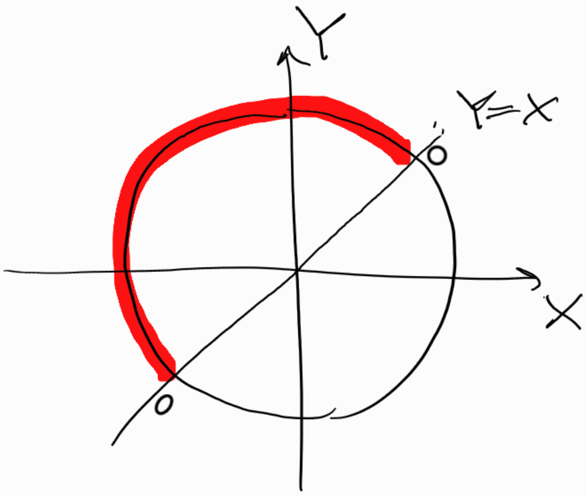
$$\frac{2\sin x\cos x - \cos x}{\cos x} < \frac{2\cos^2 x - \sin x}{\cos x}$$

$$\frac{2\sin x\cos x - \cos x - 2\cos^2 x + \sin x}{\cos x} < 0$$

$$2\cos x(\sin x - \cos x) + (\sin x - \cos x)$$

$$\frac{\overbrace{(\sin x - \cos x)}^{A}\overbrace{(2\cos x + 1)}^{B}}{\underbrace{\cos x}_{C}} < 0$$

Segno di A $\quad \underbrace{\sin x}_{\substack{\shortparallel \\ Y}} > \underbrace{\cos x}_{\substack{\shortparallel \\ X}}$



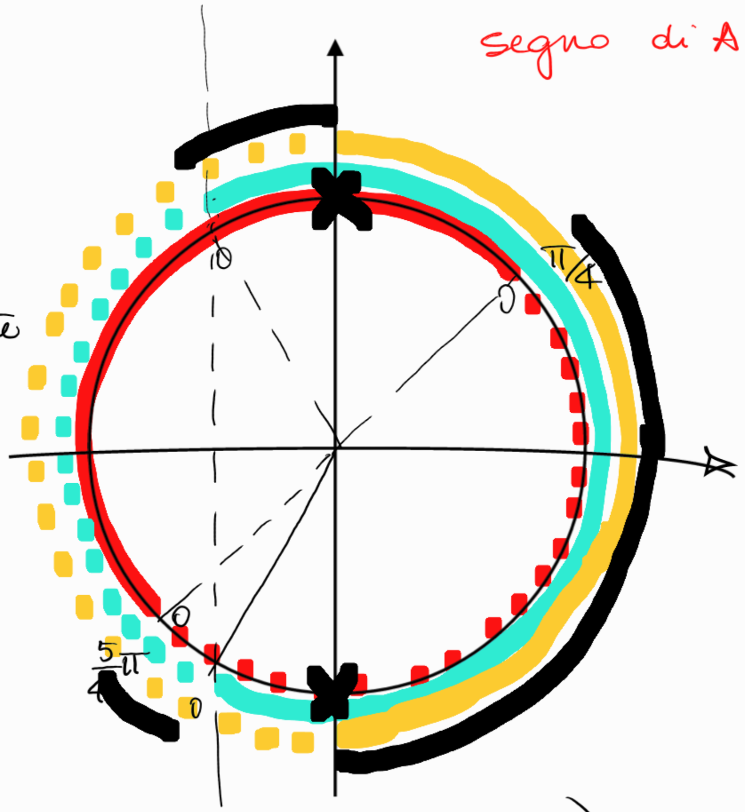
segno di B $\quad 2\cos x + 1 > 0$

$\cos x > -\dfrac{1}{2}$

segno di C $\quad \cos x > 0$

devo trovare gli x t.c.

$$\frac{A\,B}{C} < 0$$



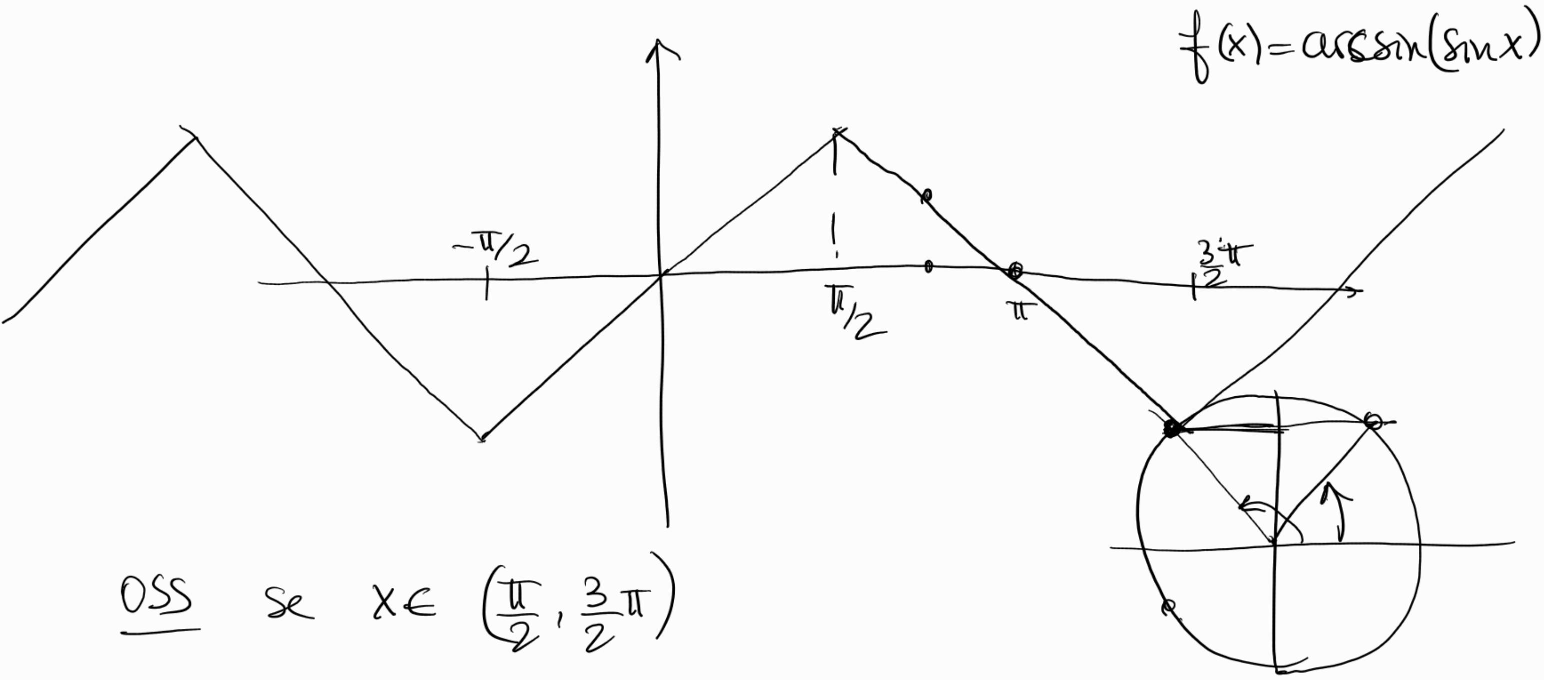
Sol^{ni} della diseq^{ne} sono mostrate in

$$\left(-\frac{\pi}{2}+2k\pi < x < \frac{\pi}{4}+2k\pi\right) \vee$$

$$\left(\frac{\pi}{2}+2k\pi < x < \frac{2}{3}\pi+2k\pi\right) \vee \left(\frac{5}{4}\pi+2k\pi < x < \frac{4}{3}\pi+2k\pi\right)$$

Disegnare il grafico di $f(x)=\arcsin(\sin x)$

N.B. $\arcsin(\sin x) = x$ solo se $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



OSS se $x \in \left(\frac{\pi}{2}, \frac{3}{2}\pi\right)$

$$\arcsin(\sin x) = \pi - x$$