La scorsa settimana abbiamo risolto.

$$\sin x - \sqrt{3}\cos x \geq \underbrace{\sqrt{3}}_{\substack{\approx \\ 1{,}7}}$$

al posto di $\sqrt{3}$ ci metto $\frac{3}{2} = 1{,}5$ (Questo cambia gli angoli: non saranno più angoli "noti")

$$\sin x - \frac{3}{2}\cos x \geq \frac{3}{2}$$

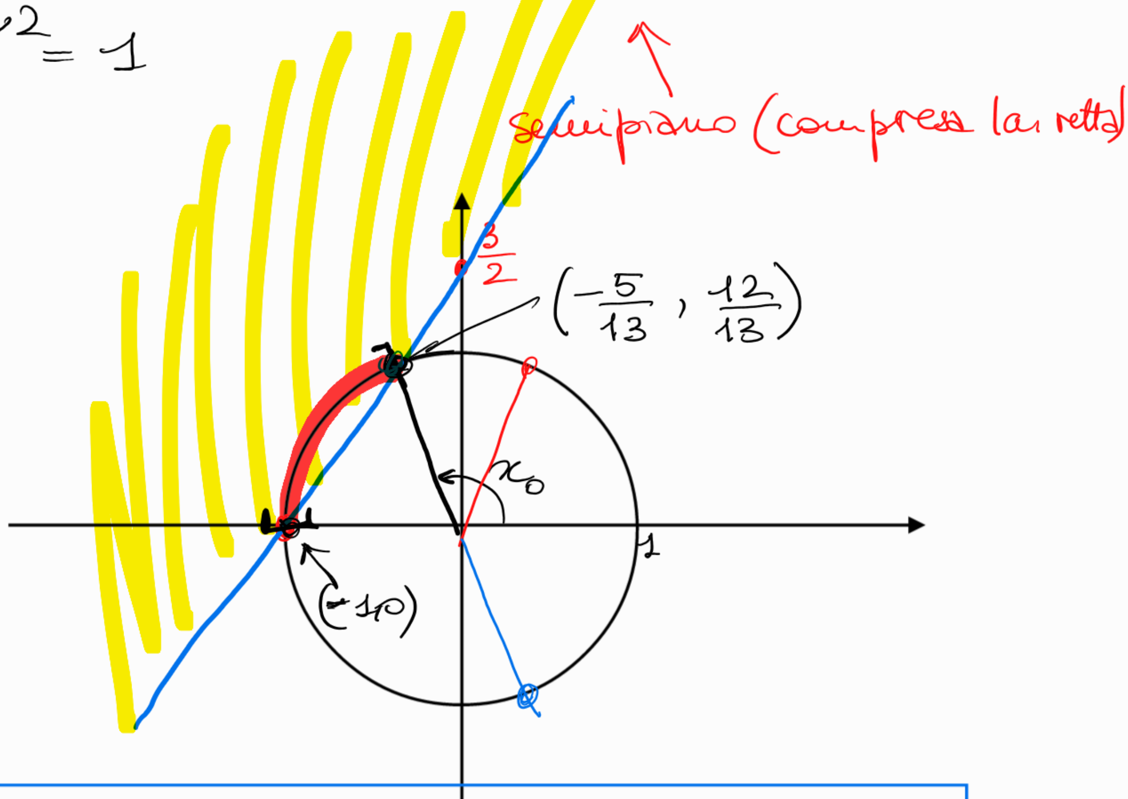
$$\boxed{2\sin x - 3\cos x \geq 3}$$

1° modo intersezione con la circ. goniometrica

$\sin x = Y$

$\cos x = X$

$$\begin{cases} 2Y - 3X \geq 3 \iff Y \geq \frac{3}{2}(X+1) \\ X^2 + Y^2 = 1 \end{cases}$$



Sol^{ni} $\boxed{x_0 + 2k\pi \leq x \leq \pi + 2k\pi \qquad k \in \mathbb{Z}}$

Per trovare x_0, trovo l'intersezione tra retta e circonf.

$$\begin{cases} Y = \frac{3}{2}(X+1) \\ X^2 + Y^2 = 1 \end{cases} \Rightarrow X^2 + \frac{9}{4}(X+1)^2 = 1$$

$$x^2+\frac{9}{4}\left(x^2+2x+1\right)=1.$$

$$4x^2+9x^2+18x+9=4$$

$$13x^2+18x+5=0$$

$$x=\frac{-9\pm\sqrt{81-65}}{13}=\frac{-9\pm4}{13}=\begin{cases}-1\\ -\frac{5}{13}\end{cases}$$

$$y=\frac{3}{2}(x+1)\begin{cases}0\\ \frac{3}{2}\left(1-\frac{5}{13}\right)=\frac{12}{13}\end{cases}$$

$$(-1,0)\qquad\left(-\frac{5}{13},\frac{12}{13}\right)$$

$$\cos x_0=-\frac{5}{13}\qquad x_0=\arccos\left(-\frac{5}{13}\right)\quad \text{Ok!}$$

$$\sin x_0=\frac{12}{13}\qquad x_0=\cancel{\arcsin\left(\frac{12}{13}\right)}\quad \pi-\arcsin\frac{12}{13}$$

$$\operatorname{tg} x_0=-\frac{12}{5}\qquad x_0=\cancel{\operatorname{arctg}\left(-\frac{12}{5}\right)}\quad \pi-\operatorname{arctg}\frac{12}{5}$$

2° modo: metodo dell'angolo aggiunto

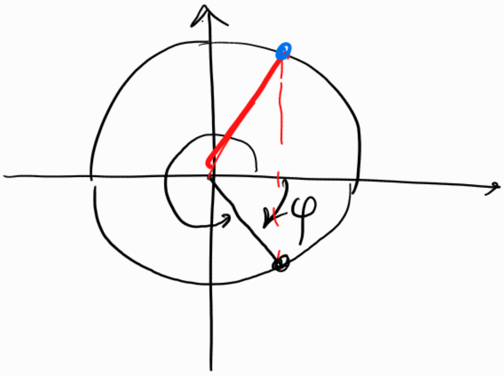
$$2\sin x-3\cos x\geq 3.$$

$$\underbrace{2}_{A}\sin x\underbrace{-3}_{B}\cos x=\sqrt{A^2+B^2}\,\sin(x+\varphi)$$

$$\begin{cases}\cos\varphi=\dfrac{A}{\sqrt{A^2+B^2}}\\[2ex] \sin\varphi=\dfrac{B}{\sqrt{A^2+B^2}}\end{cases}$$

Nel nostro caso $\sqrt{A^2+B^2}=\sqrt{4+9}=\sqrt{13}$

$$\begin{cases} \cos\varphi = \dfrac{2}{\sqrt{13}} \\ \sin\varphi = -\dfrac{3}{\sqrt{13}} \end{cases}$$



$$\varphi = \cancel{\arccos \frac{2}{\sqrt{13}}} \quad - \arccos \frac{2}{\sqrt{13}} \qquad \text{ok.}$$

$$\varphi = \arcsin\left(-\frac{3}{\sqrt{13}}\right) = -\arcsin \frac{3}{\sqrt{13}} \qquad \text{Ok}$$

$$\varphi = \operatorname{arctg}\left(-\frac{3}{2}\right) = -\operatorname{arctg} \frac{3}{2} \qquad \text{ok.}$$

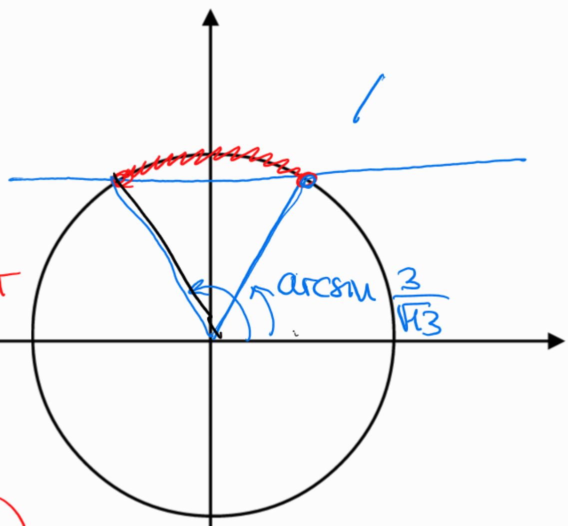
La diseq^ne^ diventa

$$\sqrt{13}\,\sin(\underbrace{x+\varphi}_{t}) \geq 3$$

$$\sin t \geq \frac{3}{\sqrt{13}}$$

$$\Updownarrow$$

$$\arcsin \frac{3}{\sqrt{13}} + 2k\pi \leq t \leq \pi - \arcsin \frac{3}{\sqrt{13}} + 2k\pi$$



$$\underset{\underset{-\varphi}{\shortparallel}}{\arcsin \frac{3}{\sqrt{13}}} + 2k\pi \leq x + \varphi \leq \underbrace{\pi - \arcsin \frac{3}{\sqrt{13}}}_{\pi + \varphi} + 2k\pi .$$

$$\boxed{-2\varphi + 2k\pi \leq x \leq \pi + 2k\pi . \qquad k \in \mathbb{Z}.}$$

Controllo che il risultato sia lo stesso di prima

$$-2\varphi \overset{?}{=} x_0$$

$$2\arcsin\left(\frac{3}{\sqrt{13}}\right) \overset{?}{=} \arccos\left(-\frac{5}{13}\right)$$

sono due angoli del 2° quadrante.

sono uguali se e solo se hanno lo stesso cos.

$$\cos\Big(2\underbrace{\arcsin\frac{3}{\sqrt{13}}}_{t}\Big) \overset{?}{=} \cos\left(\arccos -\frac{5}{13}\right) = -\frac{5}{13}$$

$$\cos(2t) = 1 - 2\sin^2 t$$

$$\cos\left(2\arcsin\frac{3}{\sqrt{13}}\right) = 1 - 2\left(\sin\left(\arcsin\frac{3}{\sqrt{13}}\right)\right)^2 =$$

$$= 1 - 2\cdot\left(\frac{3}{\sqrt{13}}\right)^2 = 1 - 2\cdot\frac{9}{13} = -\frac{5}{13}$$

sì, è la stessa soluzione.

3° modo metodo parametrico

$$t = tg\,\frac{x}{2}$$

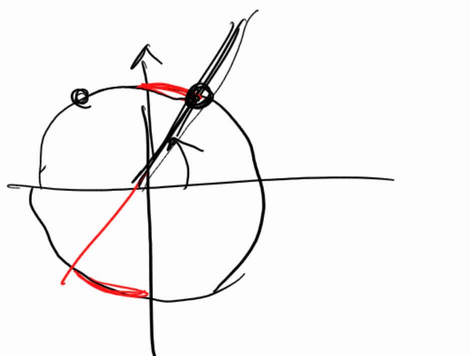
$$\sin x = \frac{2t}{1+t^2}, \quad \cos x = \frac{1-t^2}{1+t^2} \qquad \forall x \neq \pi + 2k\pi$$

$2\sin x - 3\cos x \geq 3$ diventa

$$\frac{4t}{1+t^2} - \frac{3(1-t^2)}{1+t^2} \geq 3$$

$$4t - 3 + \cancel{3t^2} \geq 3 + \cancel{3t^2}$$

$$\boxed{t \geq \frac{3}{2}} \qquad tg\,\frac{x}{2} \geq \frac{3}{2}$$



$$\operatorname{arctg}\frac{3}{2} + k\pi \leq \frac{x}{2} < \frac{\pi}{2} + k\pi$$

$$\underbrace{2\operatorname{arctg}\frac{3}{2} + 2k\pi} \leq x < \underbrace{\pi} + 2k\pi.$$

L'esclusione di $x=\pi$ è artificiale. Vediamo se è soluzione.

$$\underbrace{2\sin x}_{\substack{\shortparallel \\ 0}} - \underbrace{3\cos x}_{\substack{\shortparallel \\ 3}} \geq 3$$

$x=\pi$ vera!

Sol^{ni} sono

$$\underbrace{2\operatorname{arctg}\frac{3}{2}} + 2k\pi \leq x \leq \pi + 2k\pi$$

controllare che l'angolo sia lo stesso di prima.