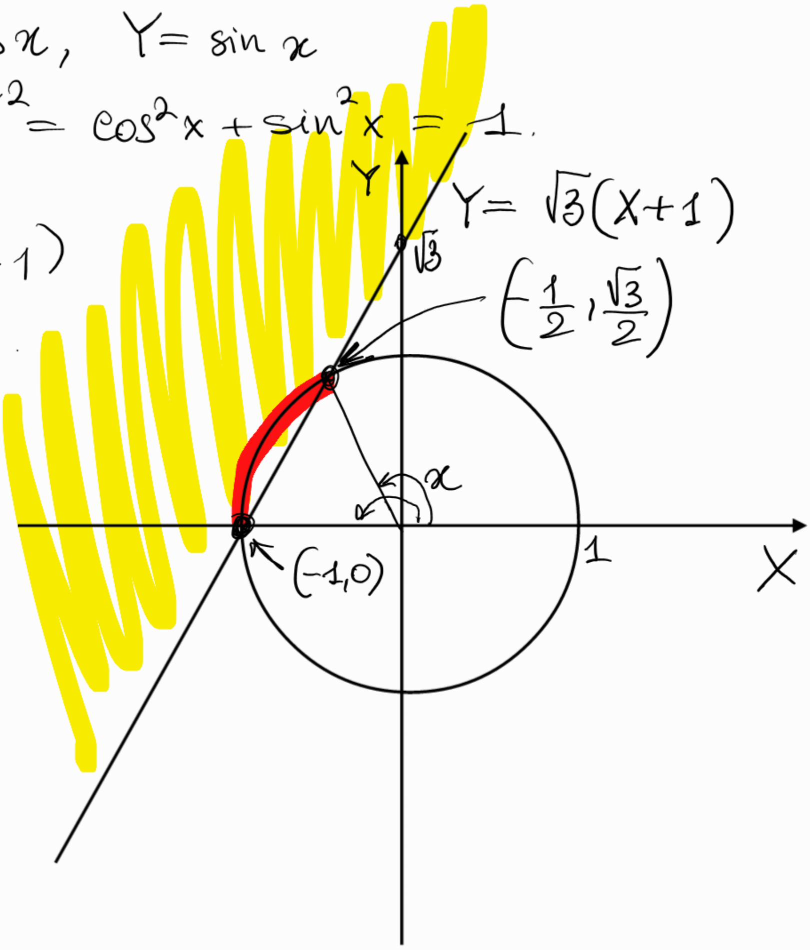
Risolvere $\boxed{\sin x - \sqrt{3}\cos x > \sqrt{3}}$

<u>1° modo:</u> Porre $X = \cos x$, $Y = \sin x$

e ricordare che $X^2 + Y^2 = \cos^2 x + \sin^2 x = 1$.

Si ottiene il sistema

$$\begin{cases} Y > \sqrt{3}(X+1) \\ X^2 + Y^2 = 1 \end{cases}$$



Trovo le intersezioni di retta e circ.

$$Y = \sqrt{3}(X+1)$$

$$X^2 + 3(X+1)^2 = 1$$

$$X^2 + 3X^2 + 6X + 3 = 1$$

$$4X^2 + 6X + 2 = 0$$

$$2X^2 + 3X + 1 = 0 \qquad X_{1,2} = \frac{-3 \pm \sqrt{9-8}}{4} =$$

$$= \frac{-3 \pm 1}{4} = \begin{cases} -1 \\ -\frac{1}{2} \end{cases}$$

$$X = -1 \Rightarrow Y = \sqrt{3}(-1+1) = 0$$

$$X = -\frac{1}{2} \Rightarrow Y = \sqrt{3}\left(-\frac{1}{2}+1\right) = \frac{\sqrt{3}}{2}.$$

$(-1,0)$ corrisponde a $x = \pi \ (+2k\pi)$

$\left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$ corrisponde a $\begin{cases} \cos x = -\frac{1}{2} \\ \operatorname{sen} x = \frac{\sqrt{3}}{2} \end{cases} \Rightarrow x = \frac{2}{3}\pi + 2k\pi$

Sol^{ne} $\quad \frac{2}{3}\pi + 2k\pi < x < \pi + 2k\pi \qquad k \in \mathbb{Z}$

2° modo consiste nell'osservare che una combinazione lineare di $\cos x$ e $\sin x$ è sempre una sinusoide, con un'ampiezza e una fase.

$$f(x) = A \sin x + B \cos x \qquad A, B \in \mathbb{R} \text{ non entrambi nulli}$$

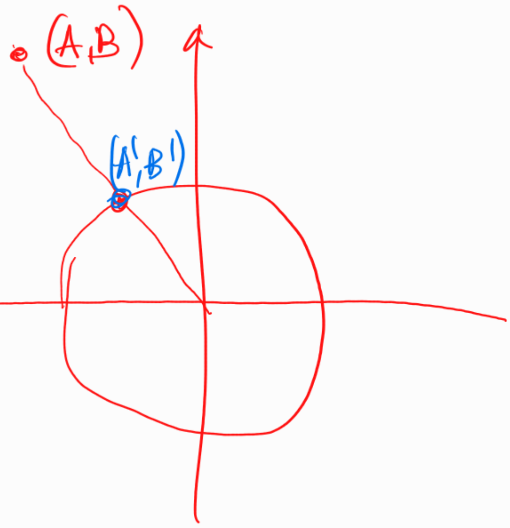
Formula per il seno di una somma:

$$\sin(x+\varphi) = \underbrace{\cos\varphi}_{=A} \sin x + \underbrace{\sin\varphi}_{=B} \cos x \qquad \forall \varphi, x \in \mathbb{R}.$$

ma questo è possibile solo se $A^2 + B^2 = 1$

$$f(x) = A \sin x + B \cos x =$$

$$= \sqrt{A^2+B^2}\left(\underbrace{\frac{A}{\sqrt{A^2+B^2}}}_{A'} \sin x + \underbrace{\frac{B}{\sqrt{A^2+B^2}}}_{B'} \cos x\right)$$



$$(A')^2 + (B')^2 = \frac{A^2}{A^2+B^2} + \frac{B^2}{A^2+B^2} = 1$$

$$f(x) = \sqrt{A^2+B^2}\left(\underbrace{\frac{A}{\sqrt{A^2+B^2}}}_{\cos\varphi} \sin x + \underbrace{\frac{B}{\sqrt{A^2+B^2}}}_{\sin\varphi,} \cos x\right)$$

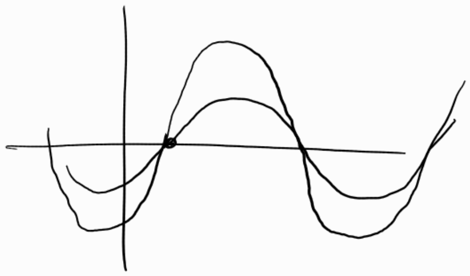
Risolvo il sistema in φ

$$\begin{cases} \cos\varphi = \dfrac{A}{\sqrt{A^2+B^2}} \\ \sin\varphi = \dfrac{B}{\sqrt{A^2+B^2}} \end{cases} \Rightarrow \text{trovo } \varphi \text{ a meno di multipli di } 2\pi.$$

$$f(x) = A\sin x + B\cos x = \sqrt{A^2+B^2}\,\sin(x+\varphi)$$

Nel nostro caso particolare

$$\sin x - \sqrt{3}\cos x = 2\sin\left(x - \frac{\pi}{3}\right)$$



$A = 1$

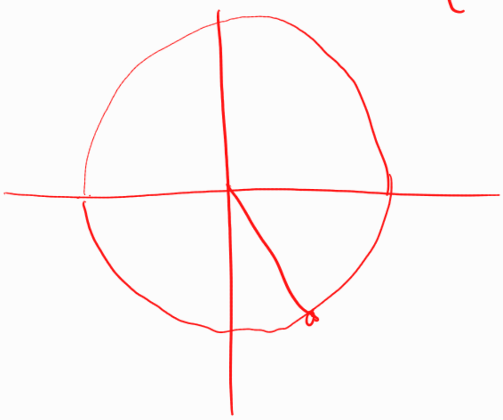
$B = -\sqrt{3}$

$$\sqrt{A^2+B^2} = \sqrt{1+3} = 2$$

φ è sol^{ne} di:

$$\begin{cases} \cos\varphi = \dfrac{A}{\sqrt{A^2+B^2}} = \dfrac{1}{2} \\ \sin\varphi = \dfrac{B}{\sqrt{A^2+B^2}} = -\dfrac{\sqrt{3}}{2} \end{cases} \Rightarrow \varphi = -\frac{\pi}{3} + 2k\pi$$

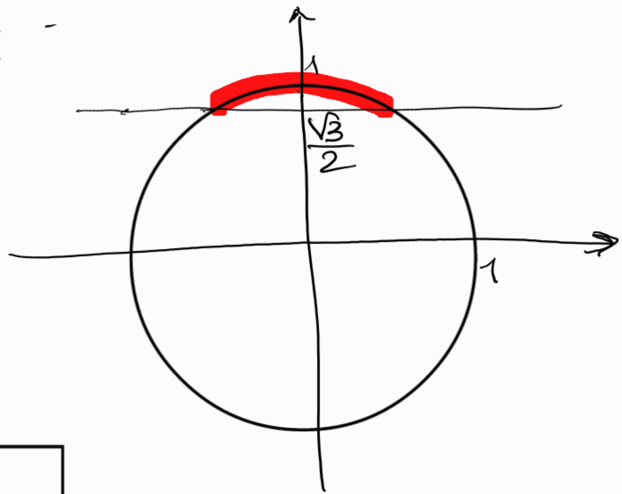
denom. $\neq 0$ perché A, B non sono entrambi nulli.



La diseq^{ne} $\sin x - \sqrt{3}\cos x > \sqrt{3}$ diventa

$$2\sin\underbrace{\left(x - \frac{\pi}{3}\right)}_{t} > \sqrt{3}$$

$$\sin t > \frac{\sqrt{3}}{2}.$$



$$\frac{\pi}{3} + 2k\pi < \underset{\substack{\parallel \\ x - \frac{\pi}{3}}}{t} < \frac{2}{3}\pi + 2k\pi$$

$$\boxed{\frac{2}{3}\pi + 2k\pi < x < \pi + 2k\pi}$$

come prima!

3° modo Trasformiamo tutto in funzione di $t = tg\frac{x}{2}$.

Dimostreremo le seguenti formule

$$\boxed{\cos x = \frac{1-t^2}{1+t^2}, \quad \sin x = \frac{2t}{1+t^2} \qquad t = tg\frac{x}{2}}$$

$\forall x$ t.c. $\cancel{\frac{x}{2}} \neq (2k+1)\frac{\pi}{\cancel{2}}$

$x \neq (2k+1)\pi$

$$\sin x - \sqrt{3}\cos x > \sqrt{3}$$

$t = tg\frac{x}{2}$

La diseq^ne diventa $\leftarrow$ $x = \pi + 2k\pi$ va studiato a parte

$$\frac{2t}{1+t^2} - \frac{\sqrt{3}(1-t^2)}{1+t^2} > \sqrt{3}$$

$$\Updownarrow$$

$$2t - \sqrt{3}(1-t^2) > \sqrt{3}(1+t^2)$$

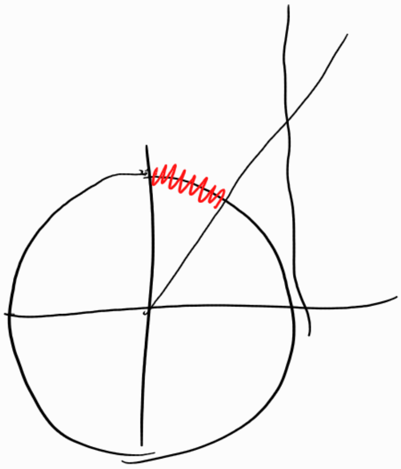
$$2t - \sqrt{3} + \cancel{\sqrt{3}t^2} > \sqrt{3} + \cancel{\sqrt{3}t^2}$$

$$2t > 2\sqrt{3}$$

$$t > \sqrt{3} \iff tg\frac{x}{2} > \sqrt{3}$$

$$\Updownarrow$$

$$\frac{\pi}{3} + k\pi < \frac{x}{2} < \frac{\pi}{2} + k\pi$$



$$\Updownarrow$$

$$\boxed{\frac{2\pi}{3} + 2k\pi < x < \pi + 2k\pi} \leftarrow$$

devo controllare cosa succede per $x = \pi + 2k\pi$ nell'eq^ne orig.

$\sin x - \sqrt{3}\cos x > \sqrt{3}$ $\qquad$ $x = \pi$

$0 \quad + \sqrt{3} \quad \overset{?}{>} \sqrt{3}$ $\qquad$ NO

non è sol^{ne}