Funzioni "trigonometriche inverse"

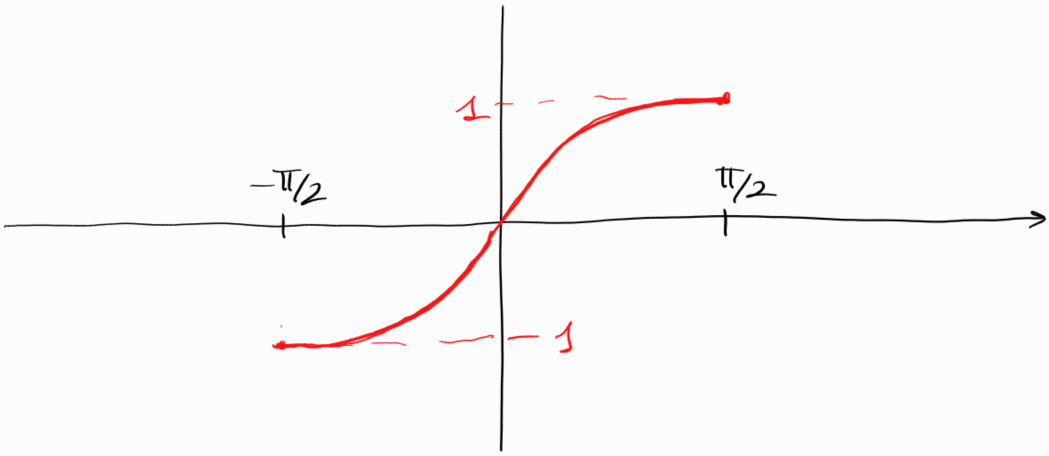
$$f(x) = \sin x \quad : \mathbb{R} \longrightarrow \mathbb{R}$$
$$x \longmapsto \sin x.$$

non è né iniettiva né suriettiva.

La suriettività si recupera prendendo $[-1,1]$ come insieme di arrivo.

$$f: \quad \mathbb{R} \to [-1,1] \qquad \text{suriettiva.}$$
$$x \mapsto \sin x$$

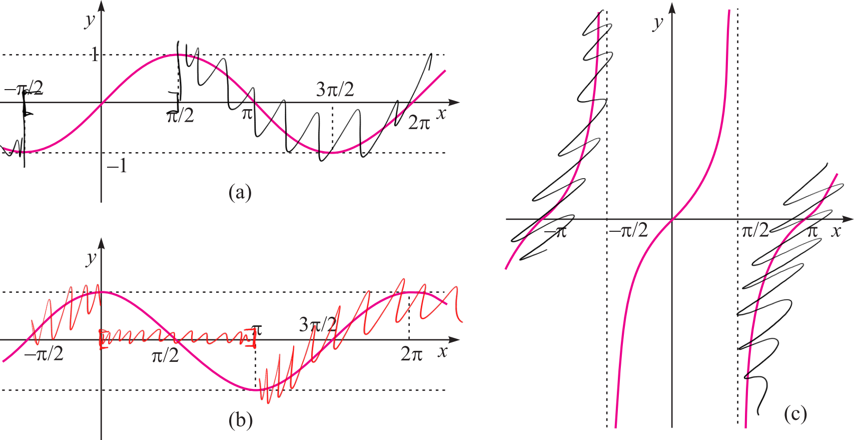
Per "recuperare" l'iniettività mi restringo a un intervallo in cui f è strett. monotòna. Per convenzione si prende $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.



$$f(x) = \sin\Big|_{\left[-\frac{\pi}{2},\frac{\pi}{2}\right]} x \; : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1,1].$$
$$x \longmapsto f(x) = \sin x$$

è biiettiva:

- iniettiva perché strettamente crescente
- suriettiva in quanto continua in un intervallo $\overset{\text{Teor.}}{\Longrightarrow}$ assume tutti i valori compresi tra $\inf f = -1$ e $\sup f = 1$.



y
1
−π/2
π/2
π
3π/2
2π
x
−1
(a)
y
−π/2
π/2
π
3π/2
2π
x
(b)
y
−π
−π/2
π/2
π
x
(c)

$$\Rightarrow \quad \forall y \in [-1,1] \quad \exists!\ x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ t.c. } \sin x = y.$$

$$f^{-1}(y) = \arcsin y : [-1,1] \longrightarrow \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$$

$$y \longmapsto \text{l'unico } x \text{ t.c. } \sin x = y$$

se $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $y \in [-1,1]$.

$$x = \arcsin y \iff y = \sin x.$$

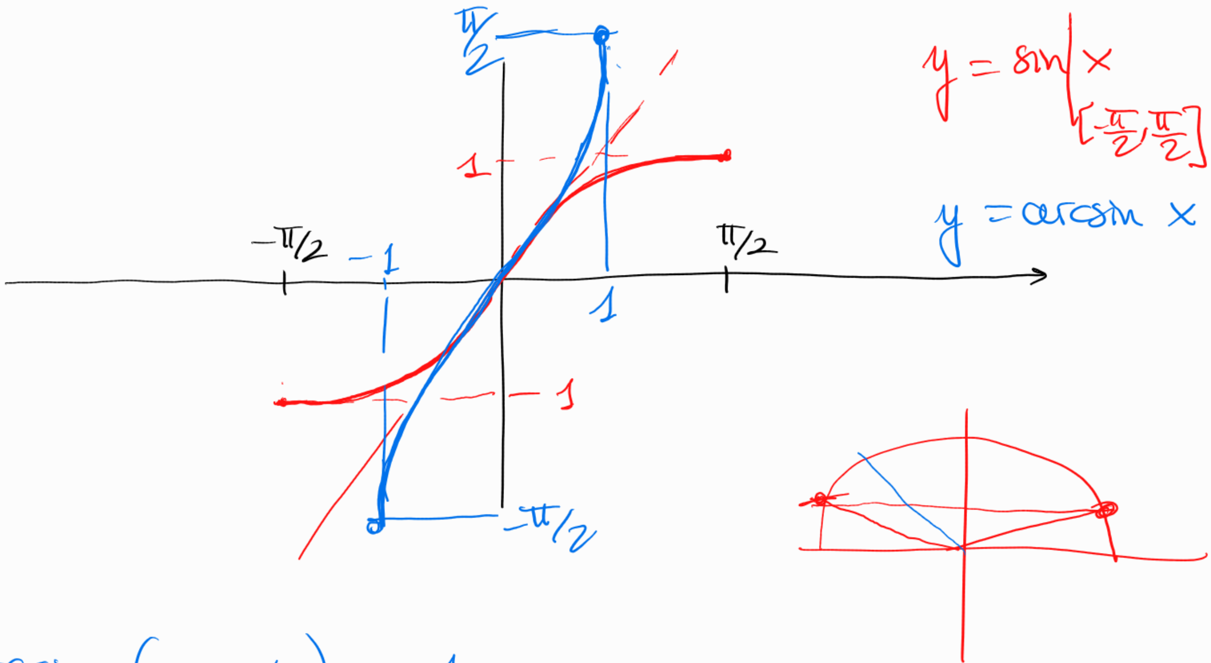
$$\arcsin 0 = 0$$

$$\cancel{\arcsin 0 = \pi}$$

$$\arcsin 0 = k\pi \quad k \in \mathbb{Z}.$$

$$\arcsin \frac{1}{2} = \frac{\pi}{6}$$

$$\arcsin\left(-\frac{\sqrt{3}}{2}\right) = -\frac{\pi}{3}$$



$$\arcsin\left(\sin \frac{1}{4}\right) = \frac{1}{4}$$

$$\arcsin(\sin 3) = \pi - 3 \quad (\text{non } 3)$$

$$\arcsin\left(\sin \frac{3}{4}\pi\right) = \frac{\pi}{4} \quad \left(\text{non } \frac{3}{4}\pi\right)$$

$\arcsin(\sin(-\pi)) = 0$

$\sin(\arcsin x) = x \qquad \forall x \in [-1,1]$.

$f(x) = \cos\big|_{[0,\pi]} x \; : \; [0,\pi] \longrightarrow [-1,1]$. biiettiva.

$\qquad\qquad x \longmapsto \cos x$.

$f^{-1}(y) = \arccos y \; : \; [-1,1] \longrightarrow [0,\pi]$

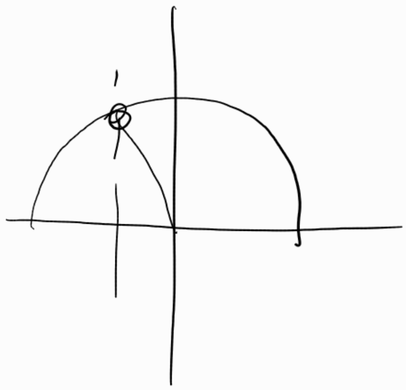
$\qquad\qquad y \longmapsto$ l'unico x t.c. $\cos x = y$.

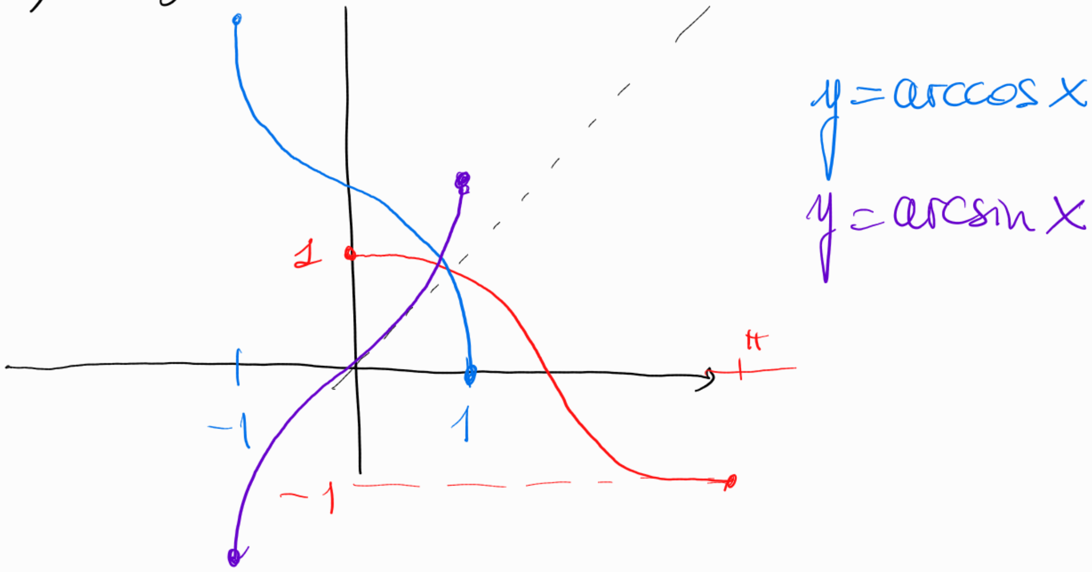
$\arccos 0 = \frac{\pi}{2}$

$\arccos 1 = 0$

$\arccos(-1) = \pi$

$\arccos\left(-\frac{1}{2}\right) = \frac{2\pi}{3}$





$$\boxed{\arccos x = \frac{\pi}{2} - \arcsin x}$$

$$f(x) = \operatorname{tg}\big|_{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)} x \; : \left(-\frac{\pi}{2},\frac{\pi}{2}\right) \to \mathbb{R} \qquad \text{biiettiva}$$

$$x \longmapsto \operatorname{tg} x$$

$$f^{-1}(y) = \operatorname{arctg} y \; : \mathbb{R} \to \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

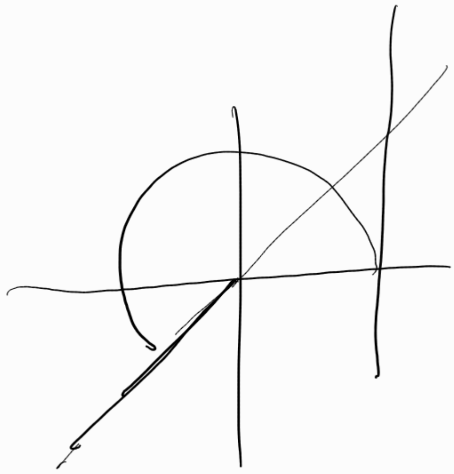
$$y \longmapsto \text{l'unico } x \text{ t.c. } \operatorname{tg} x = y$$

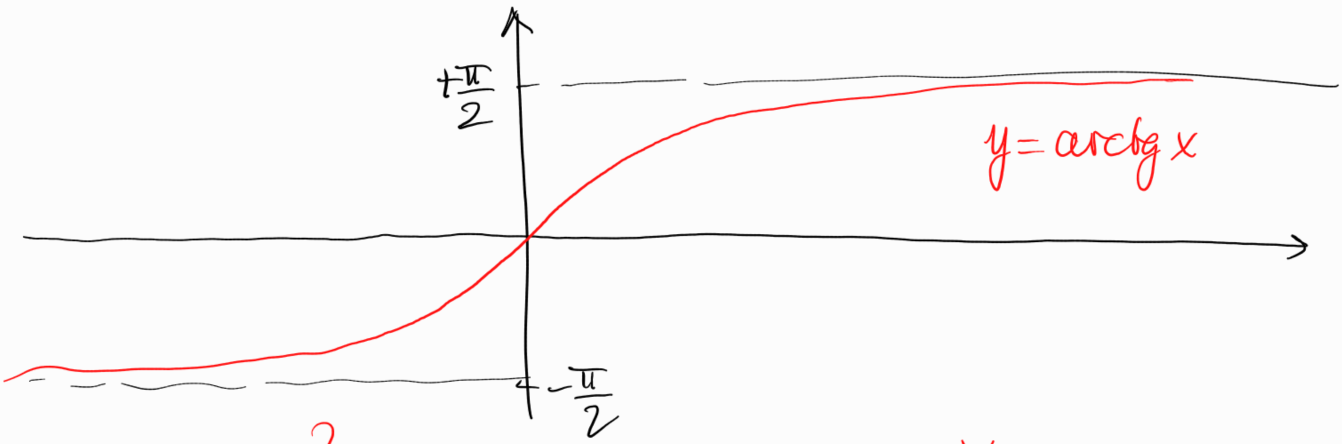
$$\operatorname{arctg} 1 = \frac{\pi}{4}$$

$$\operatorname{arctg}(-\sqrt{3}) = -\frac{\pi}{3}$$

$$\operatorname{arctg} 0 = 0$$

$$\operatorname{arctg}\left(\operatorname{tg} \frac{5}{4}\pi\right) = \frac{\pi}{4}$$





$$\operatorname{arctg}(-x) \overset{?}{=} -\operatorname{arctg} x \qquad \forall x \in \mathbb{R}.$$

$$-x = \operatorname{tg}(\operatorname{arctg}(-x)) \overset{?}{=} \operatorname{tg}(-\operatorname{arctg} x) = -\operatorname{tg}(\operatorname{arctg} x) = -x$$

Risolvere:

$$\sin x = \frac{1}{3} \iff \left(x = \arcsin\frac{1}{3} + 2k\pi\right) \vee \left(x = \pi - \arcsin\frac{1}{3} + 2k\pi\right)$$