Trovare il dominio naturale di

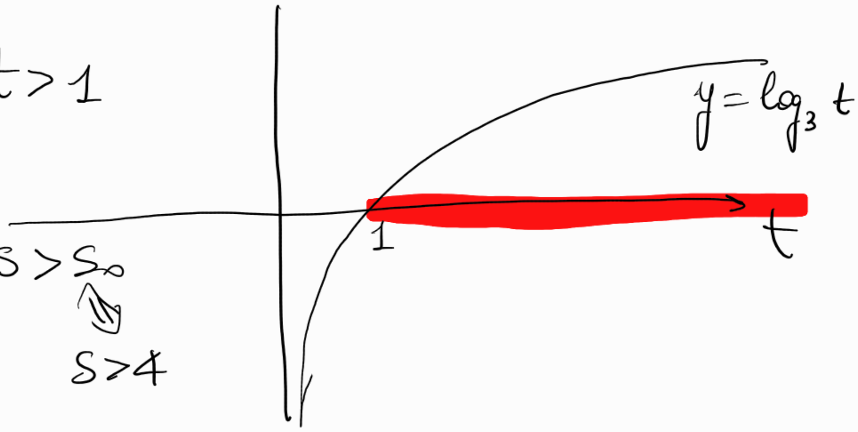
$$f(x) = \left(\log_3\left(\log_4\left(x^2-5\right)\right)\right)^{-1/4}$$

Cominciare dall'esterno. $t^{-1/4}$ è definito per $t>0$.

$$\log_3\left(\underbrace{\log_4\left(x^2-5\right)}_{t}\right) > 0$$

cioè: questa quantità <u>deve esistere</u> ed essere $>0$

$\log_3 t > 0 \iff t > 1$



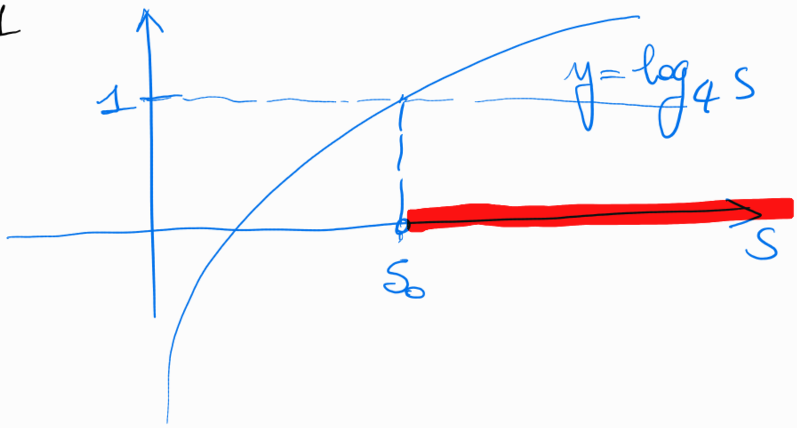

$\log_4\left(\underbrace{x^2-5}_{s}\right) > 1 \iff s > s_0$

$\Updownarrow$

$s > 4$

dove $s_0$ è t.c. $\log_4 s_0 = 1$

$$\boxed{s_0 = 4}$$




$x^2 - 5 > 4$

$x^2 > 9$

$\text{dom} f = (-\infty, -3) \cup (3, +\infty)$

$$f(x) = \underbrace{\log_{1/2} x + \log_{1/2}\left(|x|-1\right)}_{\overset{\text{vera se } x>0,\ |x|-1>0}{=}\ \log_{1/2}\left(x\left(|x|-1\right)\right)} \geq 0.$$

E' sbagliato fare subito questo passaggio perché cambia il dominio.

$$f(x) = \log_{1/2} x + \log_{1/2}(|x|-1) \geq 0 \qquad \boxed{\text{C.E. } x>1}$$

dominio $\begin{cases} x>0 \\ |x|-1>0 \end{cases}$ $\quad \begin{cases} x>0 \\ x>1 \end{cases}$ deve essere $x>1$

Invece dominio di $\log_{1/2}\left(x\left(|x|-1\right)\right)$ qual è? sarebbe $(-1,0)\cup(1,+\infty)$

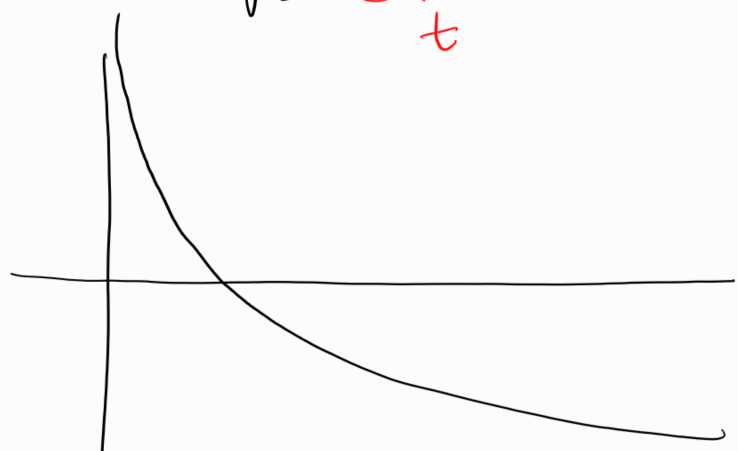
$\begin{cases} x>0 \\ x>1 \end{cases}$ $\qquad \begin{cases} x<0 \\ |x|<1 \end{cases}$ $\quad x<0 \quad -x<1 \quad x>-1$

$x>1 \qquad\qquad -1<x<0$

Dopo aver imposto $\boxed{x>1}$, posso scrivere

$$\log_{1/2} x + \log_{1/2}(|x|-1) = \log_{1/2}\big(\underbrace{x(x-1)}_{t}\big) \overset{?}{\geq} 0$$

$\log_{1/2} t > 0 \iff 0<t<1$



$y = \log_{1/2} t$

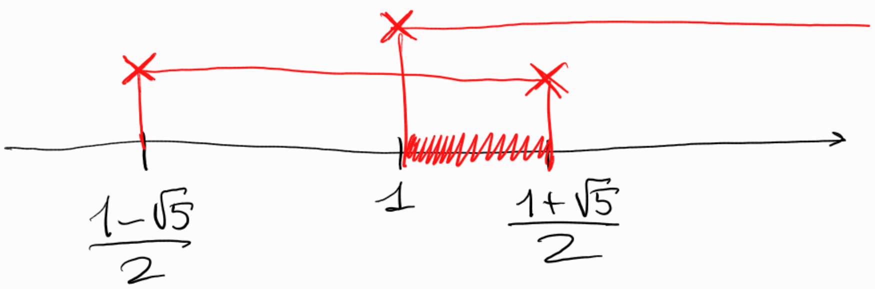
$0 < x(x-1) < 1$

vera per $x>1$

$x^2 - x - 1 < 0 \iff \boxed{\dfrac{1-\sqrt5}{2} < x < \dfrac{1+\sqrt5}{2}}$

$$x_{1,2} = \frac{1 \pm \sqrt{1+4}}{2} = \frac{1\pm\sqrt5}{2}$$

$\begin{cases} x>1 \\ \dfrac{1-\sqrt5}{2} < x < \dfrac{1+\sqrt5}{2} \end{cases}$



Sol$^{ne}$ $\boxed{1 < x < \frac{1+\sqrt{5}}{2}}$

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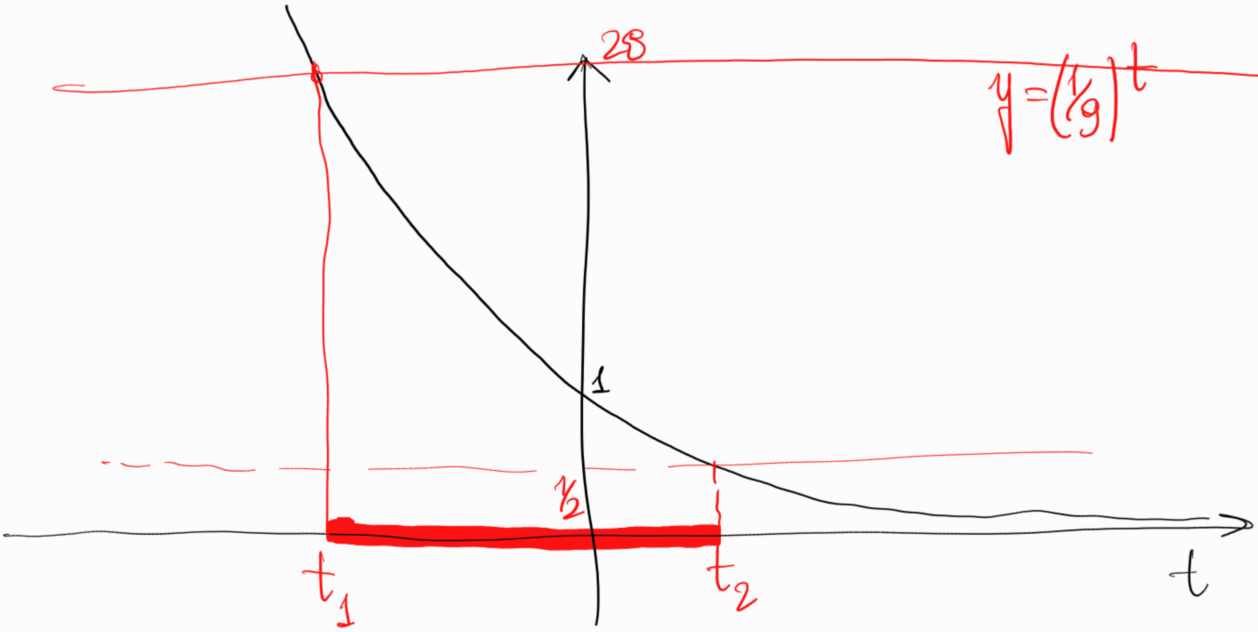
$$\boxed{\frac{1}{2} < \left(\frac{1}{9}\right)^{\underset{t}{2x-3}} < 28}$$




Sol$^{ni}$ $t_1 < t < t_2$

dove $t_1$ è t.c. $\left(\frac{1}{9}\right)^{t_1} = 28 \iff t_1 = \log_{1/9} 28$

$t_2$ è t.c. $\left(\frac{1}{9}\right)^{t_2} = \frac{1}{2} \iff t_2 = \log_{1/9} 1/2$

Sol$^{ni}$ $\log_{1/9} 28 < 2x-3 < \log_{1/9} \frac{1}{2}$

$3 + \log_{1/9} 28 < 2x < 3 + \log_{1/9} \frac{1}{2}$

$$\frac{3+\log_{1/9} 28}{2} < x < \frac{3+\log_{1/9} 1/2}{2}$$

Oppure

$$\frac{1}{2} < \left(\frac{1}{9}\right)^{2x-3} < 28$$

$$\Updownarrow$$

$$\log_{1/9}\frac{1}{2} > 2x-3 > \log_{1/9} 28$$

Applico $\log_{1/9} t$

Attenzione: è decresc.

(cambio i versi delle disug.)