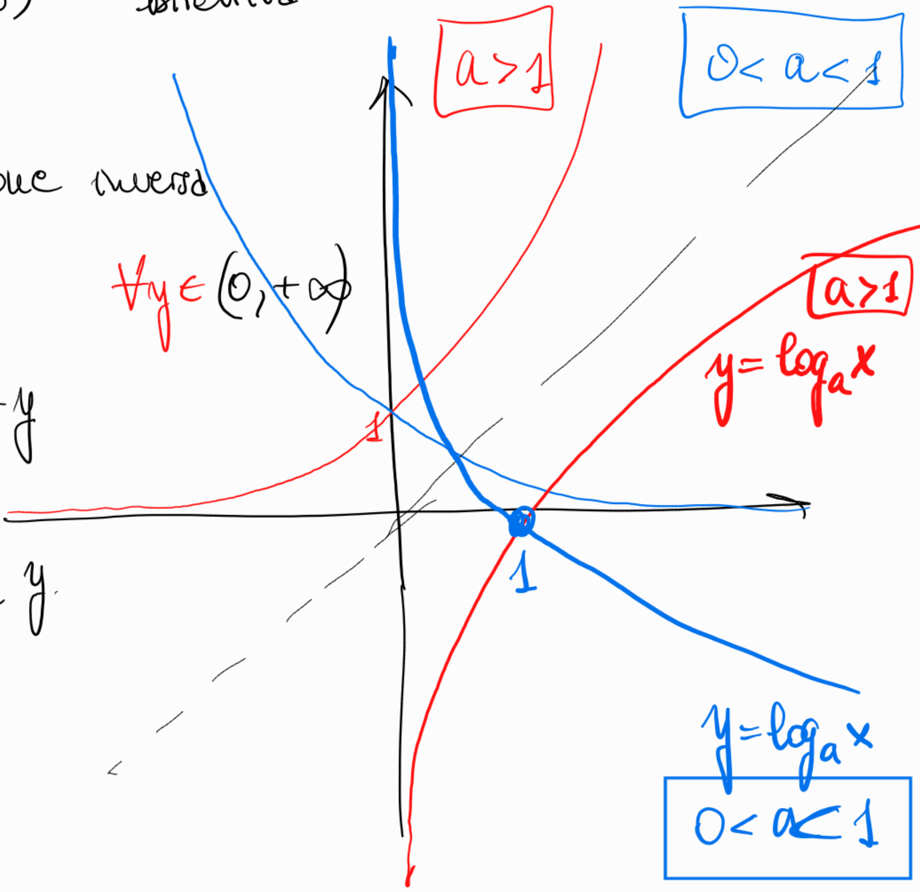
$$f : \mathbb{R} \longrightarrow (0,+\infty) \quad \text{biiettiva}$$
$$x \longmapsto a^x$$

Resta definita la funzione inversa

$f^{-1}(y) = \log_a y =$ $\forall y \in (0,+\infty)$

$=$ l'unico $x \in \mathbb{R}$ t.c. $a^x = y$

$$a^x = y \iff x = \log_a y.$$



Proprietà del logaritmo

$\log_a x$ deve essere $x \in (0,+\infty)$
$a \in (0,+\infty)$, $a \neq 1$

$$\log_3 81 = 4$$

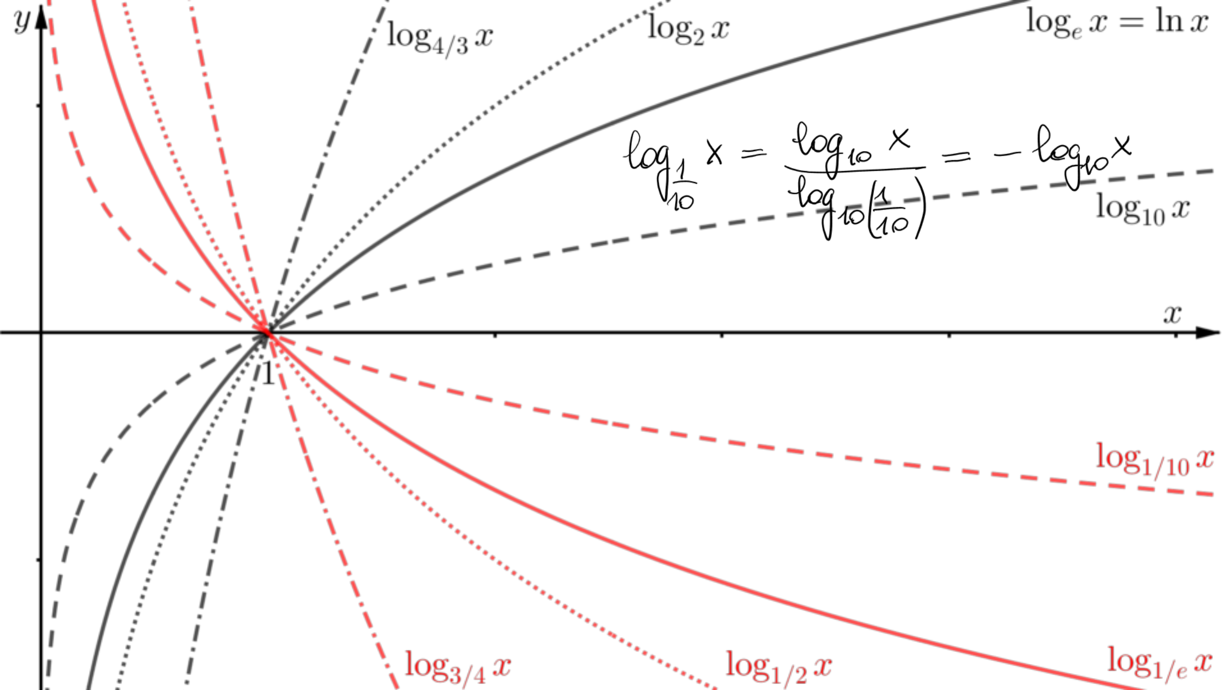
$$\log_3 \sqrt{3} = 1/2$$

$$\log_{\sqrt{3}} 3 = 2.$$

$$\log_{1/2} 64 = -6$$

$$\log_a 1 = 0 \qquad \forall a>0,\ a \neq 1.$$

$$\underline{f\left(f^{-1}(y)\right) = y} \qquad\qquad f^{-1}\left(f(x)\right) = x$$


y
x
1
$\log_{4/3} x$
$\log_2 x$
$\log_e x = \ln x$
$\log_{10} x$
$\log_{1/10} x$
$\log_{3/4} x$
$\log_{1/2} x$
$\log_{1/e} x$
$\log_{\frac{1}{10}} x = \frac{\log_{10} x}{\log_{10}\left(\frac{1}{10}\right)} = -\log_{10} x$

1) $$\boxed{\begin{aligned} a^{\log_a x} &= x \\ \log_a(a^x) &= x \end{aligned}}$$

$\forall x \in (0,+\infty)$

$\forall x \in \mathbb{R}$.

2) $$\boxed{\log_a(xy) = \log_a x + \log_a y \qquad \forall x,y>0}$$

$$\log_a(xy) = t \iff a^t = xy$$

$$\log_a(xy) = \log_a x + \log_a y \iff a^{\log_a x + \log_a y} = xy$$

$$a^{\log_a x + \log_a y} = \underbrace{a^{\log_a x}}_{x} \cdot \underbrace{a^{\log_a y}}_{y} = x \cdot y$$

Ne segue che

$$\log_a(x^2) = \log_a(x \cdot x) = 2\log_a x$$

vero se $x>0$

falso se $x<0$

definito $\forall x \neq 0$

$$\boxed{\log_a(x^2) = 2\log_a(|x|)} \qquad \forall x \neq 0$$

$$\left.\begin{aligned} \log_a(x^2) &= \log_a(x \cdot x) = 2\log_a x \\ \log_a(x^2) &= \log_a((-x)(-x)) = 2\log_a(-x) \end{aligned}\right\} 2\log_a|x| \qquad \begin{aligned} &\text{se } x>0 \\ &\text{se } x<0 \end{aligned}$$

$$\log_2(x^2-x) = \log_2(x(x-1)) = \log_2 x + \log_2(x-1)$$

In generale se non preciso dove varia x, è scorretto.

definito in $(-\infty,0) \cup (1,+\infty)$

definito in $(1,+\infty)$

$\log_2(x^2-x)$ dominio: $x^2-x>0$

cioè $x \in (-\infty, 0) \cup (1, +\infty)$

$$\log_2(\underbrace{x^2-x}_{= x(x-1)}) = \begin{cases} \log_2 x + \log_2(x-1) & x>1 \\ \log_2(-x) + \log_2(1-x) & x<0 \end{cases} = \log_2|x| + \log_2|x-1|$$

attenzione! questa ha un dominio differente $\mathbb{R} \setminus \{0,1\}$

$$\log_2(x^2-x) = \log_2|x| + \log_2|x-1| \quad \forall x \in (-\infty,0) \cup (1,+\infty)$$

OSS se a (base) $= e$ $\quad \log_a x = \underline{\log x} = \ln x$

$$\boxed{3) \quad \log_a\left(\frac{x}{y}\right) = \underbrace{\log_a x - \log_a y}_{t} \quad \forall x, y > 0.}$$

$$\log_a\left(\frac{x}{y}\right) = t \iff a^t = \frac{x}{y}$$

$$a^t = a^{\log_a x - \log_a y} = \frac{a^{\log_a x}}{a^{\log_a y}} = \frac{x}{y}$$

$$\boxed{4) \quad \log_a(x^r) = \underbrace{r \log_a x}_{t} \quad \begin{array}{c} \forall x>0 \\ \hline \forall r \in \mathbb{R}. \end{array}}$$

$$\log_a(x^r) = t \iff a^t = x^r$$

$$a^t = a^{r \log_a x} = (a^{\log_a x})^r = x^r. \qquad \square$$

5) Cambio di base dei logaritmi.

$$\log_a x = \frac{\log_b x}{\underbrace{\log_b a}_{\neq 0 \text{ perché } a \neq 1}} \qquad \forall x > 0 \quad \forall a, b > 0; \quad a, b \neq 1.$$

$$\Updownarrow$$

$$\underbrace{\log_b a \, \log_a x}_{t} \stackrel{?}{=} \log_b x$$

$$\log_b x = t \iff b^t = x$$

$$b^t = b^{\log_b a \, \log_a x} = (\underbrace{b^{\log_b a}}_{= a})^{\log_a x} = a^{\log_a x} = x$$