$$\sqrt[n]{P(x)} \gtreqless Q(x) \qquad n \text{ dispari}$$

$$\sqrt[n]{P(x)} \leq Q(x) \qquad n \text{ pari}$$

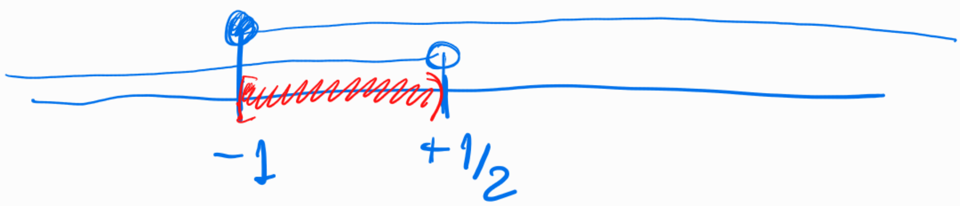
$$\sqrt[n]{P(x)} \geq Q(x) \qquad n \text{ pari}$$

$$\begin{cases} P(x) \geq 0 \\ Q(x) < 0 \end{cases} \quad \vee \quad \begin{cases} \cancel{P(x) \geq 0} \\ Q(x) \geq 0 \\ P(x) > (Q(x))^n \end{cases}$$

Questa è implicata dall'ultima

$$\sqrt{3(x+1)} > 2x-1$$

(A) $\begin{cases} 3(x+1) \geq 0 & x \geq -1 \\ 2x-1 < 0 & x < 1/2 \end{cases}$



sol (A) : $x \in \left[-1, \frac{1}{2}\right)$

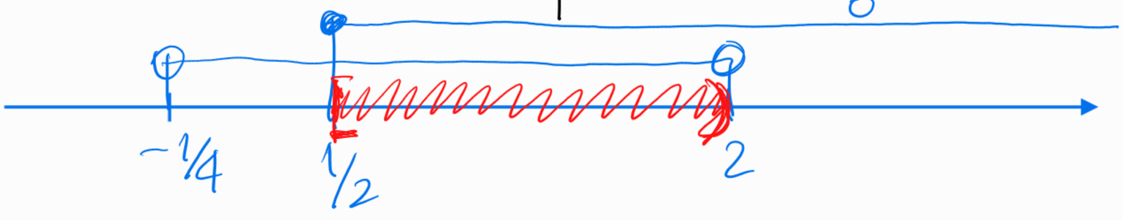
(B) $\begin{cases} 2x-1 \geq 0 & x \geq 1/2 \\ \boxed{3(x+1) > (2x-1)^2} \end{cases}$

$3x+3 > 4x^2-4x+1$

$4x^2-7x-2 < 0$

$-1/4 < x < 2$

$$x_{1,2} = \frac{7 \pm \sqrt{49+32}}{8} = \frac{7 \pm 9}{8} = \begin{cases} -1/4 \\ 2 \end{cases}$$



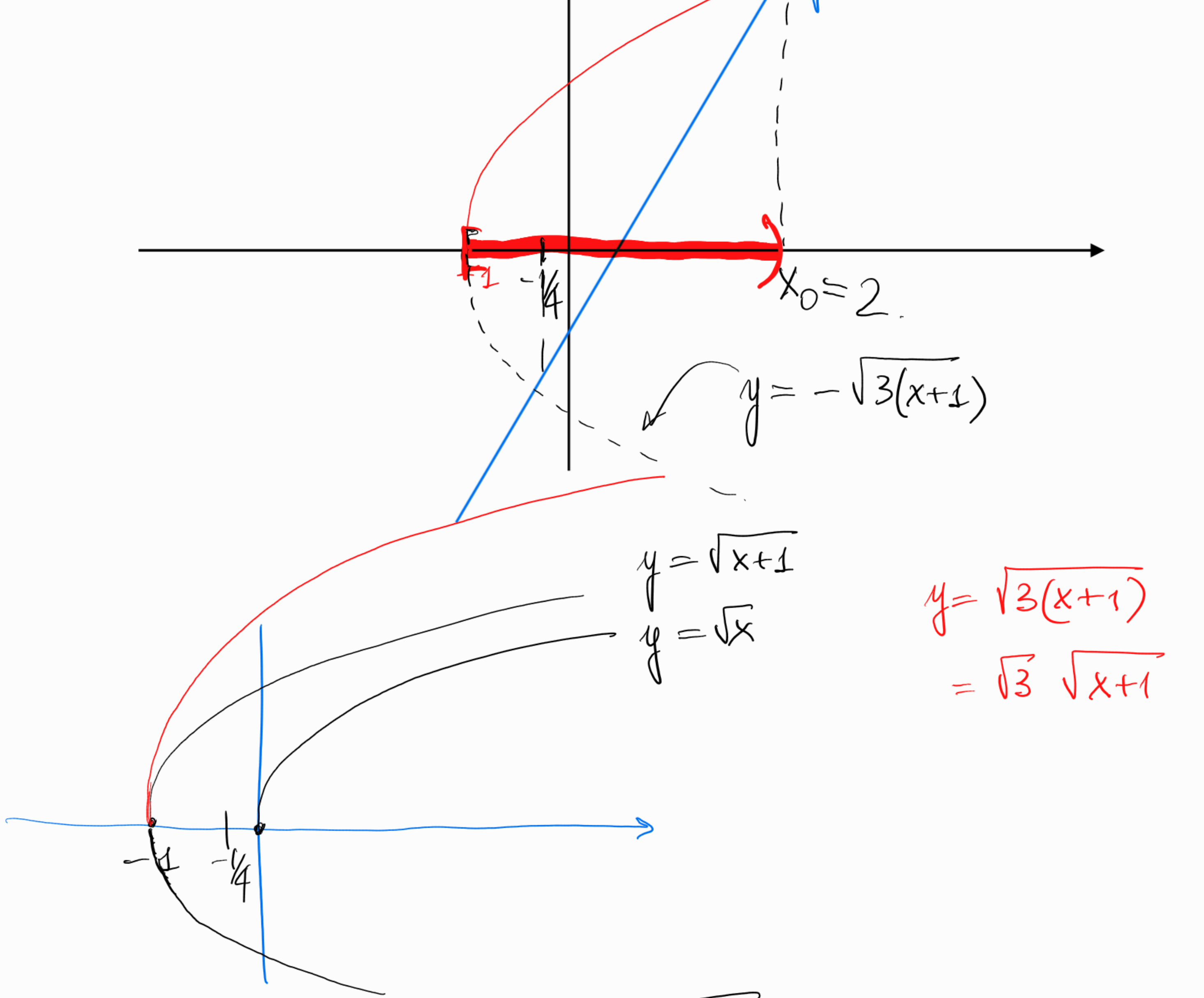
sol. B $\qquad x \in \left[\frac{1}{2}, 2\right)$

$\{Sol^{ni}\ diseq^{ne}\} = \{sol.\ A\} \cup \{sol\ B\} = \left[-1, \frac{1}{2}\right) \cup \left[\frac{1}{2}, 2\right) = [-1, 2)$

$-1 \le x < 2$.

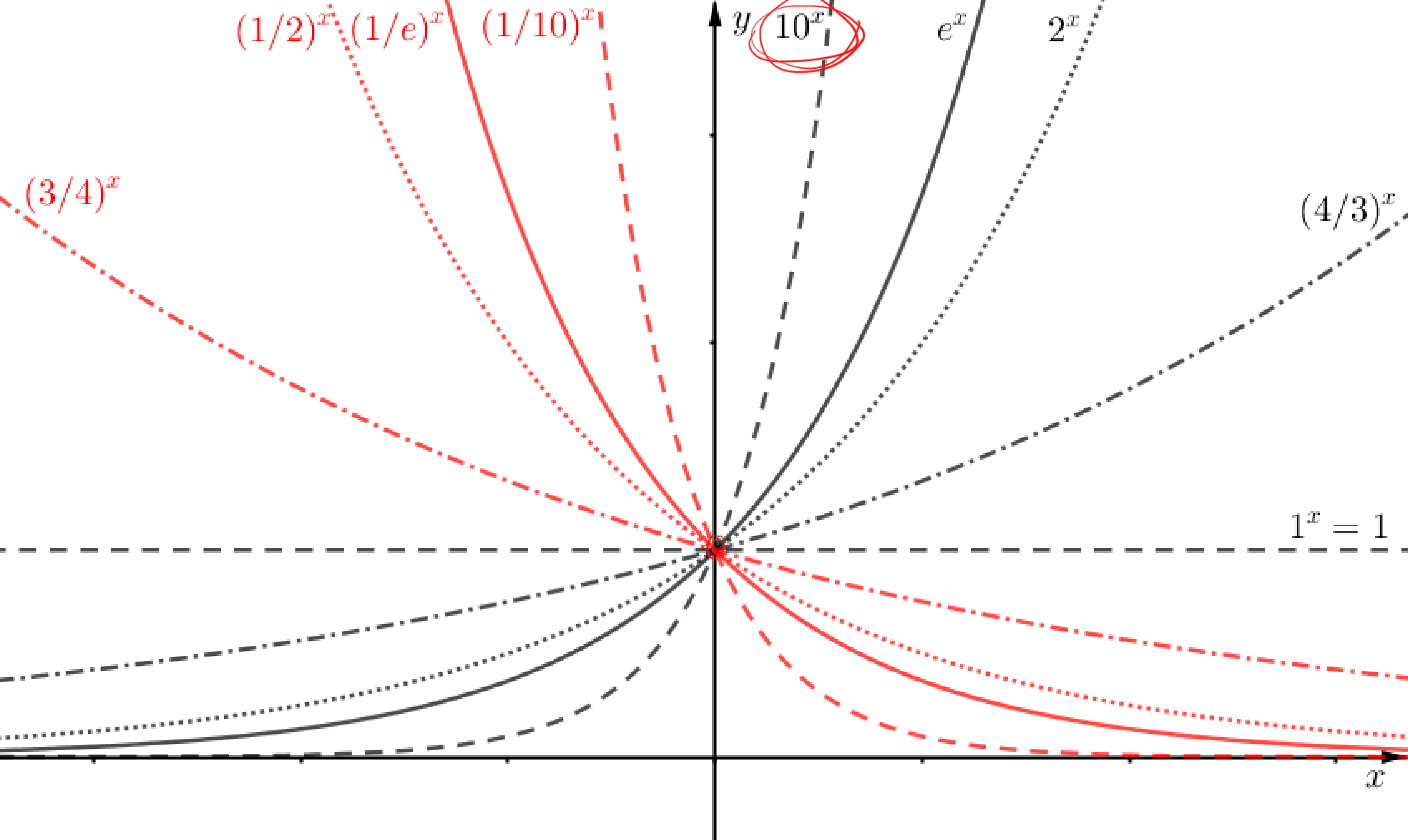
Risoluz. grafica

$\sqrt{3(x+1)} > 2x-1$



x_0 è la soluzione dell'eq^{ne} $\sqrt{3(x+1)} = 2x-1$

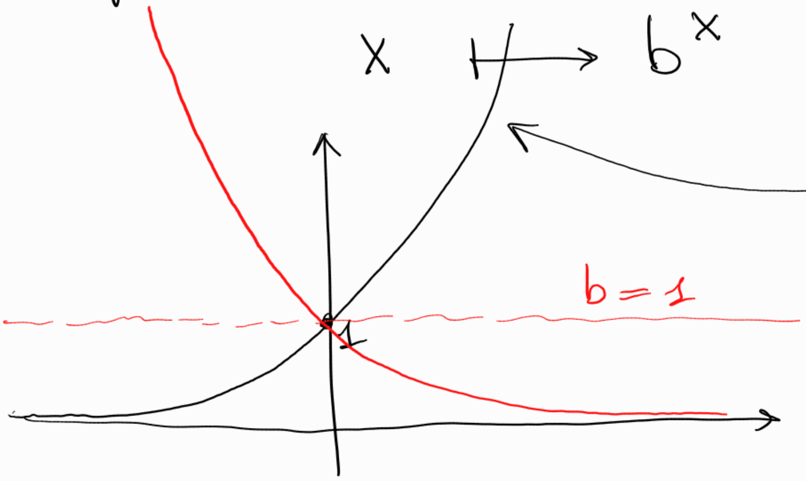
$3x+3 = (2x-1)^2 \qquad x = -\frac{1}{4} \qquad x = 2$



y
10^x
e^x
2^x
$(1/2)^x$
$(1/e)^x$
$(1/10)^x$
$(3/4)^x$
$(4/3)^x$
$1^x = 1$
x

Esponenziali: sono le funzioni del tipo $y = b^x$ con b parametro > 0.

$$f(x): \quad \mathbb{R} \longrightarrow (0,+\infty) = \mathbb{R}^+$$
$$x \longmapsto b^x$$

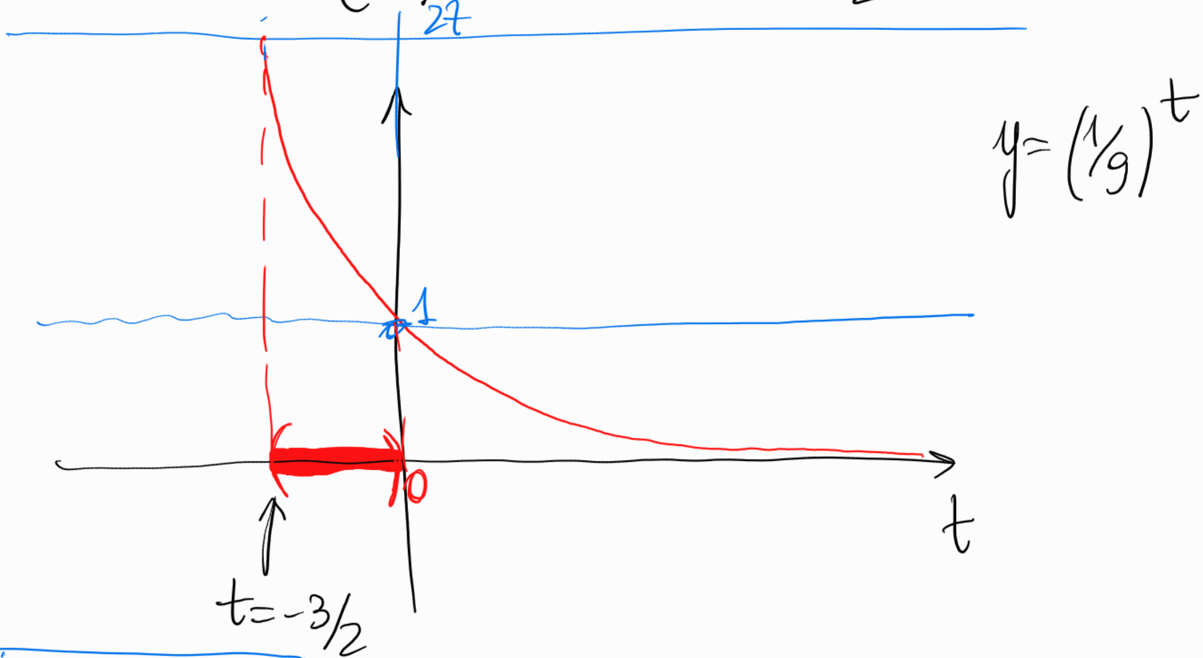


$y = b^x \quad b > 1$

$y = b^x \quad 0 < b < 1$

Vogliamo risolvere questa (doppia) disequazione

$$1 < \left(\frac{1}{9}\right)^{\underbrace{2x-3}_{t}} < 27 \iff -\frac{3}{2} < \overset{2x-3}{\overset{\parallel}{t}} < 0$$



$$\boxed{\left(\frac{1}{9}\right)^t = 27}$$

$$9^t = \frac{1}{27}$$

$$3^{2t} = \frac{1}{27} = 3^{-3}$$

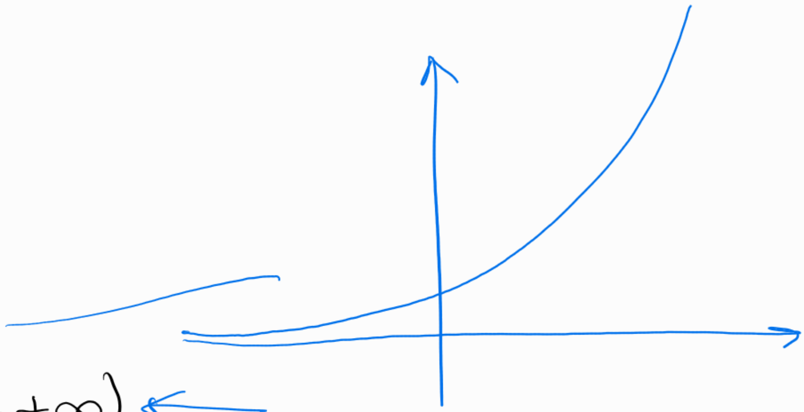
$$2t = -3 \qquad t = -3/2$$

$$-\frac{3}{2} < 2x-3 < 0$$

$$\frac{3}{2} < 2x < 3$$

$$\boxed{\frac{3}{4} < x < \frac{3}{2}}$$

Primo caso: $b>1$.



$$f: \mathbb{R} \longrightarrow (0,+\infty)$$
$$x \longmapsto b^x$$

è biiettiva.

1) è iniettiva, in quanto strettamente crescente.

$$\left.\begin{array}{l} x_1, x_2 \in \mathbb{R} \\ x_1 \neq x_2 \end{array}\right| \Rightarrow \begin{array}{l} x_1 < x_2 \Longrightarrow b^{x_1} < b^{x_2} \\ x_2 < x_1 \longrightarrow b^{x_2} < b^{x_1} \end{array}$$

2) è suriettiva. in $\mathbb{R}$ intervallo

Teorema $\Longrightarrow$ assume tutti i valori compresi tra $\inf_{\mathbb{R}} f(x) = 0$ e $\sup_{\mathbb{R}} f(x) = +\infty$

$$\exists\, f^{-1}: (0,+\infty) \longrightarrow \mathbb{R}$$
$$y \longmapsto \text{l'unico } x \in \mathbb{R} \text{ t.c. } y = f(x) = b^x$$

$f^{-1}(y)$ lo chiamo $\log_b y$ ed è l'unica $x \in \mathbb{R}$ t.c. $b^x = y$.