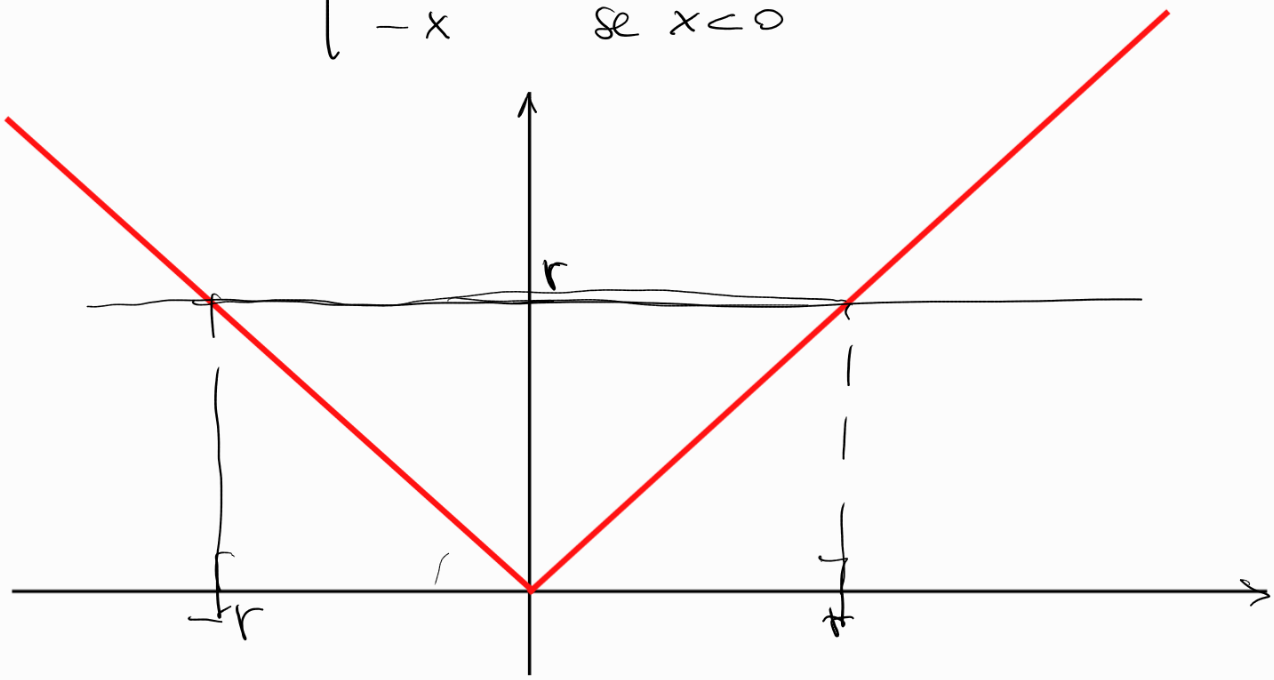
Valore assoluto

$$f(x) = |x| = \begin{cases} x & \text{se } x \geq 0 \\ -x & \text{se } x < 0 \end{cases}$$



1) $|x| \geq 0$, $|x| = 0 \iff x = 0$

2) $|x| = r > 0 \iff x = \pm r$

3) $|xy| = |x||y|$

dim) fare i vari casi

a) $x \geq 0$, $y \geq 0 \Rightarrow xy \geq 0$

$$\underbrace{|xy|}_{xy} \stackrel{?}{=} \underbrace{|x|}_{\overset{\shortparallel}{x}} \underbrace{|y|}_{\overset{\shortparallel}{y}} \quad \text{ok}$$

b) $x \geq 0$, $y < 0 \Rightarrow xy \leq 0$

$|x| = x \quad |y| = -y \quad |xy| = -xy$

$$x(-y) \stackrel{?}{=} -xy \quad \text{ok}$$

c) $x<0,\ y\geq 0$ uguale!

d) $x<0,\ y<0 \Rightarrow xy>0$

$|x|=-x,\quad |y|=-y,\quad |xy|=xy$

$xy \overset{?}{=} (-x)(-y)$ ok.

4) $$\left|\frac{x}{y}\right| = \frac{|x|}{|y|} \quad \forall x,y\in\mathbb{R},\ y\neq 0$$

(dim. simile

5) Sia $r\geq 0$. Allora

$$|x|\leq r \iff \boxed{-r\leq x\leq r}$$

Si vede dal grafico oppure si dividono i casi.

$$|x|\leq r \iff \begin{cases} x\geq 0 \\ x\leq r\end{cases} \vee \begin{cases} x<0 \\ -x\leq r\end{cases} \quad x\geq -r$$

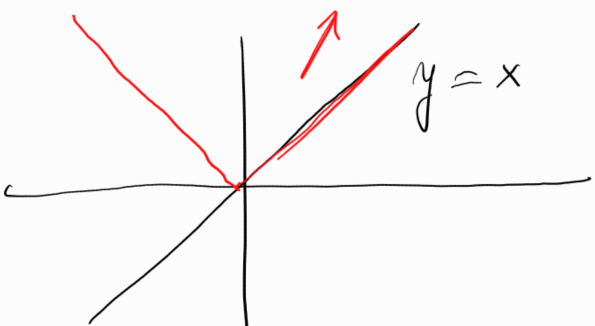
$$0\leq x\leq r \quad \vee \quad -r\leq x<0$$

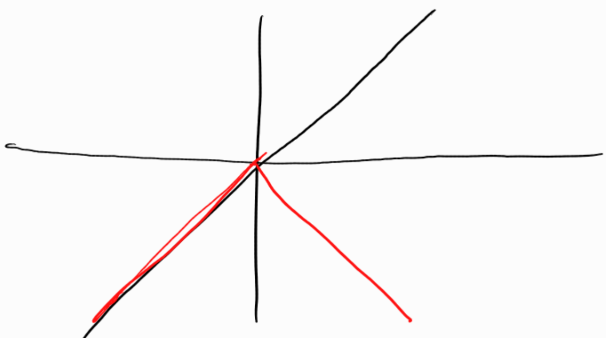
cioè $\quad -r\leq x\leq r$

5') se $r>0$, allora

$$|x|<r \iff -r<x<r$$

6) $-|x| \overset{\checkmark}{\leq} x \leq |x| \quad \forall x\in\mathbb{R}$





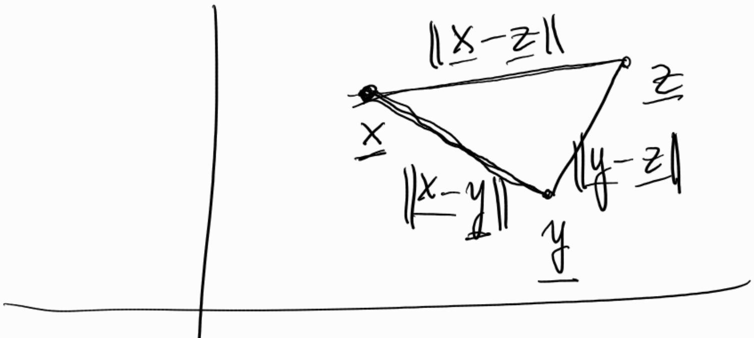
7) $|x+y| \le |x|+|y| \qquad \forall x,y \in \mathbb{R}.$ ← dis. triangolare.

$$-|x| \le x \le |x|$$
$$-|y| \le y \le |y|$$
$$\overline{\qquad\qquad\qquad\qquad\qquad}$$
$$-\underbrace{(|x|+|y|)} \le x+y \le \underbrace{|x|+|y|}_{r}$$

$$-r \le x+y \le r$$
$$\Updownarrow (5)$$
$$|x+y| \le r = |x|+|y|$$

OSS $|x-y|$ si interpreta come la distanza di x da y.

$$|x-y| = |(x-z)+(z-y)| \le |x-z| + |z-y|$$



$$\|x-y\| \le \|x-z\| + \|z-y\|$$

$$\boxed{\begin{array}{l}\text{7') Dis. triangolare per la differenza} \\ \qquad \big||x|-|y|\big| \le |x-y|\end{array}}$$

8) $|x|^2 = x^2$

$$\big||x|\big| = |x|$$

9) $E \subseteq \mathbb{R}$ è limitato $\overset{\text{def}}{\Longleftrightarrow}$ $a \leq x \leq b \quad \forall x \in E$

Si dimostra che

E è limitato $\Leftrightarrow$ $\exists M \geq 0$ t.c. $|x| \leq M \ \forall x \in E$

$\boxed{\Leftarrow}$ $|x| \leq M \Rightarrow \underbrace{-M}_{a} \leq x \leq \underbrace{M}_{b} \quad \forall x \in E$

$\boxed{\Rightarrow}$ $-7 \leq x \leq 3 \Rightarrow |x| \leq 7$

$$-|a| \leq a \leq x \leq b \leq |b| \leq \underbrace{\max\{|a|,|b|\}}_{r}$$

$$\min\{-|a|,-|b|\} \leq -|a|$$

$$\min\{-|a|,-|b|\} = -\underbrace{\max\{|a|,|b|\}}_{-r}$$

$\Updownarrow$

$|x| \leq r = \max\{|a|,|b|\} = M$