Def. funzione Gamma di Eulero.

$$\Gamma : (0, +\infty) \longrightarrow (0, +\infty)$$

$$\Gamma(s) = \int_0^{+\infty} t^{s-1} e^{-t} \, dt$$

PROPRIETÀ:

1) $\Gamma(1) = 1$

2) $\Gamma\left(\frac{1}{2}\right) = \sqrt{\pi}$

3) $\Gamma(s+1) = s\,\Gamma(s)$

OSS da (3) e da (1) segue che

$$\Gamma(n) = (n-1)! \qquad \forall n = 1, 2, \dots$$

Dim 1) ovvio!

Dim. 2) $\Gamma\left(\frac{1}{2}\right) = \int_0^{+\infty} t^{-1/2} e^{-t} \, dt = \left[t^{1/2} = r\right]$

$$= 2\int_0^{+\infty} e^{-r^2} dr = \int_{-\infty}^{+\infty} e^{-r^2} dr = \sqrt{\pi} \qquad \blacksquare$$

Dim. 3)

$$\Gamma(s+1) = \int_0^{+\infty} t^s e^{-t} \, dt = [\text{per parti}]$$

$$= \underbrace{-t^s e^{-t} \Big|_0^{+\infty}}_{= 0} + s \int_0^{+\infty} t^{s-1} e^{-t} \, dt$$

$$= s\,\Gamma(s) \qquad \blacksquare$$