One Hundred Years of Alfred Landé's g-Factor

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Abstract

Prompted by the centenary of Alfred Landé's g-factor, we reconstruct Landé's path to his discovery of half-integer angular momentum quantum numbers and of vector coupling of atomic angular momenta – both encapsulated in the g-factor – as well as point to reverberations of Landé's breakthroughs in the work of other pioneers of quantum physics.

Keywords: History of science, old quantum theory, atomic physics, optical spectra, anomalous Zeeman effect, theory of angular momentum, g-factor, Stern-Gerlach experiment.

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I. INTRODUCTION

Optical spectra of atoms (and molecules) constituted much of the empirical basis of the emerging (old) quantum theory. However, by the end of the 1910s, the complexity of the optical spectra amassed in the leading laboratories of the time presented an "embarrassment of riches" that both confused and distracted the pioneers of quantum theory. As Friedrich Hund (1896-1997) put it, Ref. [1], p. 108:

> They could not know that in order to grasp the fundamentals of the new quantum mechanics, the complicated properties of the spectra were not necessary and that to the understanding of these complicated properties of the spectra the fundamentals of quantum mechanics would contribute little. Thus, the two pursuits [of the fundamentals of quantum mechanics and of optical spectra] inhibited each other; the issues considered in each always presented several interwoven difficulties that had to do with: the fundamentals of quantum theory, spin, and the maximum shell occupancy [the exclusion principle].

Although conceived as tentative, some of the advances in understanding optical spectra proved to be of enduring value for both atomic physics and quantum mechanics. Among these were the recognition by Alfred Landé (see Box) that angular momenta combine vectorially and that angular momentum quantum numbers can take half-integer values. At first rejected by some leading figures, including his teacher, Arnold Sommerfeld (1868-1951) [2], Landé's inferences from optical spectra were vindicated by the 1925 discoveries of quantum mechanics and of electron spin.

Apart from the crisis of the old quantum theory precipitated by its inability to explain the spectrum of helium (at which Landé also had a shot), it was the anomalous Zeeman effect that mystified much of the community from its elders such as Sommerfeld, Peter Debye (1884-1966), and Niels Bohr (1885-1962) to the up-and-coming, such as Werner Heisenberg (1901-1976) and Wolfgang Pauli (1900-1958). The last would later reminisce [4]:

> The anomalous type of [magnetic] splitting was on the one hand especially fruitful because it exhibited beautiful and simple laws, but on the other hand it was hardly understandable, since very general assumptions concerning the electron, using classical theory as well as quantum theory, always led to the simple triplet. A closer investigation of this problem

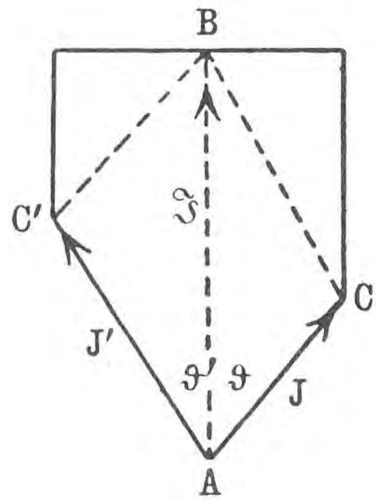

FIG. 1: Vector addition of the angular momenta **J'** (inner electron) and **J** (luminous electron) resulting in total angular momentum $\mathfrak{J}$ [3].

> left me with the feeling that it was even more unapproachable. A colleague who met me strolling rather aimlessly in the beautiful streets of Copenhagen said to me in a friendly manner, "You look very unhappy"; whereupon I answered fiercely, "How can one look happy when he is thinking about the anomalous Zeeman effect?"

The effort to make sense of the Zeeman spectra was redoubled in 1916 by Sommerfeld [5] and Debye [6]. In their independent, back-to-back papers, they expanded Bohr's model of the atom by introducing the concept of space quantization along with the quantum number, m, to characterize the projection of the orbital angular momentum on the direction of the magnetic field. Although space quantization did not make the "number mystery" [*Zahlenmysterium*] [7] presented by the anomalous Zeeman effect go away, it became a key part of a framework that finally did. Moreover, it offered itself to an experimental test – the Stern-Gerlach experiment – that corroborated the reality of space quantization and thus provided a much-needed reassurance that the old quantum theory was on the right track.

Another indispensable ingredient of the framework that would finally explain the anomalous Zeeman effect came from the recognition that atomic angular momenta are vectors, that they combine as such (vector addition), and that each is subject to quantization conditions.

Alfred Landé made use of space quantization in his 1921 papers [8, 9] on the anomalous Zeeman effect and of both space quantization and vector addition in his 1923 seminal papers [10, 11] in which he formulated an old-quantum-theoretical version of what we call today the Landé g-factor. We note that Landé invoked vector addition of angular momenta (of the inner and luminous electrons) already in his 1919 take on the spectrum of helium [3], see Fig. 1. The idea of vector addition was rekindled by Sommerfeld in 1921, whereupon Landé embraced it on his path to the g-factor two years later.

In what follows, we reconstruct, in turn, Landé's path to the half-integer angular momentum quantum numbers and to the Landé g-factor. We conclude with comments on the above made by Landé in his 1962 interview with Thomas Kuhn and John Heilbron [12] as well as point to reflections of Landé's ideas in the work of other pioneers of quantum physics.

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Box on Alfred Landé

FIG. 2: Alfred Landé in about 1920 (left) and in 1958 (right). Courtesy of Louis DiMauro (OSU) and Creative Commons.

Alfred Landé was born on 13 December 1888 in Elberfeld (today a part of the city of Wuppertal) into a liberal Jewish family. His father Hugo Landé (1859-1936) and mother Thekla, ne Landé (1864-1932) were first cousins. The father was the floor leader of the *Sozialdemokratische Partei Deutschlands* (SPD) in Elberfeld. He was involved in drafting SPD's "Erfurt Program" aimed at improving workers' lives rather than at precipitating a socialist revolution. The mother became in 1919 one of the first female members of parliament in Rhineland. Some of Alfred Landé's ancestors served as rabbis; several are buried at the Old Jewish Cemetery in Prague. Alfred was the eldest of four siblings and was considered a prodigy in mathematics and physics. He graduated from a humanistic high-school in Elberfeld at Easter 1908. By that time, Alfred became also an accomplished pianist; later on, he would earn a living for a while as a piano teacher. In 1908, he entered university to study physics and mathematics (1st semester in Marburg, 2nd-4th semester in Munich, and 5th-8th semester in Göttingen). In January 1912, he passed a state examination in Göttingen, whereby he qualified to teach physics, mathematics and chemistry at high school. In 1912, he joined Arnold Sommerfeld (1868-1951) in Munich as a PhD student in theoretical physics with the dissertation topic "On the Method of Natural Oscillations in Quantum Theory." After two semesters, he became, on Sommerfeld's recommendation, a special assistant to David Hilbert (1862-1943) in Göttingen – with the assignment to keep him [Hilbert] abreast of the developments in physics. In parallel, he completed his doctoral thesis under Sommerfeld and received his Ph.D. from Munich in July 1914, just weeks before the outbreak of World War One. Whereupon he was drafted to serve with the Red Cross on the Eastern Front and subsequently transferred to Berlin – through the mediation of Max Born (1882-1970) whom he knew from Göttingen – to the Artillery Testing Commission (A.P.K.), which was run by Rudolf Ladenburg (1882-1952) and Max Born. Still during the war, he investigated jointly with Born the compressibility of crystals, that led them to the conclusion that atoms have volume. In December 1918, Landé took the job of a music teacher at the Odenwald School in Heppenheim while continuing his work in theoretical physics. After Max Born succeeded Max von Laue (1879-1960) at the University of Frankfurt in 1919, he hired Landé as his assistant, alongside with Otto Stern (1888-1969) and Elisabeth Bormann (18951986). The same year, Landé completed his habilitation thesis "Quantum Theory of the Helium Spectrum" and was appointed *Privatdozent* on October 28, 1919. On September 17, 1920, he received the *Venia Legendi* in Frankfurt. Since 1919, Landé was preoccupied with the structure of atoms and from 1920 on with the Zeeman effect. During his time in Frankfurt, he discovered what we call today Landé's g-factor. In 1922, Landé married Elisabeth Grunewald, with whom he had two sons, Arnold and Carl. In October 1922, he accepted a call to become an *Extraordinarius* at Tübingen. In 1929, Landé was invited to lecture at the Ohio State University (OSU) in Columbus. After a repeated stay at OSU in 1931, he accepted a professorship there. He remained at OSU until his retirement on 1 October 1959. Landé published over 150 papers dealing almost exclusively with quantum physics issues, as well as 10 books and 4 handbook articles. Since about 1950, he was engaged in debates on the interpretation of quantum mechanics. Landé's two sisters, Charlotte (1890-1977) and Eva (1901-1977), thanks to his help, were able to emigrate to the U.S. before the outbreak of World War Two. However, his brother Franz (1893-1942) stayed put and was murdered in Auschwitz. Their father committed suicide in 1936 after escaping from the Nazis to Switzerland. Landé died in Columbus on 30 October 1976.

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II. HALF-INTEGER QUANTUM NUMBERS

In his 1921 duo of papers "On the Anomalous Zeeman Effect" [8, 9], Landé introduced a "working hypothesis" according to which Sommerfeld's "inner" quantum number of an atom corresponds to the quantum number J of the atom's total angular momentum (in what follows, we use modern notation for all the quantities discussed). Furthermore, Landé adopted the projection quantum number m from Sommerfeld and Debye as well as coopted from Adalbert [also Wojciech] Rubinowicz (1889-1974) [13] the selection rules $\Delta m = 0$ and $\Delta m = \pm 1$ for spectral lines polarized, respectively, parallel and perpendicular to the magnetic field. Landé writes [8]:

> While the usual space quantization in a magnetic field admits only integer values of m, one must come to grips here with rational fractions of m [justified in [9] by "anomalous" Larmor precession] such ... that adjacent values of m are separated by ± 1. Because of the $+$ and $-$ symmetry, the only possible sequence of fractions is: $m = \pm\frac{1}{2}, \pm\frac{3}{2}, \ldots, \pm\frac{2J-1}{2}$, apart from the other [integer] sequence $m = 0, \pm 1, \pm 2, \ldots, \pm J$.

At the end of 1921, Heisenberg entered the fray with his very first paper [14], in which he provided his own interpretation of half-integer quantum numbers. He conjectured that in the process of the *Aufbau* [build-up] of an atom, an electron that is being added to the existing atomic core [*Rumpf*] imparts $\frac{1}{2}\hbar$ of its orbital angular momentum $\ell\hbar$

to the core and ends up with an angular momentum $(\ell - \frac{1}{2})\hbar$. Here $\hbar \equiv h/(2\pi)$ with h Planck's constant. These ideas would be expanded upon later in a joint paper by Heisenberg and Landé [15]. As "it is hard to quarrel with success" – and there was plenty of it interpreting the Zeeman spectra of optical doublet and triplet transitions – Landé's half-integer quantum numbers would be in use until the discovery of electron spin by George Uhlenbeck and Samuel Goudsmit [16, 17].

III. LANDÉ'S g-FACTOR IN THE OLD AND MODERN QUANTUM THEORY

In his 1923 papers on the "Term Structure and the Zeeman Effect of Multiplets" [10, 11], Landé made strides towards a phenomenological understanding of the elusive anomalous Zeeman effect, which he topped off a year later with a paper where he considered its weak- and strong-field limits [18]. Landé wrote these papers in response to the discovery in 1922 of multiplets (up to octets) by Miguel Catalán (1894-1957) [19] and Hilde Gieseler [20], further experimental investigations of the Zeeman effect by Ernst Back (1881-1959) as well as a bigger picture of the above drawn by Sommerfeld [21].

Under the spell of Heisenberg's interpretation of the half-integer quantum numbers [14], Landé introduced the quantum number $r \equiv \ell - \frac{1}{2}$ for the core of the atom and went on to redefine the triad of quantum numbers characterizing an energy term (level) as follows: $r \mapsto R$, $\ell \mapsto L$, and $j \mapsto J$ (in modern notation, except for R, which is Landé's). The vector addition of the corresponding angular momenta $\mathbf{R}$, $\mathbf{L}$, and $\mathbf{J}$ of a magnitude given by the respective quantum numbers R, L, and J, in units of $\hbar$, gave rise to the the following range of possible values of the total angular momentum quantum number $J = L + R - \frac{1}{2}, L + R - \frac{3}{2}, \ldots, |L - R| + \frac{1}{2}$. Hence the multiplicity of an (R, L, J) term came out as $2R$ for $L \geq R$ or $2L$ for $L < R$. Thus, the projection quantum number could take values $m = J - \frac{1}{2}, J - \frac{3}{2}, \ldots, -J + \frac{1}{2}$, which, in a magnetic field of magnitude $|\mathbf{B}|$, led to the following term energies $E(R, L, J, m)$:

$$E(R, L, J, m) = mg(R, L, J)h\nu_0 \tag{1}$$

where

$$\nu_0 = \frac{e|\mathbf{B}|}{4\pi m_e} \tag{2}$$

is the Larmor frequency with e the charge of the electron and m_e its mass, and $g = g(R, L, J)$ is the term-dependent g-factor (or "splitting" factor, in the parlance

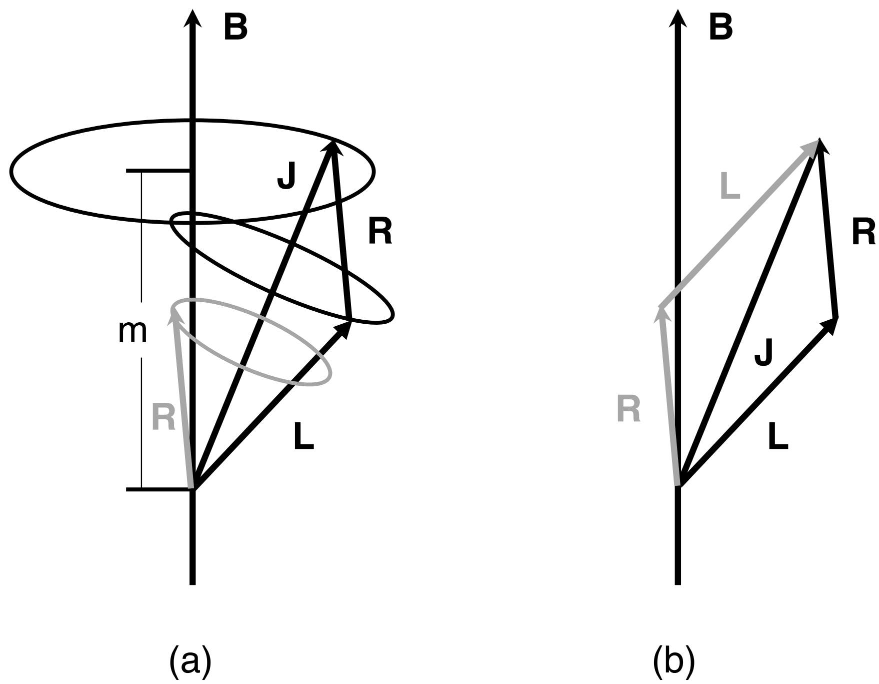

FIG. 3: (a) Weak-field coupling of atomic angular momenta **R** and **L** pertaining, respectively, to the core and orbital electrons as considered by Landé [10, 11]. The core and orbital angular momenta precess about their resultant, the total electronic angular momentum **J** which, in turn, precesses about the direction of the magnetic field **B**. The total angular momentum **J** makes a constant projection m on **B**. (b) Parallelogram pertaining to the vector addition of the angular momenta **R** and **L**. See text.

of Landé's day). Although the notion of a g-factor appears in Landé's earlier papers, only in his 1923-24 works he gives an explicit expression for it derived from the vector model of the atom.

Landé's original g-factor, Eq. (8) of Ref. [10], pertains to the weak-field limit and can be derived from the vector model, see Fig. 3, as follows: the angular momenta **R** and **L** precess about the total angular momentum **J**, contributing, respectively, (dimensionless) magnetic moments

$$g(R, J) = \frac{R}{J}\cos(\mathbf{R}, \mathbf{J}) = \frac{R}{J}\left(\frac{J^2 + R^2 - L^2}{2RJ}\right) \tag{3}$$

and

$$g(L, J) = \frac{L}{J}\cos(\mathbf{L}, \mathbf{J}) = \frac{L}{J}\left(\frac{J^2 - R^2 + L^2}{2LJ}\right) \tag{4}$$

where we made use of the law of cosines. Landé's g-factor then obtains from the sum of $g(R, J)$ and $g(L, J)$ where, however, the R-contribution from the core is taken twice:

$$g(R, L, J) = g(L, J) + 2g(R, J)R = 1 + \frac{J^2 + R^2 - L^2}{2J^2} \tag{5}$$

The doubling of the contribution from the core ensured an approximate agreement of the term energies, Eq. 1, with experiment. The agreement was further improved, to within about 5%, by replacing the squares of the angular momenta in Eq. (5) by what Landé called their geometric means:

$$\begin{aligned} \overline{g}(R, L, J) &= 1 + \frac{(J - \frac{1}{2})(J + \frac{1}{2}) + (R - \frac{1}{2})(R + \frac{1}{2}) - (L - \frac{1}{2})(L + \frac{1}{2})}{2(J - \frac{1}{2})(J + \frac{1}{2})} \\ &= 1 + \frac{J^2 - \frac{1}{4} + R^2 - L^2}{2(J^2 - \frac{1}{4})} \end{aligned} \tag{6}$$

Landé and Back noted later [22], p. 41:

> This formula has arisen empirically through theoretical considerations and is fully confirmed by the measurements of Catalán [Ref. [19]] and Back [Ref. [23]] on the manganese spectrum and H. Gieseler [Ref. [20]] on the chromium spectrum.

In order to recast Eq. (6) in modern form, we have to replace the quantum numbers R, L, and J with $S + \frac{1}{2}$, $L + \frac{1}{2}$, and $J + \frac{1}{2}$ and introduce the anomalous gyromagnetic ratio of the electron, g_S. Then

$$\tilde{g}(R, L, J) = \frac{J(J+1) - S(S+1) + L(L+1)}{2J(J+1)} + g_S \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \tag{7}$$

where $g_S \approx 2.0023$, Ref. [26]. Setting $g_S = 2$, gives

$$\tilde{g}(R, L, J) \approx 1 + \frac{J(J+1) + S(S+1) - L(L+1)}{2J(J+1)} \tag{8}$$

We note that the product of the type $K(K + 1)$ was recognized as being the eigenvalue of a (disembodied) angular momentum squared, $\mathbf{K}^2$, only after the discovery of quantum mechanics [27].

Landé's derivation and inference of the g-factor for the weak-field case was informed by the optical Zeeman spectra as well as by Pauli's analysis of the anomalous Zeeman effect in the strong-field limit (the Pachen-Back effect) [24, 25], see Fig. 4. For that case, Pauli concluded that rather than coupling with one another, the core

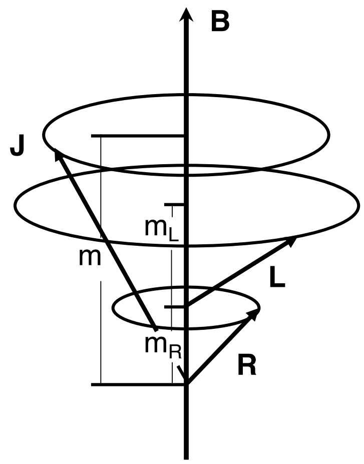

FIG. 4: Strong-field coupling of atomic angular momenta **R** and **L** pertaining, respectively, to the core and orbital electrons as considered by Pauli [24, 25] and Landé [18]. The core and orbital angular momenta precess independently about the direction of the magnetic field **B** with constant projections m_R and m_L, respectively. The total electronic angular momentum **J** makes a constant projection m on **B** such that $m = m_L + m_R$. See text.

and orbital angular momenta **R** and **L** couple directly to the magnetic field, making projections m_R and m_L such that $m = m_R + m_L$, and inferred that this coupling gives rise to a term energy

$$E(m_R, m_L, m) = (m_L + 2m_R)h\nu_0 \tag{9}$$

i.e., that the contribution from the core needs to be taken twice in order to achieve agreement with experiment. Here's Pauli's comment on the significance of this finding [4]:

> I could not find a satisfactory solution at that time, but succeeded, however, in generalizing Landé's analysis for the simpler case (in many respects) of very strong magnetic fields [Ref. [24]]. This early work was of decisive importance for the finding of the exclusion principle.

In his interview with Thomas Kuhn John Heilbron, Landé would note [12]:

> I remember a visit of Pauli to Tübingen [in 1925] which however came immediately at the time of the exclusion principle $\cdots$ Apparently at that time he was already looking for confirmation of the exclusion principle. He found some spectrum of Back's in which there was a line missing which

> should have been there, and the reason that it was missing was of course the exclusion principle. And he [Pauli] stayed in my home. We had a party at night, and after the party Pauli worked on in the kitchen, and the next morning he told me about the exclusion principle. So I claimed that he discovered the exclusion principle in my kitchen. But there are seven other cities with seven other physicists who ··· make similar claims.

In the context of the Paschen-Back effect and the Barnett [28, 29] and Einstein-de Haas [30] experiments, Landé noted [31]:

> In particular, a number of questions from the field of magnetism are awaiting clarification, especially the question of whether the magnetic moment of an electron system (moving charges [e]) can be calculated in the usual way from the mechanical torque (moving masses [m_e]), as required by the equation attached to Larmor's theorem.

A summary of the understanding of the Zeeman effect on the eve of the discovery of quantum mechanics and of electron spin is given in a monograph penned by Landé along with his erstwhile competitor Ernst Back [22].

IV. IN CONCLUSION

Landé's ideas about angular momentum coupling (vector addition) were applied to the individual electrons of an atom by Henry Norris Russell (1877-1957) and Frederick Albert Saunders (1875-1963) [32]. Russel and Saunders showed that these add up to Landé's angular momenta R and L.

Landé's vector model lived on also in Goudsmit's and Uhlenbeck's "spin" paper [17]. In addition, they noted about the difficulties of tackling the anomalous Zeeman effect:

> However, these difficulties disappear at once when, as assumed, the electron has a spin and the ratio between magnetic moment and angular momentum of this spin is different from that corresponding to the revolution of the electron in an orbit large compared with its own size. On this assumption the spin axis of an electron not affected by other forces would precess with a frequency different from the Larmor rotation. It is easily shown that the resultant motion of the atom for magnetic fields of small

> intensity will be of just the type revealed by Landé's analysis. If the field is so strong that its influence on the precession of the spin axis is comparable with that due to the orbital motion in the atom, this motion will be changed in a way which directly explains the gradual transformation of the multiplet structure for increasing fields known as the Paschen-Back effect.

As highly prescient appears in hindsight Landé's interpretation of the Stern-Gerlach experiment, whose arduous implementation he witnessed from an office adjacent to Stern's and Gerlach's laboratory in Frankfurt. In the Stern-Gerlach experiment, a beam of silver atoms in the ^2S ground state was split into two beams by passage through an inhomogeneous magnetic field, whereby the magnitude of the splitting corresponded to a magnetic dipole moment of about one Bohr magneton, $\mu_B = h\nu_0/|\mathbf{B}| = e\hbar/(2m_e)$. The prevalent interpretation of the outcome of the experiment at the time ascribed the origin of the splitting to the orbital angular momentum $L = 1$ of the silver atoms, whose magnetic moment was presumed to be one Bohr magneton. Landé, however, would not budge [33]. Instead, he appealed to his theory of the anomalous Zeeman effect, noting that for $L = 1$, the silver beam would be split into a triplet of beams, corresponding to $m = -1, 0, +1$. Since a splitting into two beams had been observed, Landé inferred that the state of the silver atom was in fact a doublet whose components had $m = -\frac{1}{2}$ and $\frac{1}{2}$, but because of the anomalous g-factor of 2, each component carried a magnetic dipole of one Bohr magneton. As we know today, the magnetic moment of $Ag(^2S)$ is due to spin of $\frac{1}{2}$ and the anomalous gyromagnetic ratio of the electron of about 2. That things looked as if the silver atoms had a magnetic dipole moment of one Bohr magneton could be characterized as "an uncanny conspiracy of nature" [34].

To which we add that the system of conjectures of Landé and others invoked in their attempts to understand the anomalous Zeeman effect amounted in some respects to as much of a "triumph over logic" – Abraham Pais's term [35], p. 146 – as Bohr's model of the atom.

In his 1962 interview with Thomas Kuhn and John Heilbron, Landé said [12]:

> And from that [Ref. [3]] to the vector model is only a small step. This is already a vector model – two axes precessing around their common resultant ... But I think that this paper of mine here, "Eine Quantenregel für die räumliche Orientierung von Elektronenringen," may be the first

in which this model is used extensively. Well, the angular momentum always played the leading role in quantization, in Sommerfelds and Wilsons quantum rule. This is much more important than the quantization of energy ... And one tried this and that, and it gradually became clearer that these quantum numbers could be associated with a vector model. Some people think more in models, and other people more in terms of mathematical symmetries, matrices ... My case is only to think in models, certainly. I am not a mathematician.

Kuhn: Do you recall by any chance what kind of model you were thinking of which helped get the g-factor?

Landé: Oh yes, the g factor is quite at the end of this whole vector business ... The only model consideration in the case of the g factor was that there was something – the core – which had twice as much magnetic moment than it ought to have [had]. Of course there were model considerations, the whole vector model is a model ... This is here the first paper on the anomalous Zeeman effect [Ref. [8]]. Here is already almost the whole story – the g factor is in it.

It was a long and winding road from the "number mystery" of the anomalous Zeeman effect to the realization that the mystery was largely due to electron's own magnetic moment. Nevertheless, Landé's discovery of half-integer angular momentum quantum numbers and of vector coupling of atomic angular momenta – both encapsulated in the Landé g-factor – were milestones in the development of quantum physics.

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