Trovare una formula iterativa per

$$I_n = \int_0^1 1 \cdot (x^4-2)^n \, dx$$

Per parti:

$$f'(x) = 1 \Rightarrow f(x) = x$$

$$g(x) = (x^4-2)^n \Rightarrow g'(x) = 4n\, x^3 (x^4-2)^{n-1}$$

$$I_n = \underbrace{x\,(x^4-2)^n \Big|_0^1}_{(-1)^n} - 4n \int_0^1 x^4 \, (x^4-2)^{n-1}$$

$$= (-1)^n - 4n \int_0^1 \underbrace{(x^4-2)(x^4-2)^{n-1} dx}_{= I_n} - 4n \cdot 2 \underbrace{\int_0^1 (x^4-2)^{n-1} dx}_{I_{n-1}}$$

a 1° membro.

$$(4n+1)\, I_n = (-1)^n - 8n\, I_{n-1}$$

$$I_n = \frac{1}{4n+1} \left[(-1)^n - 8n\, I_{n-1} \right]$$

Calcolare, se esiste, la primitiva di $f(x) = \sin(2x-3)\, tg^2(2x-3)$ che vale 1 per $x = 3/2$.

$$\int \sin(2x-3)\, tg^2(2x-3)\, dx =$$

$$t = 2x-3$$

$$2dx = dt$$

$$= \frac{1}{2} \int \sin t \; tg^2 t \; dt = \frac{1}{2} \int \frac{\sin^3 t}{\cos^2 t} dt =$$

$$= \frac{1}{2}\int \frac{\sin^2 t \,\sin t}{\cos^2 t}\,dt = \frac{1}{2}\int \frac{1-\cos^2 t}{\cos^2 t}\sin t \,dt =$$

$\cos t = s$

$-\sin t\,dt = ds.$

$$= -\frac{1}{2}\int \frac{1-s^2}{s^2}\,ds =$$

$$= -\frac{1}{2}\int\left(\frac{1}{s^2}-1\right)ds = -\frac{1}{2}\left(-\frac{1}{s}-s\right)+c =$$

$$= \frac{1}{2}\left(\frac{1}{s}+s\right)+c = \frac{1}{2}\left(\frac{1}{\cos t}+\cos t\right)+c.$$

$$= \underbrace{\frac{1}{2}\left(\frac{1}{\cos(2x-3)}+\cos(2x-3)\right)+c}_{f(x)}.$$

$$f\left(\frac{3}{2}\right) = 1.$$

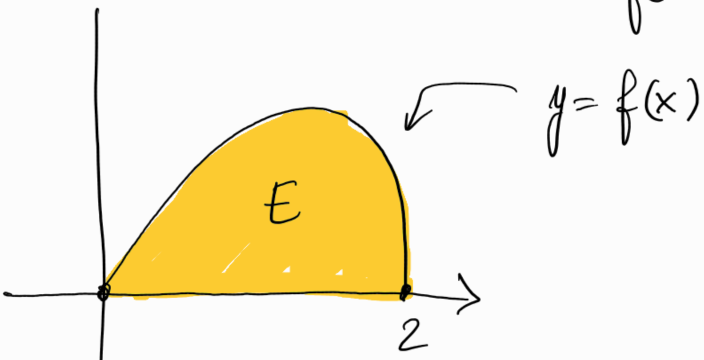
$$\frac{1}{2}(1+1)+c = 1 \Rightarrow c=0.$$

Sol^{ne} $\quad \frac{1}{2}\left(\frac{1}{\cos(2x-3)}+\cos(2x-3)\right)$

In alternativa: la funzione cercata è:

$$f(x) = \int_{3/2}^{x} \sin(2t-3)\,tg^2(2t-3)\,dt + 1$$

Calcolare l'area della regione di piano contenuta nel primo quadrante che si trova al di sotto del grafico di $f(x) = x \operatorname{arctg}\sqrt{4-x^2}$



Area di E = $\boxed{\int_0^2 f(x)\,dx = \int_0^2 x \operatorname{arctg}\sqrt{4-x^2}\,dx =}$

$$= -\frac{1}{2}\int_0^2 (-2x)\operatorname{arctg}\left(\sqrt{4-x^2}\right)dx =$$

$4-x^2=t \qquad x=0 \Rightarrow t=4$

$-2x\,dx = dt \qquad x=2 \Rightarrow t=0$

$t=\sqrt{4-x^2}$

$t^2 = 4-x^2$

$\cancel{2}t\,dt = -\cancel{2}x\,dx$

$$= -\frac{1}{2}\int_4^0 \operatorname{arctg}(\sqrt{t})\,dt$$

$$= \frac{1}{2}\int_0^4 1\cdot\operatorname{arctg}(\sqrt{t})\,dt = (*)$$

$f'(t)=1 \Rightarrow f(t)=t$

$g(t)=\operatorname{arctg}(\sqrt{t}) \Rightarrow g'(t) = \frac{1}{1+t}\,\frac{1}{2\sqrt{t}}$

$$(*) = \frac{1}{2}\left[\underbrace{t\operatorname{arctg}(\sqrt{t})\Big|_0^4}_{4\operatorname{arctg}2} - \int_0^4 \frac{t}{1+t}\,\frac{1}{2\sqrt{t}}\,dt\right]$$

sost. $\sqrt{t}=v \qquad \frac{dt}{2\sqrt{t}} = dv \qquad t=v^2$

$t=4 \Rightarrow v=2 \qquad t=0 \Rightarrow v=0$

$$= 2\,\text{arctg}\,2 - \frac{1}{2}\int_0^2 \frac{(v^2+1)-1}{1+v^2}\,dv =$$

$$= 2\,\text{arctg}\,2 - \frac{1}{2}\int_0^2 \left(1 - \frac{1}{1+v^2}\right)dv =$$

$$= 2\,\text{arctg}\,2 - \frac{1}{2}\left[2 - \text{arctg}\,v\Big|_0^2\right] =$$

$$= 2\,\text{arctg}\,2 - 1 + \frac{1}{2}\text{arctg}\,2 = \frac{5}{2}\text{arctg}\,2 - 1.$$

Calcolare $\displaystyle\int_a^b \underbrace{\arccos\left(\frac{1}{\sqrt{3x-4}}\right)}_{f(x)} dx$ con a,b scelti a piacere distinti.

$$\text{Dom } f(x) = \left\{x \in \mathbb{R} : \frac{1}{\sqrt{3x-4}} \in [-1,1]\right\} =$$

$$= \left\{x \in \mathbb{R} : \sqrt{3x-4} \geq 1\right\} = \left\{x \geq 5/3\right\} = \left[\frac{5}{3}, +\infty\right)$$

Calcolo l'integrale indefinito.

sost. $\sqrt{3x-4} = t \qquad 3x-4 = t^2 \qquad x = \dfrac{t^2+4}{3}$

$$dx = \frac{2t}{3}dt$$

$$\int \arccos\left(\frac{1}{\sqrt{3x-4}}\right)dx = \frac{2}{3}\int t \arccos\left(\frac{1}{t}\right) dt =$$

$$f'(t) = 2t \Rightarrow f(t) = t^2$$

$$g(t) = \arccos\left(\frac{1}{t}\right) \Rightarrow g'(t) = +\frac{1}{\sqrt{1-\frac{1}{t^2}}}\,\frac{1}{t^2} =$$

$$= \frac{t}{\sqrt{t^2-1}}\,\frac{1}{t^2} = \frac{1}{t\sqrt{t^2-1}}$$

$$= \frac{1}{3}\left[t^2 \arccos\left(\frac{1}{t}\right) - \frac{1}{2}\int \frac{2t^{\cancel{2}}}{\cancel{t}\sqrt{t^2-1}}\,dt \right] =$$

$t^2-1=v$

$2t\,dt = dv$

$$= \frac{1}{3}\, t^2 \arccos\left(\frac{1}{t}\right) - \frac{1}{6}\underbrace{\int \frac{dv}{\sqrt{v}}}_{2\sqrt{v} = 2\sqrt{t^2-1}} =$$

$$= \frac{1}{3}\left[t^2 \arccos\frac{1}{t} - \sqrt{t^2-1}\right] + c$$

$t = \sqrt{3x-4}$

$t^2 - 1 = 3x-4-1$

$= 3x-5$

$$= \frac{1}{3}\left[(3x-4)\arccos\frac{1}{\sqrt{3x-4}} - \sqrt{3x-5}\right] + c.$$

Possiamo prendere per es. $a = \frac{5}{3}$, $b = \frac{8}{3}$

$$\int_{\frac{5}{3}}^{8/3} f(x)\,dx = \frac{1}{3}\left[(3x-4)\arccos\frac{1}{\sqrt{3x-4}} - \sqrt{3x-5}\right]_{5/3}^{8/3} =$$

$$= \frac{1}{3}\left[4 \arccos\frac{1}{2} - \cancel{\arccos 1} - \sqrt{3}\right] =$$

$$= \frac{1}{3}\left[\frac{4}{3}\pi - \sqrt{3}\right]$$