$$\int \cos(\log x)\,dx = \int \cos t \; e^t\,dt$$

$\log x = t$

$x = e^t$

$dx = e^t dt$

$\frac{dx}{x} = dt$

$dx = \underbrace{\left(\frac{dx}{x}\right)}_{dt} \underbrace{x}_{e^t} = e^t dt$

$$\int f'(x)\,g(x)\,dx = f(x)\,g(x) - \int f(x)\,g'(x)\,dx$$

$$\int \underbrace{e^t}_{g} \underbrace{\cos t}_{f'}\,dt = \sin t \; e^t - \int e^t \sin t \; dt =$$

$f'(t) = \cos t \Rightarrow f(t) = \sin t$

$g(t) = e^t \Rightarrow g'(t) = e^t$

$f'(t) = \sin t \Rightarrow f(t) = -\cos t$

$g(t) = e^t \Rightarrow g'(t) = e^t$

$$= e^t \sin t - \left[-e^t \cos t + \int e^t \cos t \; dt\right]$$

$$= e^t \sin t + e^t \cos t - \underbrace{\int e^t \cos t \; dt}_{\text{a } 1^o \text{ membro}}$$

$$\cancel{2}\int e^t \cos t = \frac{e^t(\sin t + \cos t)}{2} + c = \quad [t = \log x]$$

$$= \frac{x}{2}\left(\sin(\log x) + \cos(\log x)\right) + c$$

a) Calcolare $\frac{1}{3}\int 3x^2 \sqrt{16+x^6}\,dx = \frac{1}{3}\int \sqrt{16+t^2}\,dt$

$x^3 = t$

$3x^2dx = dt$

$$\int 1\cdot\sqrt{16+t^2}\,dt = t\sqrt{16+t^2} - \int \frac{(t^2+16)-16}{\sqrt{16+t^2}}dt =$$

$$\left[\begin{array}{l} \int f'g = fg - \int fg' \\ f'(t) = 1 \Rightarrow f(t) = t \\ g(t) = \sqrt{16+t^2} \Rightarrow g'(t) = \frac{2t}{2\sqrt{16+t^2}} = \frac{t}{\sqrt{16+t^2}} \end{array}\right]$$

$$= t\sqrt{16+t^2} \boxed{- \int \sqrt{16+t^2}\,dt} + 16\int \frac{dt}{\sqrt{16+t^2}}$$

$$\Rightarrow \int \sqrt{16+t^2}\,dt = \frac{1}{2}\, t\sqrt{16+t^2} + 8\int \frac{dt}{\sqrt{16+t^2}}$$

$$\left[\int \frac{ds}{\sqrt{1+s^2}} = \text{settsinh}\, s + c = \log\left(s+\sqrt{1+s^2}\right) + c\right]$$

$$\int \frac{dt}{\sqrt{16+t^2}} = \frac{1}{4}\int \frac{dt}{\sqrt{1+\frac{t^2}{16}}} = \frac{1}{4}\int \frac{dt}{\sqrt{1+\left(\frac{t}{4}\right)^2}} =$$

$$= \frac{1}{\cancel{4}}\cdot\cancel{4}\ \text{settsinh}\left(\frac{t}{4}\right) + c = \log\left(\frac{t}{4}+\sqrt{1+\frac{t^2}{16}}\right) + c$$

$$= \log\left(\frac{t}{4} + \frac{\sqrt{16+t^2}}{4}\right) + c$$

$$= \log\left(t+\sqrt{16+t^2}\right) \underbrace{-\log 4 + c}_{=c_1}$$

$$\frac{1}{3}\int 3x^2 \sqrt{16+x^6}\,dx = \frac{1}{3}\int \sqrt{16+t^2}\,dt$$

$$= \frac{1}{3}\left(\frac{t}{2}\sqrt{16+t^2} + 8\log\left(t+\sqrt{16+t^2}\right)\right) =$$

$$= \frac{1}{6}x^3\sqrt{16+x^6} + \frac{8}{3}\log\left(x^3+\sqrt{16+x^6}\right) + c.$$

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b) Calcolare l'area dell'insieme

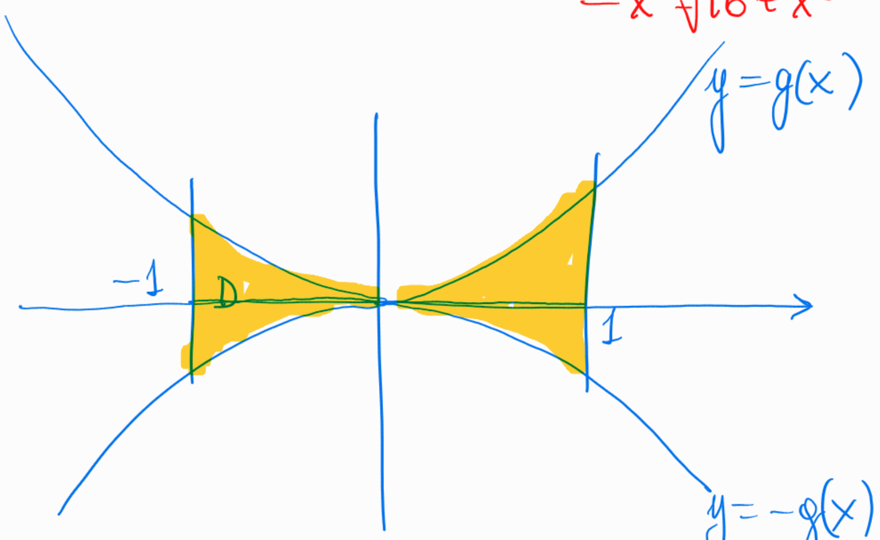
$$D = \{(x,y)\in\mathbb{R}^2 : \; |x|\le 1, \; |y| \le x^2\sqrt{16+x^6}\}.$$

$$-1 \le x \le 1$$

$$-x^2\sqrt{16+x^6} \le y \le \underbrace{x^2\sqrt{16+x^6}}$$

$g(x) \ge 0$

$>0 \;\forall x \ne 0$



$$\text{Area } D = 4\int_0^1 x^2\sqrt{16+x^6}\,dx =$$

$$= \frac{2}{3}x^3\sqrt{16+x^6}\Big|_0^1 + \frac{32}{3}\left(\log\left(x^3+\sqrt{16+x^6}\right)\right)\Big|_0^1 =$$

$$= \frac{2}{3}\sqrt{17} + \frac{32}{3}\left(\log\left(1+\sqrt{17}\right) - \log 4\right)$$

Calcolare l'area di

$$D = \left\{ (x,y) \in \mathbb{R}^2 : \ -\frac{\pi}{2} < x < \frac{\pi}{2}, \quad 0 \leq y \leq \underbrace{-\sqrt{3}\,\operatorname{tg} x - \operatorname{tg}^2 x}_{g(x)} \right\}$$

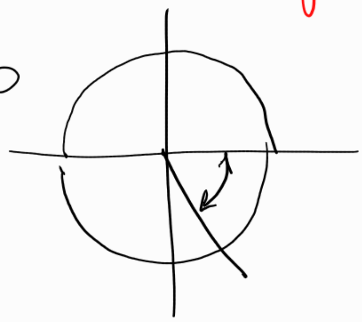
devo controllare per quali $x \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ si ha $g(x) \geq 0$.

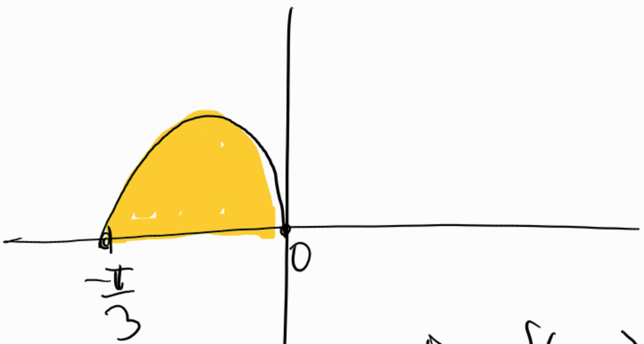
- $\sqrt{3}\,\operatorname{tg} x + \operatorname{tg}^2 x \leq 0$

$$\operatorname{tg} x = s$$

$$s^2 + \sqrt{3}\,s \leq 0 \iff -\sqrt{3} \leq \underbrace{s}_{= \operatorname{tg} x} \leq 0$$

$$-\sqrt{3} \leq \operatorname{tg} x \leq 0 \iff -\frac{\pi}{3} \leq x \leq 0$$





$$D = \left\{ (x,y) : -\frac{\pi}{3} \leq x \leq 0, \quad 0 \leq y \leq g(x) \right\}$$

$$\text{Area } D = \int_{-\pi/3}^{0} g(x)\,dx = \int_{-\pi/3}^{0} \left(-\sqrt{3}\,\operatorname{tg} x - \operatorname{tg}^2 x\right) dx$$

$$= -\sqrt{3} \int_{-\pi/3}^{0} \operatorname{tg} x\,dx - \int_{-\pi/3}^{0} \operatorname{tg}^2 x\,dx = (*)$$

$$\int tg\,x\,dx = \int \frac{\sin x}{\cos x}\,dx = \qquad \cos x = t \quad -\sin x\,dx = dt$$

$$= -\int \frac{dt}{t} = -\log|t| + c =$$

$$= -\log|\cos x| + c$$

$$\int tg^2 x\,dx = \int \left(\left(tg^2 x + 1 \right) - 1 \right) = tg\,x - x + c.$$

$$(*) = \sqrt{3}\,\log|\cos x|\Big|_{-\pi/3}^{0} - tg\,x\Big|_{-\pi/3}^{0} + x\Big|_{-\pi/3}^{0} =$$

$$= \sqrt{3}\left(\log 2\right) - \sqrt{3} + \frac{\pi}{3}$$