$$f(x) = x + \log\left(\frac{6-x}{|x|}\right)$$

Dominio: $\quad \dfrac{6-x}{|x|} > 0 \iff x \neq 0\,,\ x < 6.$

$$\text{Dominio} = (-\infty, 0) \cup (0, 6).$$

Continuità e limiti significativi

f è continua nel suo dominio

$$\lim_{x\to-\infty} f(x) = \lim_{x\to-\infty}\left(x + \log\left(\frac{6-x}{|x|}\right)\right) =$$

$$= \lim_{x\to-\infty}\left(\underbrace{x}_{\to -\infty} + \underbrace{\log\Big(\underbrace{\frac{x-6}{x}}_{\to 1}\Big)}_{\to \log 1 = 0}\right) = -\infty.$$

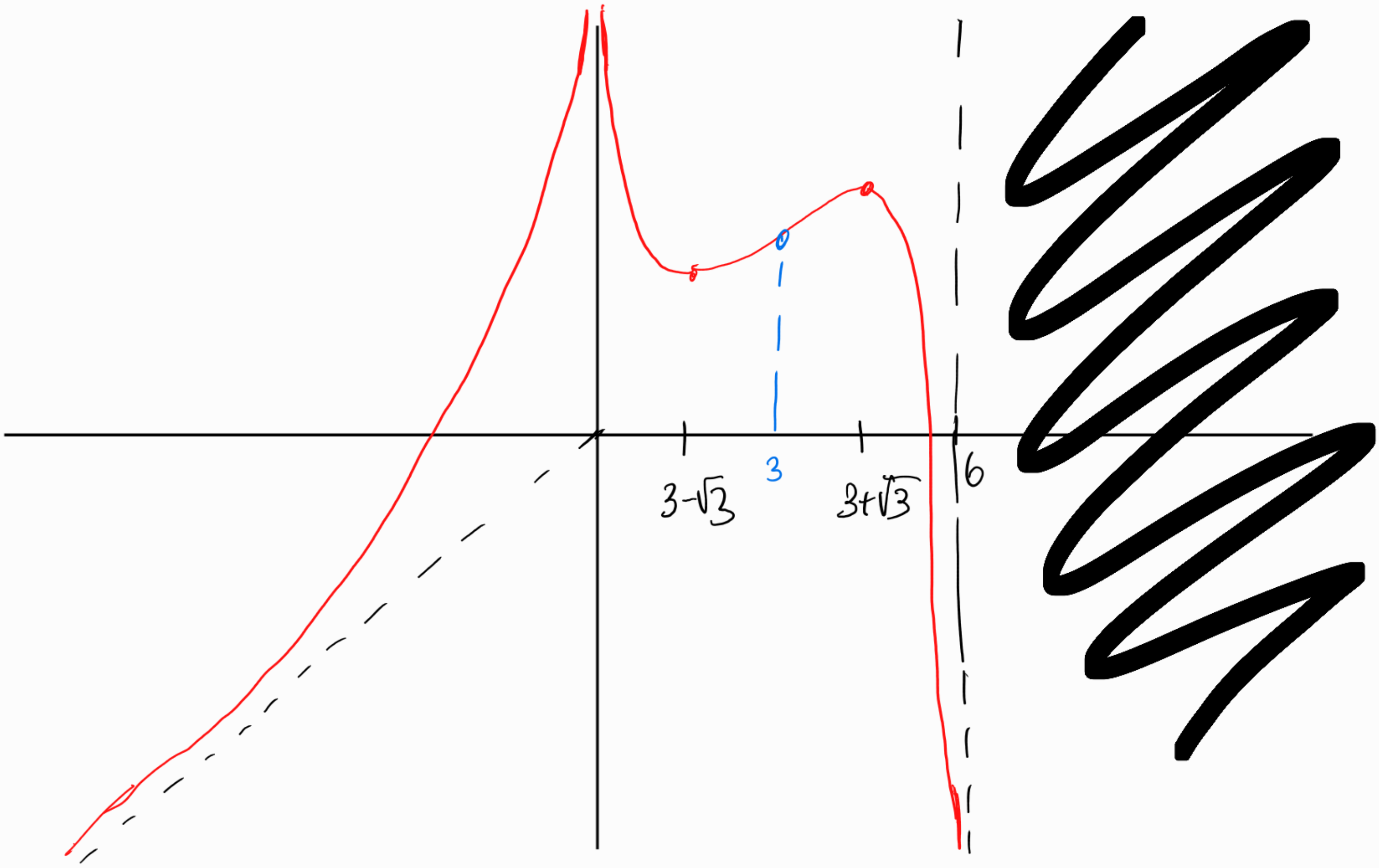
$y = x$ è asintoto obliquo per $x \to -\infty$.

$$\lim_{x\to 0} f(x) = \lim_{x\to 0}\left(\underbrace{x}_{\to 0} + \underbrace{\log\Big(\underbrace{\frac{6-x}{|x|}}_{\to +\infty}\Big)}_{\to +\infty}\right) = +\infty$$

$x = 0$ è asintoto verticale

$$\lim_{x\to 6^-} f(x) = \lim_{x\to 6^-}\left(\underbrace{x}_{\to 6} + \underbrace{\log\Big(\underbrace{\frac{6-x}{|x|}}_{\to 0^+}\Big)}_{\to -\infty}\right) = -\infty$$

$x = 6$ è asintoto verticale



$$f(x) = x + \underbrace{\log\left(\frac{6-x}{|x|}\right)} = x + \log(6-x) - \log|x|$$

$$\boxed{f'(x) = 1 - \frac{1}{6-x} - \frac{1}{x}}$$

$$(\log|x|)' = \frac{1}{x}$$

f è derivabile nel dominio

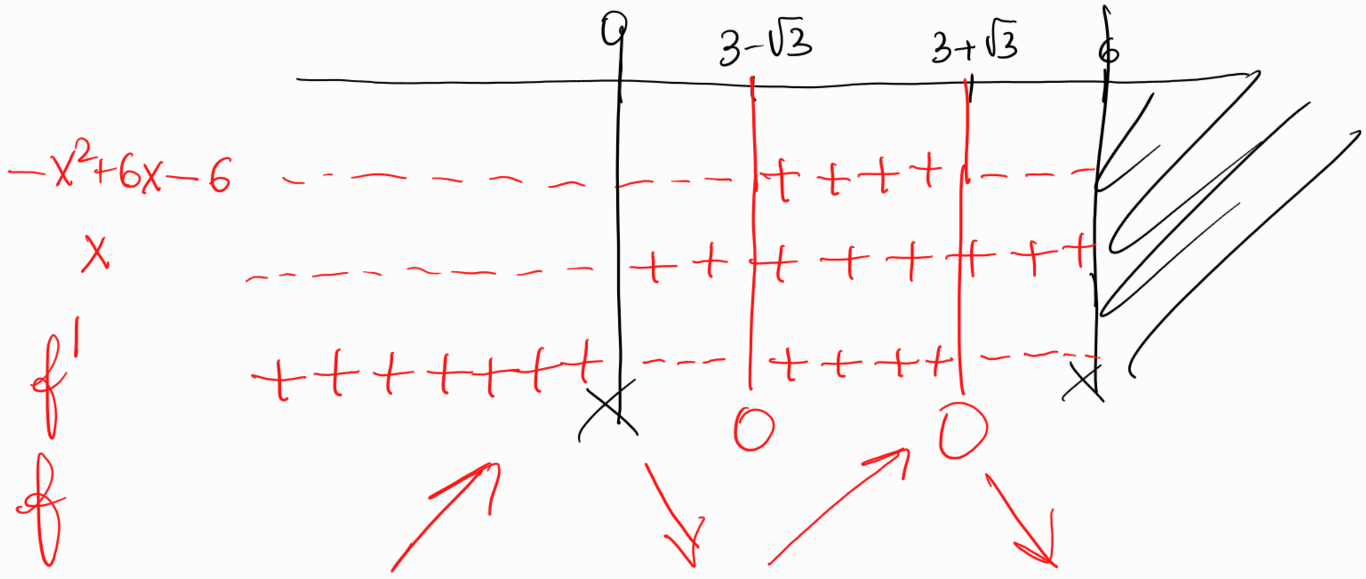
$$f'(x) = \frac{6x - x^2 - \cancel{x} - 6 + \cancel{x}}{(6-x)x} = \frac{-x^2+6x-6}{(6-x)x}$$

$$f'(x) = 0 \iff x^2 - 6x + 6 = 0$$

$$x = 3 \pm \sqrt{9-6} = 3 \pm \sqrt{3}$$

accettabili

$$f'(x) > 0 \qquad \frac{-x^2+6x-6}{\underbrace{(6-x)}_{>0}\, x} > 0$$



$f'(x)>0 \qquad x \in (-\infty, 0) \cup (3-\sqrt{3},\ 3+\sqrt{3}).$

$f'(x)<0 \qquad x \in (0, 3-\sqrt{3}) \cup (3+\sqrt{3}, 6)$

f strettamente crescente in $(-\infty, 0)$ e in $[3-\sqrt{3}, 3+\sqrt{3}]$

f " decrescente in $(0, 3-\sqrt{3}]$ e in $[3+\sqrt{3}, 6)$

$x = 3-\sqrt{3}$ è pto di min. locale

$x = 3+\sqrt{3}$ " " " max. locale.

$$f'(x) = 1 - \frac{1}{6-x} - \frac{1}{x}$$

$$f''(x) = -\frac{1}{(6-x)^2} + \frac{1}{x^2} = \frac{-x^2+(6-x)^2}{(6-x)^2x^2} = \frac{-x^2+36-12x+x^2}{(6-x)^2x^2}$$

$$= \frac{12(3-x)}{(6-x)^2x^2}$$

$f''(x)=0 \iff x=3$

$f''(x)>0 \iff x<3$

$<0 \iff x>3$

f convessa in $(-\infty, 0)$ e in $[0, 3]$

f concava in $[3, 6)$

$x = 3$ pto di flesso.

$$f(x) = \sin x \, e^{\sqrt{2}\cos x}$$

- dominio: $\mathbb{R}$.

periodicità: periodica di periodo 2π.

Continua, anzi derivabile, in $\mathbb{R}$.

Non ammette limiti a $\pm\infty$.

f dispari.

Basta studiarla in $[0, \pi]$, poi prolungarla in modo dispari in $[-\pi, 0]$ e poi prolungarla per periodicità.

Studio in $[0, \pi]$

$f(x) = 0 \iff (x = 0) \vee (x = \pi)$ $\quad f(x) > 0 \; \forall x \in (0, \pi)$

$$f(x) = \sin x \, e^{\sqrt{2}\cos x}$$

$$f'(x) = e^{\sqrt{2}\cos x}\left(-\sqrt{2}\sin^2 x + \cos x\right) =$$

$$= e^{\sqrt{2}\cos x}\left(\sqrt{2}\cos^2 x + \cos x - \sqrt{2}\right).$$

$$f'(x) = 0 \iff \sqrt{2}\cos^2 x + \cos x - \sqrt{2} = 0$$

$$\cos x = \frac{-1 \pm \sqrt{1+8}}{2\sqrt{2}} = \frac{-1 \pm 3}{2\sqrt{2}} = \begin{cases} \cancel{-\frac{2}{\sqrt{2}}} \\ \frac{1}{\sqrt{2}} \end{cases}$$

$$\cos x = \frac{1}{\sqrt{2}} \iff \boxed{x = \frac{\pi}{4}}$$

$$f'(x) > 0 \iff \sqrt{2}\cos^2 x + \cos x - \sqrt{2} \geq 0$$

$$\Updownarrow$$

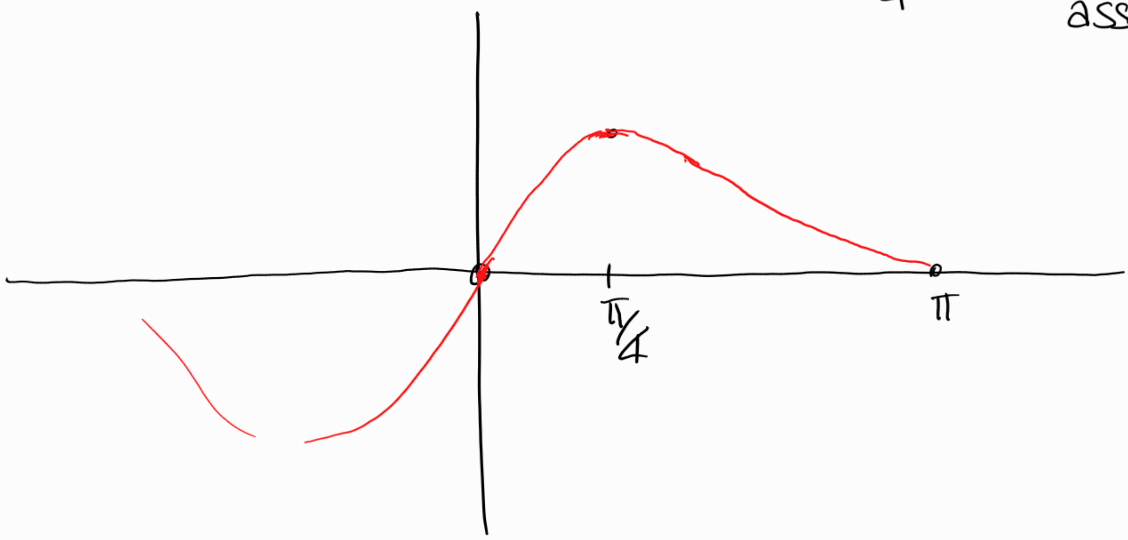
$$\cancel{\cos x < -\sqrt{2}} \ \vee \ \cos x > \frac{1}{\sqrt{2}}$$

$$\Updownarrow$$

$$0 \leq x < \frac{\pi}{4}$$

$$f'(x) < 0 \iff \frac{\pi}{4} < x \leq \pi.$$

$x = \frac{\pi}{4}$ pto di max assoluto.



$$f'(x) = e^{\sqrt{2}\cos x}\left(-\sqrt{2}\sin^2 x + \cos x\right) =$$

$$f''(x) = e^{\sqrt{2}\cos x}\left(-\sqrt{2}\sin x\left(\sqrt{2}\sin^2 x + \cos x\right) + \left(-2\sqrt{2}\sin x\cos x - \sin x\right)\right)$$

$$= e^{\sqrt{2}\cos x}\sin x\left(-2\sin^2 x - \sqrt{2}\cos x - 2\sqrt{2}\cos x - 1\right)$$

$$= e^{\sqrt{2}\cos x}\sin x\left(-2\sin^2 x - 3\sqrt{2}\cos x - 1\right)$$

$$= e^{\sqrt{2}\cos x}\sin x\left(2\cos^2 x - 3\sqrt{2}\cos x - 3\right)$$