Codice OPIS	A.A.	Compilati	Dettagli				
			Corso di Studi	Modulo	Insegnamento	Canale	Anno
AI173BR0	2024	33	INGEGNERIA AEROSPAZIALE (30837)		ANALISI MATEMATICA I (1015374)	A - K	1
GKKK2UZG	2024	0	INGEGNERIA AEROSPAZIALE (30837)		LABORATORIO DI MATEMATICA (AAF1524)	NESSUNA CANALIZZAZIONE	1

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$$\lim_{x\to 0^+} \boxed{\frac{2-2\cos(\ln(1+x))+(e^x-1)^2-2x^2}{x^4+x^\alpha}} \quad f(x)$$

Den
$$x^4+x^\alpha \begin{cases} \sim x^\alpha & \text{se } \alpha<4 \\ = 2x^4 & \alpha=4 \\ \sim x^4 & \text{se } \alpha>4 \end{cases}$$

OSS se $\alpha \le 0$ il limite vale 0.

$$e^x-1 = x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+o(x^4) \qquad x\to 0$$

$$(e^x-1)^2 = x^2+\frac{x^4}{4}+x^3+\frac{x^4}{3}+o(x^4)$$

$$= x^2+x^3+\frac{7}{12}x^4+o(x^4)$$

$\cos(\ln(1+x))$

$$\log(1+x) = \boxed{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+o(x^4)}$$

$$\cos t = 1-\frac{t^2}{2}+\frac{t^4}{24}+o(t^4) \qquad t\to 0$$

$$\cos(\ln(1+x)) = 1-\frac{1}{2}\left(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+o(x^4)\right)^2+$$

$$+\frac{1}{24}\left(\text{———}\right)^4+$$

$$+o(x^4) =$$

$$= 1-\frac{1}{2}\left(x^2+\frac{x^4}{4}-x^3+\frac{2}{3}x^4\right)$$

$$+\frac{1}{24}x^4+o(x^4)$$

(da finire...)

$$= 1 - \frac{x^2}{2} + \frac{x^3}{2} + x^4\underbrace{\left(-\frac{1}{8} - \frac{1}{3} + \frac{1}{24}\right)}_{\frac{-3-8+1}{24} = -\frac{10}{24} = -\frac{5}{12}} + o(x^4)$$

per $x \to 0$

$$\text{Num} = 2 - 2\cos(\ln(1+x)) + (e^x - 1)^2 - 2x^2 =$$

$$= \cancel{2} - \cancel{2} + \cancel{x^2} - \cancel{x^3} + \frac{5}{6}x^4$$

$$+ \cancel{x^2} + \cancel{x^3} + \frac{7}{12}x^4 + o(x^4) - \cancel{2x^2} =$$

$$= \frac{17}{12}x^4 + o(x^4) \sim \frac{17}{12}x^4 \qquad x \to 0$$

$$\lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\frac{17}{12}x^4}{x^\alpha + x^4}$$

se $\alpha > 4$ $\quad (x^\alpha + x^4 \sim x^4)$ $\quad \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\frac{17}{12}x^4}{x^4} = \frac{17}{12}$

$\alpha = 4$ $\quad (x^\alpha + x^4 = 2x^4)$ $\quad \lim_{x \to 0^+} f(x) = \frac{17}{24}$

$\alpha < 4$ $\quad (x^\alpha + x^4 \sim x^\alpha)$ $\quad \lim_{x \to 0^+} f(x) = \lim_{x \to 0^+} \frac{\frac{17}{12}x^4}{x^\alpha} =$

$$= \lim_{x \to 0^+} \frac{17}{12} x^{4-\alpha} = 0$$

Calcolare l'ordine di infinito/infinitesimo delle seguenti funzioni:

$$f(x) = \frac{\operatorname{arctg} x}{x^3 + 5\sin^2 x}$$

per $x \to 0^+$ e per $x \to +\infty$.

$\boxed{x \to 0^+}$

$$f(x) = \frac{\arctan x}{x^3 + 5\sin^2 x} = x^2\left(x + 5\,\frac{\sin^2 x}{x^2}\right) \sim 5x^2$$

$\arctan x \sim x$; $x \to 0$; $\frac{\sin^2 x}{x^2} \to 1$; $x + 5\frac{\sin^2 x}{x^2} \to 5$

$$f(x) \sim \frac{x}{5x^2} = \frac{1}{5x}$$ infinito di ordine 1 per $x \to 0^+$

$\boxed{x \to +\infty}$

$$f(x) = \frac{\arctan x}{x^3 + 5\sin^2 x}$$

$\arctan x \to \pi/2$

$$x^3 + 5\sin^2 x = x^3\left(1 + \frac{5\sin^2 x}{x^3}\right) \sim x^3$$

$\frac{5\sin^2 x}{x^3} \to 0$; $1 + \frac{5\sin^2 x}{x^3} \to 1$

$$f(x) \sim \frac{\pi}{2}\cdot\frac{1}{x^3}$$ infinitesimo di ordine 3. $x \to +\infty$

$$g(x) = x^2 - 3x^4 \log x =$$ per $x \to 0^+$

$$= x^2\left(1 - 3x^2\log x\right) \sim x^2$$ infinitesimo di ordine 2.

$3x^2 \log x \to 0$

$$h(x) = \frac{3}{x^2} + \sin\left(\frac{\alpha}{x^2}\right) + \frac{\beta}{x^6}$$ $x \to +\infty$

$\alpha, \beta \in \mathbb{R}$.

$\frac{\alpha}{x^2} \to 0$

$$\sin t = t - \frac{t^3}{6} + \frac{t^5}{120} + o(t^5)$$ $t \to 0$.

$t = \frac{\alpha}{x^2} \to 0$

$$\sin\frac{\alpha}{x^2} = \frac{\alpha}{x^2} - \frac{\alpha^3}{6x^6} + \frac{\alpha^5}{120x^{10}} + o\left(\frac{1}{x^{10}}\right)$$ $x \to +\infty$

$$h(x)=\frac{3+\alpha}{x^2}+\frac{1}{x^6}\left(\beta-\frac{\alpha^3}{6}\right)+\frac{\alpha^5}{120\,x^{10}}+o\left(\frac{1}{x^{10}}\right)$$

se $\begin{cases}\alpha\neq-3\\ \beta\in\mathbb{R}\end{cases}$ $\quad h(x)\sim\frac{3+\alpha}{x^2}$ infinitesimo di ordine 2.

$\alpha=-3$

$$h(x)=\frac{1}{x^6}\left(\beta+\frac{9}{2}\right)-\frac{243}{120}\,\frac{1}{x^{10}}+o\left(\frac{1}{x^{10}}\right)$$

se $\begin{cases}\alpha=-3\\ \beta\neq-\frac{9}{2}\end{cases}$ $h(x)$ è un infinitesimo di ordine 6.

se $\begin{cases}\alpha=-3\\ \beta=-\frac{9}{2}\end{cases}$ $\quad h(x)\sim-\frac{81}{40}\,\frac{1}{x^{10}}$ infinitesimo di ordine 10.

$$\lim_{n\to+\infty}\left(n^2+\sqrt{n}\right)^{-\frac{1}{\log n}}=\left((+\infty)^0\right)$$

$$=\lim_{n\to+\infty}e^{-\frac{\log(n^2+\sqrt{n})}{\log n}}=e^{-2}$$

$$\lim_{n\to+\infty}\left(-\frac{\log(n^2+\sqrt{n})}{\log n}\right)=\lim_{n\to+\infty}\left(-\frac{2\log n}{\log n}\right)=-2$$

$$\log\left(n^2+\sqrt{n}\right)=\log\left(n^2\left(1+\frac{1}{n^{3/2}}\right)\right)=\underbrace{\log(n^2)}_{\downarrow\ +\infty}+\underbrace{\log\left(1+\frac{1}{n^{3/2}}\right)}_{\downarrow\ 0}=$$

$$=\log(n^2)\underbrace{\left(1+\underbrace{\frac{\log\left(1+\frac{1}{n^{3/2}}\right)}{\log(n^2)}}_{=\,o(1)}\right)}_{\downarrow\ 1}\sim\log(n^2)=2\log n$$

$$\lim_{x\to 0} \frac{\sqrt{1+x} + \sqrt{1-x} - 2}{\cos x - 1} \quad \sim -\frac{x^2}{2}$$

$$\sqrt{1+x} = (1+x)^{1/2}$$

$$(1+x)^\alpha = 1 + \alpha x + \frac{\alpha(\alpha-1)}{2} x^2 + o(x^2) \quad \text{per } x\to 0$$

$\alpha = 1/2$

$$(1+x)^{1/2} = 1 + \frac{x}{2} - \frac{x^2}{8} + o(x^2)$$

$x \to 0$

$$\sqrt{1-x} = (1-x)^{1/2} = 1 - \frac{x}{2} - \frac{x^2}{8} + o(x^2)$$

etc....