$$\lim_{x\to 0^+} \frac{2\alpha + e^{2x} - \sin(2x)}{x^\alpha (1-\cos x)} = \left(\frac{2\alpha+1}{0^+}\right) = +\infty \quad \boxed{\alpha > 0}$$

$e^{2x} \to 1$, $\sin(2x) \to 0$, $x^\alpha(1-\cos x) \to 0^+$

$$\lim_{x\to 0} \left(\frac{1}{x^2} - \cot^2 x\right) = \lim_{x\to 0}\left(\frac{1}{x^2} - \frac{\cos^2 x}{\sin^2 x}\right) = (+\infty - \infty)$$

$\cos^2 x \to 1$, $\frac{1}{x^2} \to +\infty$, $\sin^2 x \to 0^+$

$$= \lim_{x\to 0} \frac{\sin^2 x - x^2\cos^2 x}{x^2 \sin^2 x} = \left(\frac{0}{0}\right)$$

$\sin^2 x \sim x^2$

cioè $\frac{\sin^2 x}{x^2} \to 1$ per $x \to 0$

$$= \boxed{\lim_{x\to 0} \frac{\sin^2 x - x^2\cos^2 x}{x^4}} = \frac{2}{3}$$

$$\sin x = x - \frac{x^3}{6} + o(x^4) \qquad x \to 0$$

$$\sin^2 x = \left(x - \frac{x^3}{6} + o(x^4)\right)^2 = x^2 + \frac{x^6}{36} - \frac{x^4}{3} + o(x^4)$$

$\frac{x^6}{36} = o(x^4)$

$$\sin^2 x = x^2 - \frac{x^4}{3} + o(x^4)$$

$$\cos x = 1 - \frac{x^2}{2} + o(x^2)$$

$$\cos^2 x = \left(1 - \frac{x^2}{2} + o(x^2)\right)^2 = 1 - x^2 + o(x^2)$$

$$x^2\cos^2 x = x^2\left(1 - x^2 + o(x^2)\right) = x^2 - x^4 + o(x^4)$$

$$\text{Num} = \sin^2 x - x^2\cos^2 x = \cancel{x^2} - \frac{x^4}{3} + o(x^4) - \cancel{x^2} + x^4 + o(x^4) =$$

$$= \frac{2}{3}x^4 + o(x^4) \sim \frac{2}{3}x^4$$

Oppure si poteva fare così:

$$\lim_{x\to0}\frac{\sin^2x-x^2\cos^2x}{x^4}=\lim_{x\to0}\frac{(\sin x-x\cos x)\boxed{(\sin x+x\cos x)}^{\sim 2x}}{x^4}=$$

$$\sin x+x\cos x=x\Big(\underbrace{\frac{\sin x}{x}}_{\to 1}+\underbrace{\cos x}_{\to 1}\Big)\sim 2x$$

$$=2\lim_{x\to0}\boxed{\frac{\sin x-x\cos x}{x^3}}=\frac{2}{3}$$

$$\sin x-x\cos x=x-\frac{x^3}{6}+o(x^3)-x\left(1-\frac{x^2}{2}+o(x^2)\right)=$$

$$=\cancel{x}-\frac{x^3}{6}+o(x^3)\cancel{-x}+\frac{x^3}{2}=\frac{x^3}{3}+o(x^3)\sim$$

$$\sim\frac{x^3}{3}$$

Oppure con L'Hôpital:

$$\lim_{x\to0}\frac{\sin^2x-x^2\cos^2x}{x^4}=\left(\frac{0}{0}\right)\overset{!}{\underset{H}{=}}$$

$$=\lim_{x\to0}\frac{\cancel{2}\sin x\cos x-\cancel{2}x\cos^2x+x^2\cancel{2}\cos x\sin x}{\underset{2}{\cancel{4}}x^3}=$$

$$=\lim_{x\to0}\frac{\overset{1}{\cos x}(\sin x-x\cos x+x^2\sin x)}{2x^3}=\left(\frac{0}{0}\right)\overset{!}{\underset{H}{=}}$$

$$=\lim_{x\to0}\frac{\cancel{\cos x}-\cancel{\cos x}+x\sin x+2x\sin x+x^2\cos x}{6x^2}=$$

$$=\lim_{x\to0}\frac{3\sin x+x\cos x}{6x}=\lim_{x\to0}\left(\overset{\to 1/2}{\frac{\sin x}{2x}}+\overset{\to 1/6}{\frac{\cos x}{6}}\right)=\frac{2}{3}$$

$$\lim_{x\to 0^+} \frac{2-2\cos(\ln(1+x))+(e^x-1)^2-2x^2}{x^4+x^\alpha}$$

<u>Den</u>

$$x^4+x^\alpha \begin{cases} \sim x^\alpha & \text{se } \alpha<4 \\ = 2x^4 & \alpha=4 \\ \sim x^4 & \text{se } \alpha>4 \end{cases}$$

OSS se $\alpha \le 0$ il limite vale 0.

$$e^x-1 = x+\frac{x^2}{2}+\frac{x^3}{6}+\frac{x^4}{24}+o(x^4) \qquad x\to 0$$

$$(e^x-1)^2 = x^2+\frac{x^4}{4}+x^3+\frac{x^4}{3}+o(x^4)$$

$$= x^2+x^3+\frac{7}{12}x^4+o(x^4)$$

$$\cos(\ln(1+x))$$

$$\log(1+x) = \boxed{x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+o(x^4)}$$

$$\cos t = 1-\frac{t^2}{2}+\frac{t^4}{24}+o(t^4) \qquad t\to 0$$

$$\cos(\ln(1+x)) = 1-\frac{1}{2}\Big(x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+o(x^4)\Big)^2+$$

$$+\frac{1}{24}\Big(\qquad\qquad\Big)^4+$$

$$+o(x^4) =$$

$$= 1-\frac{1}{2}\Big(x^2+\frac{x^4}{4}-x^3+\frac{2}{3}x^4\Big)$$

$$+\frac{1}{24}x^4+o(x^4)$$

(da finire...)