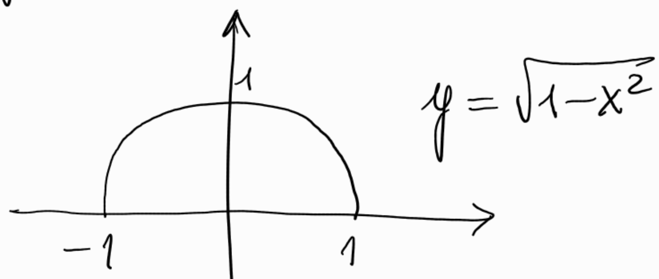
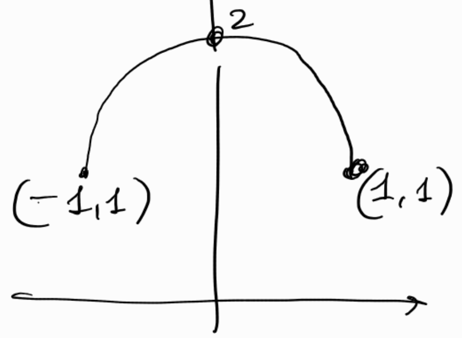
Grafico (approssimato) di $f(x) = \dfrac{1}{1+\sqrt{1-x^2}}$ dominio $[-1,1]$

$y = \sqrt{1-x^2} \implies y^2 = 1-x^2$



$\boxed{y = 1+\sqrt{1-x^2}}$



$y = \dfrac{1}{1+\sqrt{1-x^2}}$

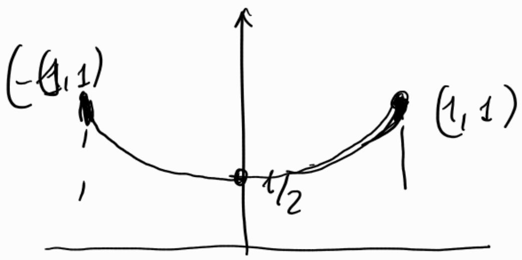
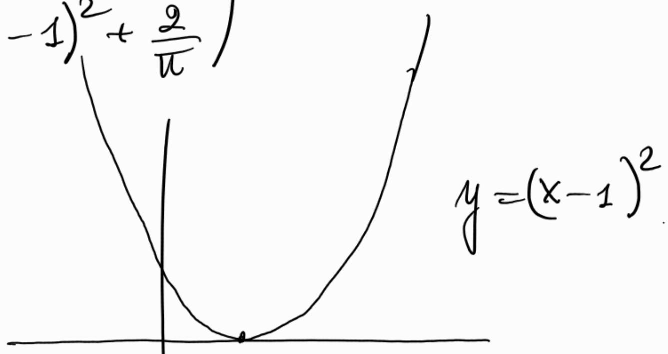
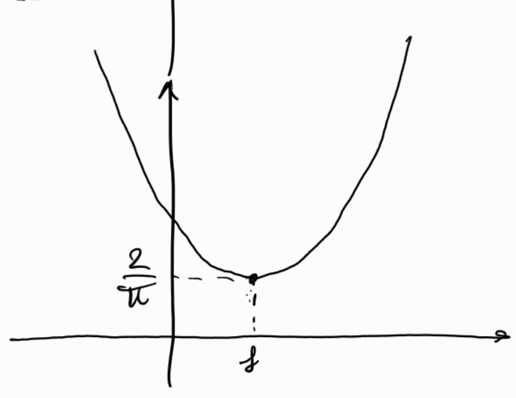


Grafico di $f(x) = \cos\left(\dfrac{1}{(|x|-1)^2+\frac{2}{\pi}}\right)$

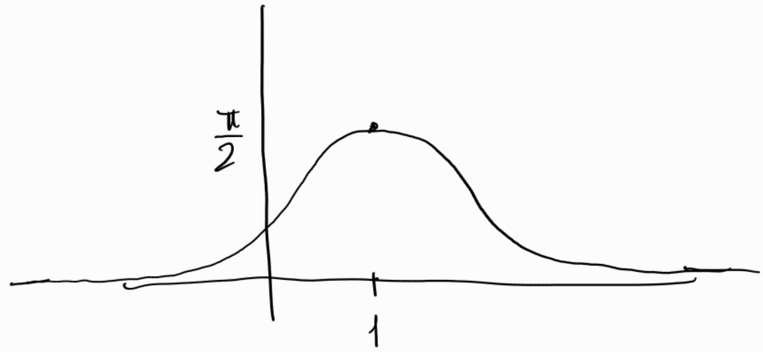
$y = (x-1)^2$



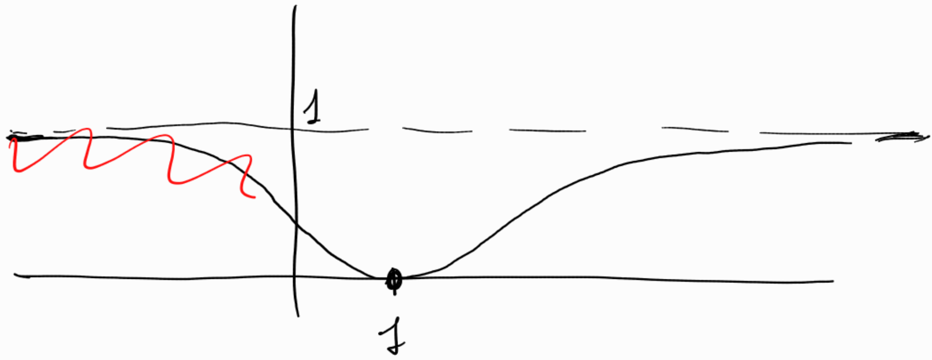
$y = (x-1)^2+\frac{2}{\pi}$



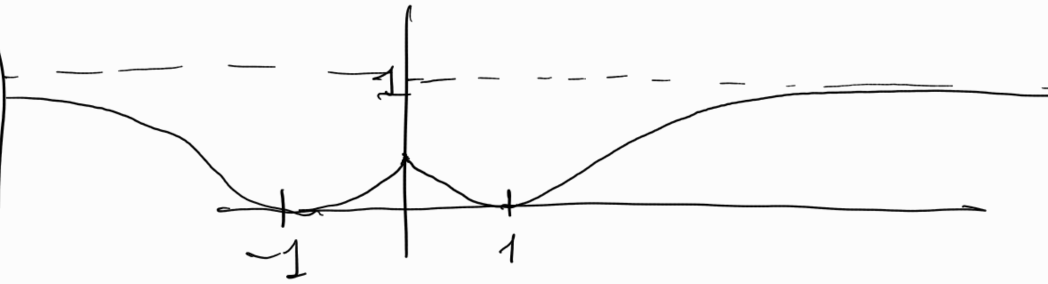
$$\boxed{y = \frac{1}{(x-1)^2+\frac{2}{\pi}}}$$



$$y = \cos\left(\frac{1}{(x-1)^2+\frac{2}{\pi}}\right)$$



$$y = \cos\left(\frac{1}{(|x|-1)^2+\frac{2}{\pi}}\right)$$



$$f(x) = \log \frac{|\cos x| - \sin(2x)}{\sqrt{\pi^2 - 9\,\mathrm{arctg}^2\left(\frac{x\sqrt{3}}{x+1}\right)}}$$

se esiste, è positiva.

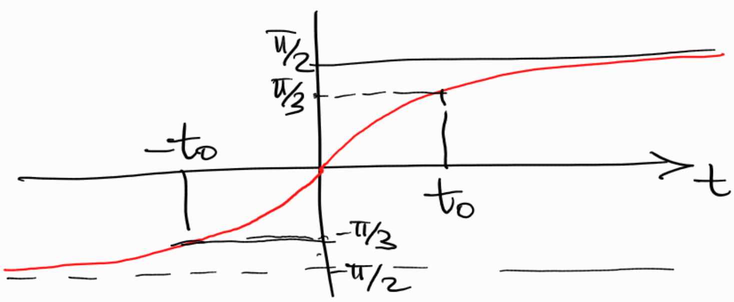
Devo imporre
$$\begin{cases} |\cos x| - \sin(2x) > 0 & (A) \\ \pi^2 - 9\,\mathrm{artg}^2\left(\frac{x\sqrt{3}}{x+1}\right) > 0 & (B) \end{cases}$$

(B)
$$\mathrm{arctg}^2\Big(\underbrace{\frac{x\sqrt{3}}{x+1}}_{t}\Big) < \frac{\pi^2}{9} \iff$$

$$-\frac{\pi}{3} < \mathrm{arctg}\, t < \frac{\pi}{3}$$

$$\Updownarrow$$

$$-t_0 < t < t_0$$



$$\mathrm{arctg}\, t_0 = \pi/3 \Rightarrow t_0 = \sqrt{3}$$

$$-\sqrt{3} < \underset{\frac{\sqrt{3}\,x}{x+1}}{t} < \sqrt{3}$$

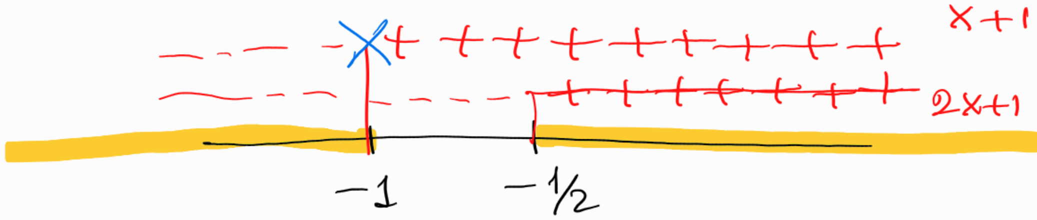
$$\Updownarrow$$

$$-\sqrt{3} < \frac{\sqrt{3}\,x}{x+1} < \sqrt{3}$$

$$\Updownarrow$$

$$-1 \underset{(\alpha)}{<} \frac{x}{x+1} \underset{(\beta)}{<} 1$$

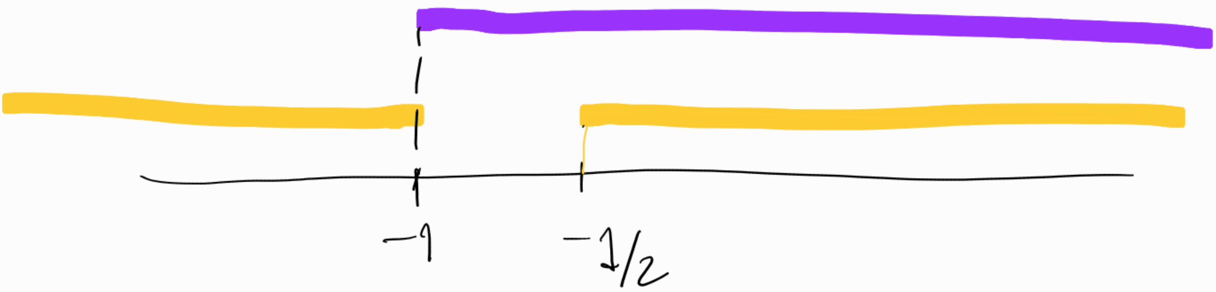
(α) $\quad \frac{x}{x+1} > -1 \iff \frac{x}{x+1} > \frac{-x-1}{x+1} \iff \frac{2x+1}{x+1} > 0$



(α) $\iff (x < -1) \vee \left(x > -\frac{1}{2}\right)$

(β) $\quad \frac{\cancel{x}}{x+1} < \frac{\cancel{x}+1}{x+1} \qquad \frac{1}{x+1} > 0 \iff x > -1$

(α) ∧ (β) $\iff \boxed{x > -\frac{1}{2}}$



(A) $\quad |\cos x| - \operatorname{sen}(2x) > 0$ ha periodo π, basta studiarla in un intervallo di ampiezza π.

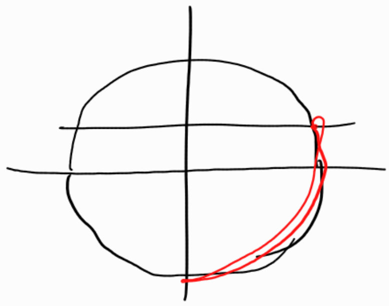
per es. $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ dove $\cos x \geq 0$

tolgo il valore assoluto

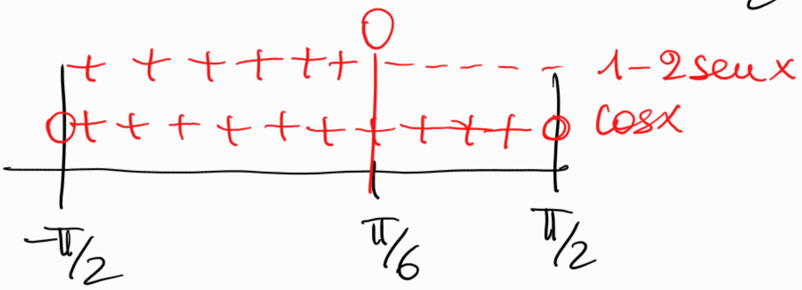
$$\cos x - \underbrace{\sin 2x}_{\substack{\shortparallel \\ 2\cos x \sin x}} > 0$$

$$\cos x \,(1 - 2\sin x) > 0$$

$\cos x > 0 \qquad -\frac{\pi}{2} < x < \frac{\pi}{2}$.

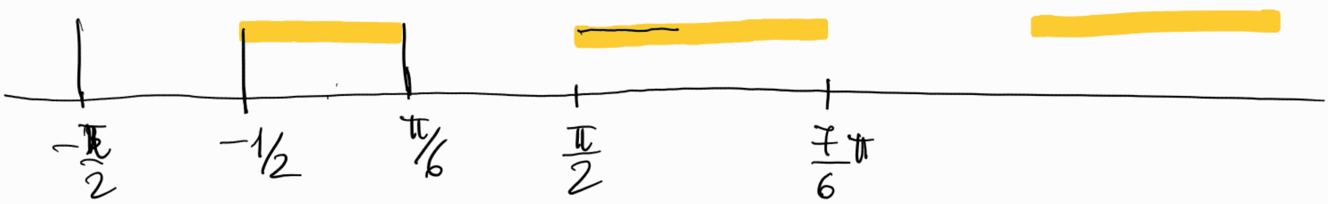


$1 - 2\sin x > 0 \iff \sin x < \frac{1}{2} \iff \frac{\pi}{2} \leq x < \frac{\pi}{6}$



$$\boxed{-\frac{\pi}{2} + k\pi < x < \frac{\pi}{6} + k\pi} \quad (A)$$

$$\boxed{x > -\frac{1}{2}} \quad (B)$$



$$-\frac{1}{2} < x < \frac{\pi}{6}$$

Dominio $\quad -\frac{1}{2} < x < \frac{\pi}{6} \;\vee\; \left(-\frac{\pi}{2} + k\pi < x < \frac{\pi}{6} + k\pi\right) \quad k \in \mathbb{N},\ k \geq 1$