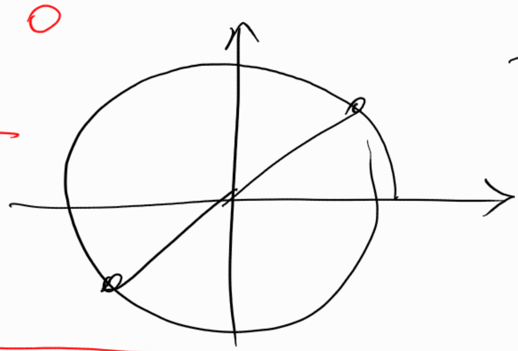
$$\lim_{n\to+\infty} \underbrace{(3n+2)}_{\downarrow\, +\infty} \underbrace{\sin\left(\frac{\pi n-5}{n+7}\right)}_{\underbrace{\downarrow\, \sin\pi = 0}_{\downarrow\, 0^+}} = (0\cdot\infty)$$

$$= \lim_{n\to+\infty} \underbrace{(3n+2)}_{\sim\, 3n} \sin\left(\pi + \underbrace{\left(\frac{\pi n-5}{n+7}-\pi\right)}_{\downarrow\, 0}\right) =$$

$$\left[\ \sin(\pi+x) = -\sin x \qquad \frac{7\pi+5}{n} \sim \frac{7\pi+5}{n+7}\ \right]$$



$$= \lim_{n\to+\infty} 3n \sin\left(\underbrace{\pi - \frac{\pi n-5}{n+7}}\right) = \lim_{n\to+\infty} 3\cancel{n}\cdot\frac{7\pi+5}{\cancel{n}} = 21\pi+15$$

$$\frac{\cancel{\pi n}+7\pi-\cancel{\pi n}+5}{n+7} = \frac{7\pi+5}{n+7} \longrightarrow 0$$

$$\lim_{n\to+\infty} n\left(\underbrace{1-\cos\frac{1}{\sqrt n}+e^{-n}}_{\downarrow\, 0}\right) = (\infty\cdot 0) =$$

$$= \lim_{n\to+\infty} \cancel{n}\,\frac{\cancel{1}}{\cancel{n}}\underbrace{\left[\underbrace{\boxed{\frac{1-\cos\frac{1}{\sqrt n}}{1/n}}}_{\downarrow\, \frac{1}{2}} + \underbrace{\frac{e^{-n}}{1/n}}_{=\,\frac{n}{e^n}\,\to\, 0}\right]}_{\to\, 1/2} = \frac{1}{2}.$$

Ordinare i seguenti infiniti, per $x \to 0^+$

$$f(x) = \frac{\log\left(1 - \operatorname{tg}\left(x + \frac{\pi}{2}\right)\right)}{x^2}, \quad g(x) = \log_2\left(3 + \frac{1}{x^4} + 2^{1/x}\right)$$

$$h(x) = \frac{\overbrace{\operatorname{arctg}(x^2)}^{\sim x^2}}{\underbrace{\operatorname{sen}^4 x}_{\sim x^4}} \sim \frac{1}{x^2}$$

$$3 + \frac{1}{x^4} + 2^{1/x} = 3 + t^4 + 2^t = 2^t\left(1 + \frac{3}{2^t} + \frac{t^4}{2^t}\right) = 2^t(1 + o(1))$$

$$t = \frac{1}{x} \to +\infty$$

$$3 + \frac{1}{x^4} + 2^{1/x} = 2^{1/x}(1 + o(1)) \qquad \text{per } x \to 0^+$$

$$\log_2\left(3 + \frac{1}{x^4} + 2^{1/x}\right) = \log_2\left(2^{1/x}(1 + o(1))\right) =$$

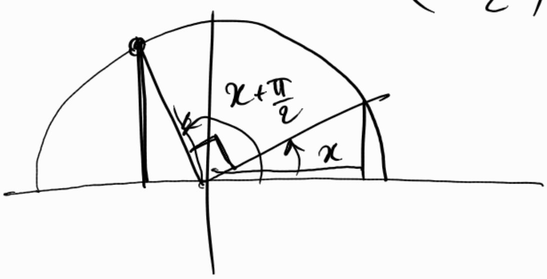
$$= \underbrace{\log_2 2^{1/x}}_{= 1/x} + \underbrace{\log_2(1 + o(1))}_{= o(1)} = \frac{1}{x} + o(1) = \frac{1}{x}\left(1 + x\, o(1)\right)$$

$$\sim \frac{1}{x}$$

$g(x)$ è infinito di ordine 1.

$$f(x) = \frac{\log\left(1 - \overbrace{\operatorname{tg}\left(x + \frac{\pi}{2}\right)}^{\to -\infty}\right)}{x^2}$$

$$-\operatorname{tg}\left(x + \frac{\pi}{2}\right) = -\frac{\sin\left(x + \frac{\pi}{2}\right)}{\cos\left(x + \frac{\pi}{2}\right)} = \frac{\overbrace{\cos x}^{\sim 1}}{\underbrace{\sin x}_{\sim x}} \sim \frac{1}{x} \qquad x \to 0^+$$



$$\sin\left(x + \frac{\pi}{2}\right) = \cos x$$

$$\cos\left(x + \frac{\pi}{2}\right) = -\sin x$$

$$1-\operatorname{tg}\left(x+\frac{\pi}{2}\right)=1+\frac{1}{x}\left(1+o(1)\right)=\frac{1}{x}\Big(\underbrace{x}_{=o(1)}+1+o(1)\Big)=$$

$$=\frac{1}{x}\left(1+o(1)\right)$$

$$\log\left(1-\operatorname{tg}\left(x+\frac{\pi}{2}\right)\right)=\log\left(\frac{1}{x}\left(1+o(1)\right)\right)=$$

$$=\log\frac{1}{x}+\log\left(1+o(1)\right)=$$

$$=-\log x+\underbrace{\log\left(1+o(1)\right)}_{=o(1)}=-\log x\Big(1+\underbrace{\frac{o(1)}{-\log(x)}}_{=o(1)}\Big)\sim-\log x$$

$f(x)\sim-\dfrac{\log x}{x^2}$ è infinito di ordine superiore a $\dfrac{1}{x^2}$ per $x\to 0^+$ e di ordine inferiore risp. a $\dfrac{1}{x^\alpha}$ $\forall\alpha>2$

In conclusione, in ordine crescente di infinito

$$g(x),\ h(x),\ f(x)$$

Ordinare i seguenti infinitesimi, per $x\to+\infty$

$$f(x)=\operatorname{tg}\Big(\underbrace{\frac{x}{2x^2+\sqrt{x}}}_{\downarrow\, 0}\Big)\sim\frac{x}{2x^2+\sqrt{x}}\sim\frac{1}{2x}$$

$$g(x)=\sqrt{9x^4+5}-3x^2=3x^2\Big(\underbrace{\sqrt{1+\underbrace{\frac{5}{9x^4}}_{\downarrow\,0}}-1}_{\sim\frac{5}{18x^4}}\Big)\sim\frac{3x^2\cdot 5}{18x^4}=$$

$$=\frac{5}{6x^2}$$

$\sqrt{1+t}-1\sim\dfrac{t}{2}$ $\quad t\to 0$

$$h(x) = x^{1-\log x} = \frac{1}{x^{\log x - 1}}$$ è inf$^{\text{mo}}$ di ordine superiore a $\frac{1}{x^\alpha}$ $\forall \alpha \geq 0$.

$$\frac{\left(\frac{1}{x^{\log x - 1}}\right)}{\left(1/x^\alpha\right)} = \underbrace{x}_{\downarrow\, +\infty}^{\overbrace{\alpha - \log x + 1}^{\downarrow\, -\infty}} \longrightarrow 0$$

$$k(x) = \frac{1}{2\cos x + x^2 \log(2^x + 7)}$$

$$2\cos x + x^2 \log(2^x + 7)$$

$$x^2 \log(2^x + 7) = x^2 \log\left(2^x(1 + o(1))\right) = x^2\Big[\underbrace{\log 2^x}_{x \log 2} + \log(1 + o(1))\Big] =$$

$$= x^3 \left[\log 2 + o(1)\right]$$

$$2\cos x + x^2 \log(2^x + 7) = 2\cos x + x^3\left[\log 2 + o(1)\right] =$$

$$= x^3\Big[\log 2 + \underbrace{\frac{2\cos x}{x^3}}_{o(1)} + o(1)\Big] = x^3\left[\log 2 + o(1)\right] \sim x^3 \log 2$$

$$k(x) \sim \frac{1}{x^3 \log 2}.$$

In ordine crescente di infinitesimo: f, g, k, h

$$\lim_{n\to+\infty}\left(\cos\frac{1}{n^2}\right)^{\sqrt{4n^8+7n+3}} = (1^{+\infty}).$$

$$= \lim_{n\to+\infty} e^{\sqrt{4n^8+7n+3}\,\log\left(\cos\frac{1}{n^2}\right)} = e^{-1} = \frac{1}{e}$$

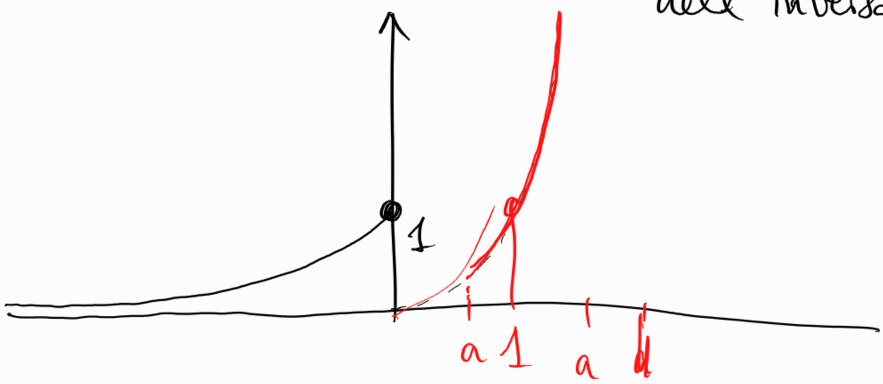
$$\underbrace{\sqrt{4n^8+7n+3}}_{\sim\, 2n^4}\ \underbrace{\log\left(\cos\frac{1}{n^2}\right)}_{=} \sim 2n^4\left(-\frac{1}{2n^4}\right) = -1$$

$$\log\Big(1+\underbrace{\left(\cos\frac{1}{n^2}-1\right)}_{\to 0}\Big) \sim \cos\frac{1}{n^2}-1 \sim -\frac{1}{2n^4}$$

Dire per quali valori di $a \overset{>0}{\cancel{\in\mathbb{R}}}$, data

$$f(x) = \begin{cases} e^x & \text{se } x \le 0 \\ x^2 & \text{se } x > a \end{cases}$$

Trovare per quali valori di a f è biiettiva come funzione da $\mathrm{dom}\, f$ a $\mathrm{im}\, f$, e poi studiare la continuità dell'inversa

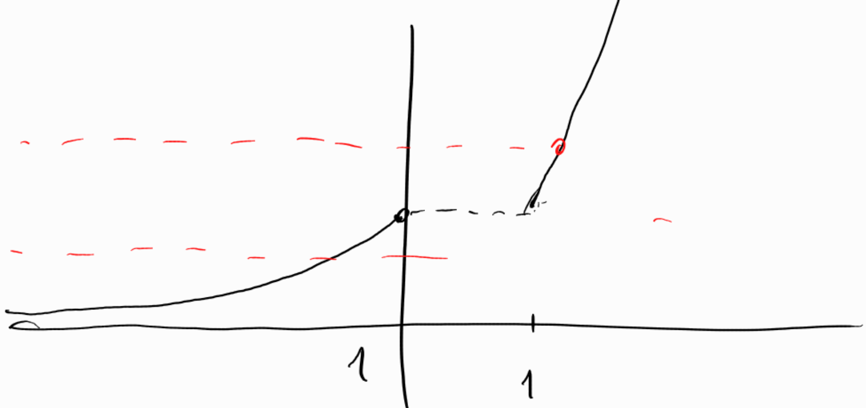


OSS se $\boxed{0<a<1}$, f non è iniettiva perché assume due volte il valore 1

$f(0) = f(1) = 1$.

se $a = 1$

$a = 1$



$\alpha = 1$ f è iniettiva, $\text{dom } f = (-\infty, 0] \cup (1, +\infty)$.

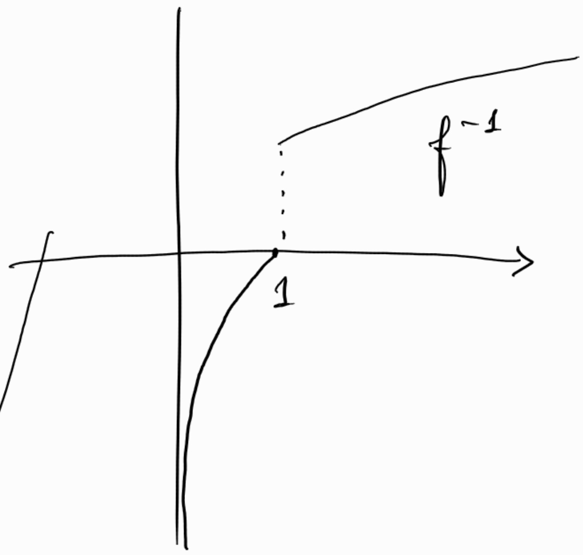
$\text{im } f = (0, +\infty)$

f biiettiva da $(-\infty, 0] \cup (1, +\infty)$ a $(0, +\infty)$.

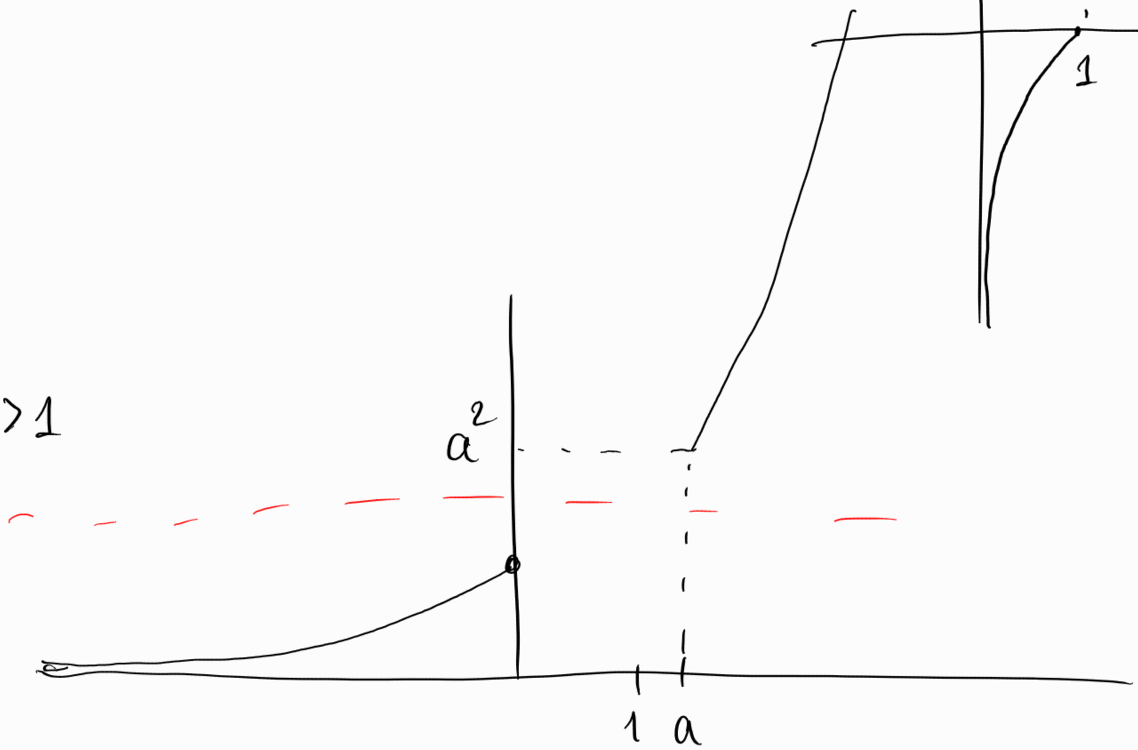
$f^{-1} : (0, +\infty) \to (-\infty, 0] \cup (1, +\infty)$.

f^{-1} non è continua (la sua immagine non è un intervallo)

$$f^{-1}(y) = \begin{cases} \log y & \text{se } 0 < y \leq 1 \\ \sqrt{y} & \text{se } y > 1 \end{cases}$$



$a > 1$



$f : (-\infty, 0] \cup (a, +\infty)$ è iniettiva in quanto strett. crescente

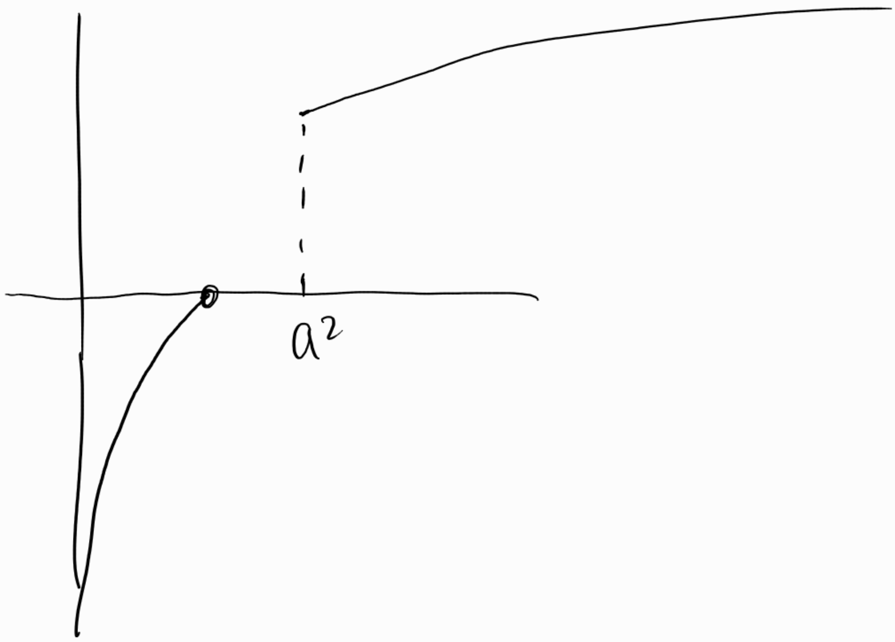
$\text{im } f = (0, 1] \cup (a^2, +\infty)$

$f : (-\infty, 0] \cup (a, +\infty) \longrightarrow (0, 1] \cup (a^2, +\infty)$ è biiettiva.

$f^{-1} : (0, 1] \cup (a^2, +\infty) \longrightarrow (-\infty, 0] \cup (a, +\infty)$

$$f^{-1}(y) = \begin{cases} \log y & \text{se } 0 < y \leq 1 \\ \sqrt{y} & \text{se } y > a^2 \end{cases}$$

è continua.



Derivabilità di $f(x) = \sqrt{-x^2 + 5x - 4}$

dominio: $-x^2 + 5x - 4 \geq 0$

$x^2 - 5x + 4 \leq 0$ $\quad\boxed{1 \leq x \leq 4}$

f continua in $[1, 4]$.

$$f'(x) = \frac{1}{2\sqrt{-x^2+5x-4}} \cdot (-2x+5) = \frac{-2x+5}{2\sqrt{-x^2+5x-4}}$$

$\forall x \in (1, 4)$

Derivabilità in $x = 1$ e $x = 4$?

$\boxed{x=1}$ $\quad f'(1)=f'_+(1)=\lim_{h\to 0^+} \frac{f(1+h)-\cancel{f(1)}}{h} =$

$$= \lim_{h\to 0^+} \frac{\sqrt{-(1+h)^2+5(1+h)-4}}{h} =$$

$$= \lim_{h\to 0^+} \frac{\sqrt{-1-h^2-2h+\cancel{5}+5h-\cancel{4}}}{h} = \lim_{h\to 0^+} \frac{\overbrace{\sqrt{3h-h^2}}^{\sim\sqrt{3h}}}{h} = +\infty$$

f non è derivabile in $x=1$ (pto a tg. verticale)

$\boxed{x=4}$

$$f'(4)=f'_-(4)=\lim_{h\to 0^-} \frac{f(4+h)-\cancel{f(4)}}{h} =$$

$$= \lim_{h\to 0^-} \frac{\sqrt{-(4+h)^2+5(4+h)-4}}{h} =$$

$$= \lim_{h\to 0^-} \frac{\sqrt{-\cancel{16}-h^2-8h+\cancel{20}+5h-\cancel{4}}}{h} = \lim_{h\to 0^-} \frac{\sqrt{-3h-h^2}}{h} =$$

$$= -\infty$$

f non derivabile in $x=4$ (pto a tg. verticale)

$$\sim \frac{\sqrt{-3h}}{h} = -\sqrt{\frac{-3h}{h^2}} = -\sqrt{\frac{-3}{h}} \to -\infty$$

<u>OSS</u> Il grafico di f è una semicirconferenza

$$y=\sqrt{-x^2+5x-4}$$

$$y^2=-x^2+5x-4 \qquad x^2-5x+\frac{25}{4}+y^2=\frac{25}{4}-4$$

$$\left(x-\frac{5}{2}\right)^2+y^2=\frac{9}{4}=\left(\frac{3}{2}\right)^2$$

Circonferenza di centro $\left(\frac{5}{2}, 0\right)$ e raggio $\frac{3}{2}$