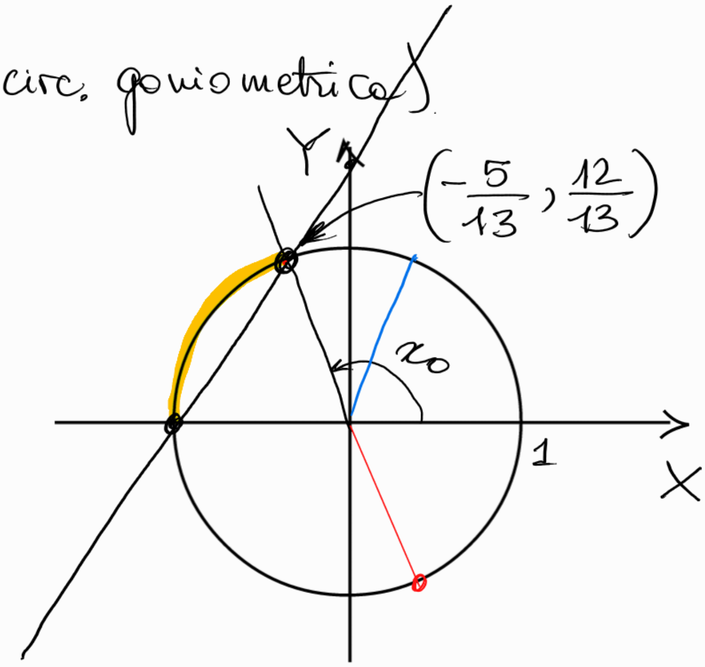
$$\boxed{2\sin x - 3\cos x \geq 3}$$

1° modo (intersezione con la circ. goniometrica)

$\sin x = Y$

$\cos x = X$

$$\begin{cases} Y \geq \frac{3}{2}(X+1) \\ X^2 + Y^2 = 1 \end{cases}$$



$$x_0 + 2k\pi \leq x \leq \pi + 2k\pi$$

Chi è x_0?

$x_0 = \arccos\left(-\frac{5}{13}\right)$ corretto.

$x_0 = \cancel{\arcsin\left(\frac{12}{13}\right)} = \pi - \arcsin\left(\frac{12}{13}\right)$

$x_0 = \cancel{\operatorname{arctg}\left(-\frac{12}{5}\right)} = \pi + \operatorname{arctg}\left(-\frac{12}{5}\right) = \pi - \operatorname{arctg}\left(\frac{12}{5}\right)$

2° Con l'angolo aggiunto

$f(x) = 2\sin x - 3\cos x =$

$f(x) = A\sin x + B\cos x = \sqrt{A^2+B^2}\,\sin(x+\varphi)$

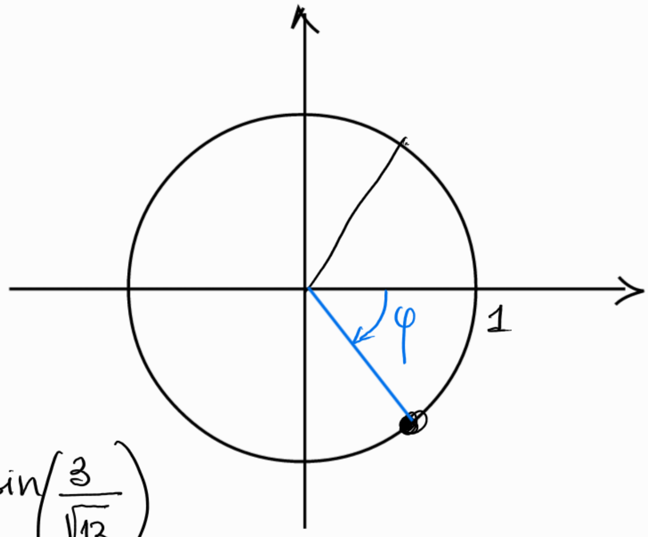
e φ è sol^{ne} di $\begin{cases} \cos\varphi = \frac{A}{\sqrt{A^2+B^2}} \\ \sin\varphi = \frac{B}{\sqrt{A^2+B^2}} \end{cases}$

$f(x) = 2\sin x - 3\cos x$

$A = 2 \qquad B = -3$

$\sqrt{A^2+B^2} = \sqrt{13}$

$$\begin{cases} \cos\varphi = \dfrac{2}{\sqrt{13}} \\ \sin\varphi = \dfrac{-3}{\sqrt{13}} \end{cases}$$



$$\varphi = -\arccos\frac{2}{\sqrt{13}} =$$

$$= \arcsin\left(-\frac{3}{\sqrt{13}}\right) = -\arcsin\left(\frac{3}{\sqrt{13}}\right)$$

$$= \operatorname{arctg}\left(-\frac{3}{2}\right) = -\operatorname{arctg}\left(\frac{3}{2}\right)$$

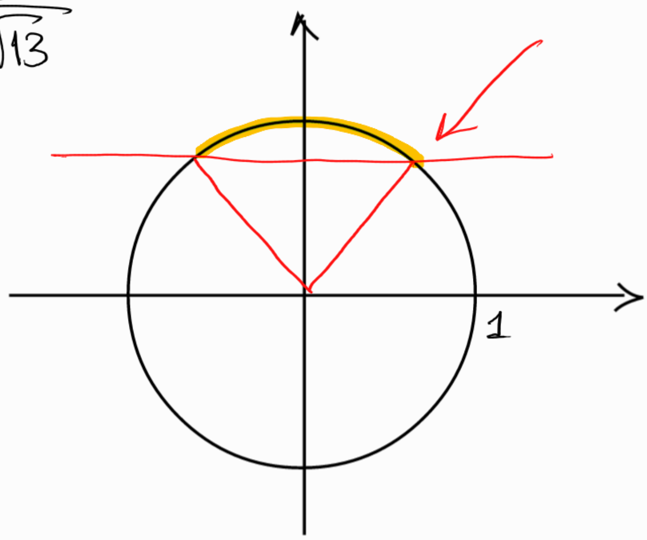
$$2\sin x - 3\cos x = \sqrt{13}\,\sin(x+\varphi)$$

La diseq^ne diventa

$$\sqrt{13}\,\sin(x+\varphi) \geq 3$$

$$\sin(\underbrace{x+\varphi}_{t}) \geq \frac{3}{\sqrt{13}}$$

$$\Updownarrow$$



$$\underbrace{\arcsin\left(\frac{3}{\sqrt{13}}\right)}_{-\varphi} + 2k\pi \leq \underset{\substack{\parallel \\ x+\varphi}}{t} \leq \underbrace{\pi - \arcsin\left(\frac{3}{\sqrt{13}}\right)}_{\pi+\varphi} + 2k\pi.$$

$$-\varphi + 2k\pi \leq x+\varphi \leq \pi+\varphi + 2k\pi.$$

$$\underbrace{-2\varphi}_{\substack{\shortparallel \\ 2\arcsin\frac{3}{\sqrt{13}}}} + 2k\pi \leq x \leq \pi + 2k\pi.$$

Per controllare che il risultato sia lo stesso di prima, devo controllare che

$$2\arcsin\frac{3}{\sqrt{13}} = \arccos\left(-\frac{5}{13}\right)$$

si tratta di due angoli in $[0,\pi]$.

Sono uguali se e solo se il loro coseno è uguale.

$$\cos\left(2\arcsin\frac{3}{\sqrt{13}}\right) \stackrel{?}{=} -\frac{5}{13}$$

$$\cos(2t) = 1 - 2\sin^2 t$$

$$\cos\left(2\arcsin\frac{3}{\sqrt{13}}\right) = 1 - 2\left(\sin\left(\arcsin\frac{3}{\sqrt{13}}\right)\right)^2 =$$

$$= 1 - 2\left(\frac{3}{\sqrt{13}}\right)^2 = 1 - 2\cdot\frac{9}{13} = \frac{13-18}{13} = -\frac{5}{13}$$

3° modo Sost. $t = \operatorname{tg}\frac{x}{2}$.

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

Come si dimostrano.

$$\sin(2x) = 2\sin x\cos x = 2\,\underbrace{\frac{\sin x}{\cos x}}_{\substack{\shortparallel \\ \operatorname{tg} x}}\,\underbrace{\cos^2 x}_{\substack{\shortparallel \\ \frac{1}{1+\operatorname{tg}^2 x}}} = \frac{2\operatorname{tg} x}{1+\operatorname{tg}^2 x}$$

$$\frac{1}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = 1 + tg^2 x$$

$2x \hookrightarrow x$

$$\sin x = \frac{2\, tg\left(\frac{x}{2}\right)}{1 + tg^2\left(\frac{x}{2}\right)} = \frac{2t}{1+t^2}$$

$$\cos(2x) = 2\cos^2 x - 1 = \frac{2}{1+tg^2 x} - 1 = \frac{1 - tg^2 x}{1 + tg^2 x}$$

$$\cos^2 x = \frac{1}{1+tg^2 x}$$

metto x al posto di $2x$

$$\cos x = \frac{1-t^2}{1+t^2}$$

$$\frac{x}{2} \neq (2k+1)\frac{\pi}{2}$$

$$x \neq \pi + 2k\pi$$

$$\boxed{t = tg\,\frac{x}{2}}$$

$$\boxed{2\sin x - 3\cos x \geq 3}$$

$$\frac{4t}{1+t^2} - \frac{3(1-t^2)}{1+t^2} \geq 3$$

molt. per $1+t^2 > 0$

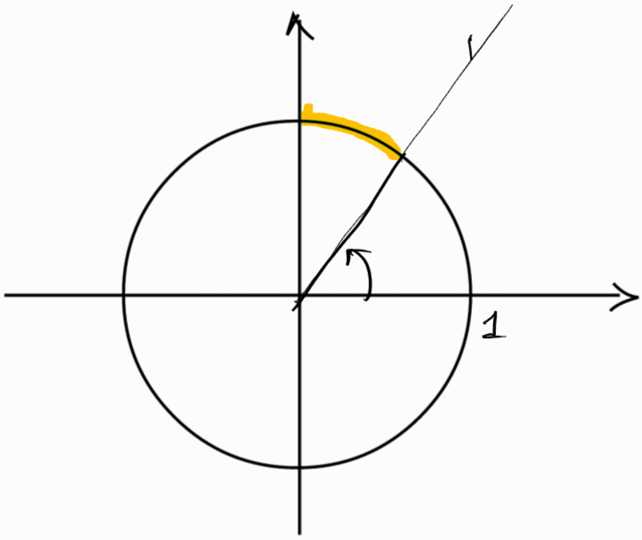
$$4t - 3 + \cancel{3t^2} \geq 3 + \cancel{3t^2}$$

$$4t \geq 6$$

$$t \geq \frac{3}{2}$$

$$tg\left(\frac{x}{2}\right) \geq \frac{3}{2}. \qquad \frac{x}{2} = s$$

$$tg\, s \geq \frac{3}{2}.$$



$$\arctg\frac{3}{2} + k\pi \le s < \frac{\pi}{2} + k\pi$$
$$s = \frac{x}{2}$$

$$2\arctg\frac{3}{2} + 2k\pi \le x \le \pi + 2k\pi$$

controllare che sia lo stesso numero di prima

<u>OSS</u> abbiamo artificialmente escluso $x = \pi + 2k\pi$ che sono soluzioni!

e quindi vanno riaggiunte!