$$\boxed{\text{Risolvere} \quad \sin x - \sqrt{3}\cos x > \sqrt{3}.}$$

1° modo Ricordiamo che $\cos x$ e $\sin x$ sono risp. ascissa e ordinata dei punti sulla circonf. goniometrica

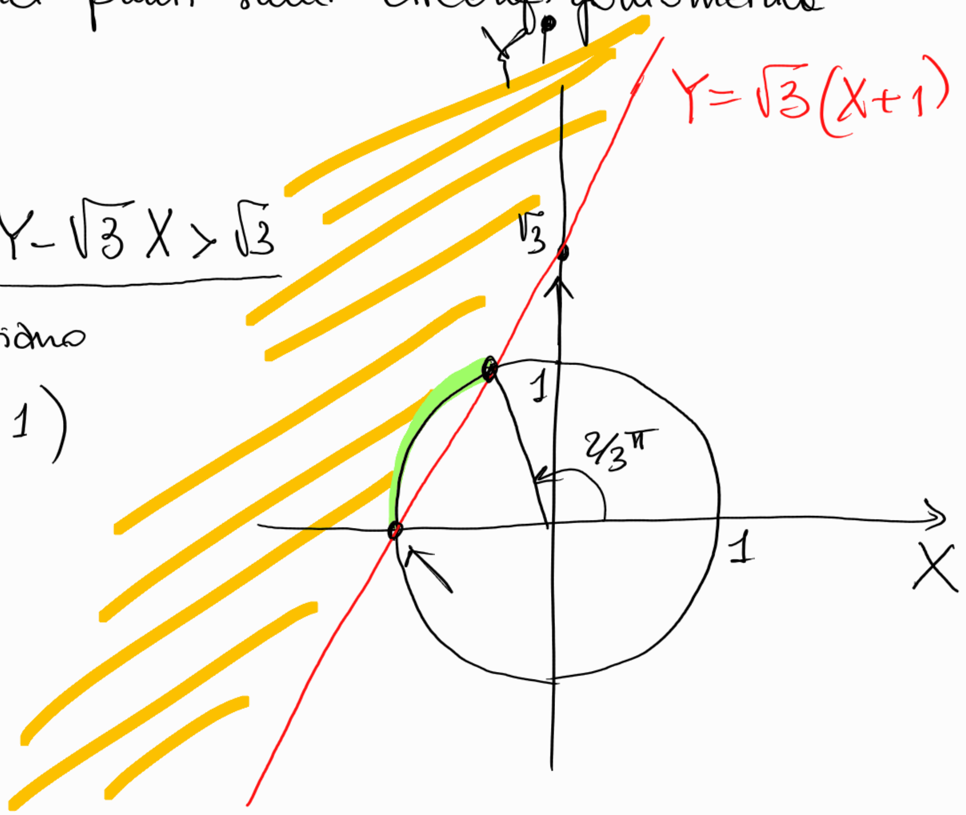
$\cos x = X$

$\sin x = Y$

La diseq^ne^ diventa $Y - \sqrt{3}X > \sqrt{3}$

che descrive il semi-piano

$$\begin{cases} Y > \sqrt{3}(X+1) \\ X^2 + Y^2 = 1 \end{cases}$$



Trovo le intersezioni fra la retta e la circonferenza

$$\begin{cases} \boxed{Y = \sqrt{3}(X+1)} \\ X^2 + Y^2 = 1 \end{cases}$$

$X^2 + 3(X+1)^2 = 1$

$X^2 + 3X^2 + 6X + 3 = 1$

$4X^2 + 6X + 2 = 0$

$2X^2 + 3X + 1 = 0$

$$X = \frac{-3 \pm \sqrt{9-8}}{4} = \frac{-3 \pm 1}{4} = \begin{cases} -1 \\ -\frac{1}{2} \end{cases}$$

$X = 1 \Rightarrow Y = 0 \qquad (1,0)$

$X = -\frac{1}{2} \Rightarrow Y = \frac{\sqrt{3}}{2} \qquad \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

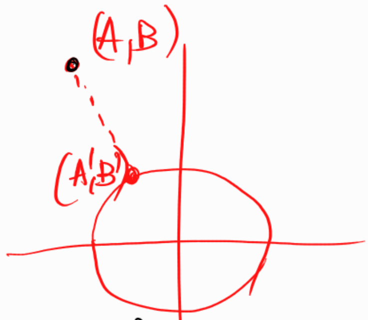
Soluzioni della diseq^ne^

$$\frac{2}{3}\pi + 2k\pi < x < \pi + 2k\pi \qquad k \in \mathbb{Z}$$

2° modo

$\sin x - \sqrt{3}\cos x > \sqrt{3}.$

$f(x) = A \sin x + B \cos x =$



$$\sin(x+\varphi) = \sin x \underbrace{\cos\varphi}_{A} + \cos x \underbrace{\sin\varphi}_{B}.$$

In generale questo non si può fare. per A,B generici

$$f(x) = \sqrt{A^2+B^2}\left(\frac{A}{\sqrt{A^2+B^2}}\sin x + \frac{B}{\sqrt{A^2+B^2}}\cos x\right)$$

$\frac{A}{\sqrt{A^2+B^2}} = \cos\varphi = A'$, $\frac{B}{\sqrt{A^2+B^2}} = \sin\varphi = B'$

ora $(A')^2 + (B')^2 = \frac{A^2}{A^2+B^2} + \frac{B^2}{A^2+B^2} = 1$

Cerco φ t.c.

$$\boxed{\begin{cases} \frac{A}{\sqrt{A^2+B^2}} = \cos\varphi \\ \frac{B}{\sqrt{A^2+B^2}} = \sin\varphi \end{cases}}$$

Individua φ a meno di multipli di 2π.

e quindi $f(x) = A \sin x + B \cos x =$

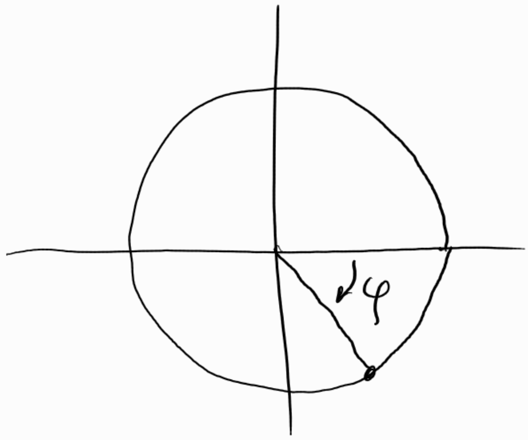
$$= \sqrt{A^2+B^2}\,(\underbrace{\cos\varphi \sin x + \sin\varphi \cos x}_{\sin(x+\varphi)}) =$$

$$= \underbrace{\sqrt{A^2+B^2}}_{\text{ampiezza}} \sin(\underbrace{x+\varphi}_{\text{fase}})$$

$$\underset{A}{1} \sin x \underbrace{-\sqrt{3}}_{B} \cos x = 2\sin(x+\varphi) = 2\sin\left(x - \frac{\pi}{3}\right)$$

dove φ è soluzione di

$$\begin{cases} \cos\varphi = \dfrac{A}{\sqrt{A^2+B^2}} = \dfrac{1}{2} \\ \sin\varphi = \dfrac{B}{\sqrt{A^2+B^2}} = -\dfrac{\sqrt{3}}{2} \end{cases}$$



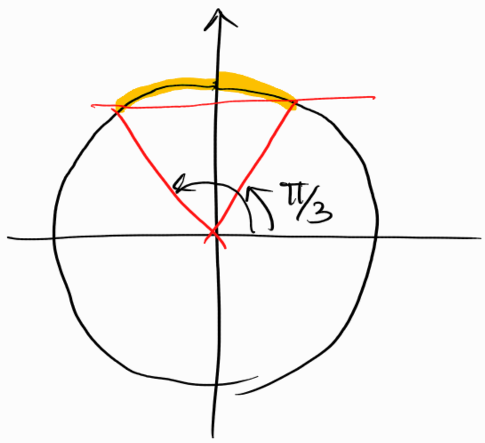
$$\varphi = -\pi/3$$

la diseq^{ne} diventa

$$\boxed{2\sin\left(x-\frac{\pi}{3}\right) > \sqrt{3}}$$

$$\sin\underbrace{\left(x-\frac{\pi}{3}\right)}_{t} > \frac{\sqrt{3}}{2}$$

$\sin t > \dfrac{\sqrt{3}}{2}$



$$\frac{\pi}{3} + 2k\pi < \underset{\substack{\shortparallel \\ x-\frac{\pi}{3}}}{t} < \frac{2}{3}\pi + 2k\pi$$

$$\boxed{\frac{2}{3}\pi + 2k\pi < x < \pi + 2k\pi}$$

3° modo. $t = tg\,\dfrac{x}{2}$. devo imporre $\dfrac{x}{2} \neq \dfrac{\pi}{2} + k\pi$

$$\boxed{x \neq \pi + 2k\pi}$$

$$\cos x = \frac{1-t^2}{1+t^2} \qquad \sin x = \frac{2t}{1+t^2}$$

$\sin x - \sqrt{3}\cos x > \sqrt{3}$ diventa nella nuova variabile t

$$\frac{2t}{\underset{\substack{\vee \\ 0}}{1+t^2}} - \frac{\sqrt{3}\,(1-t^2)}{\underset{\substack{\vee \\ 0}}{1+t^2}} > \sqrt{3},$$

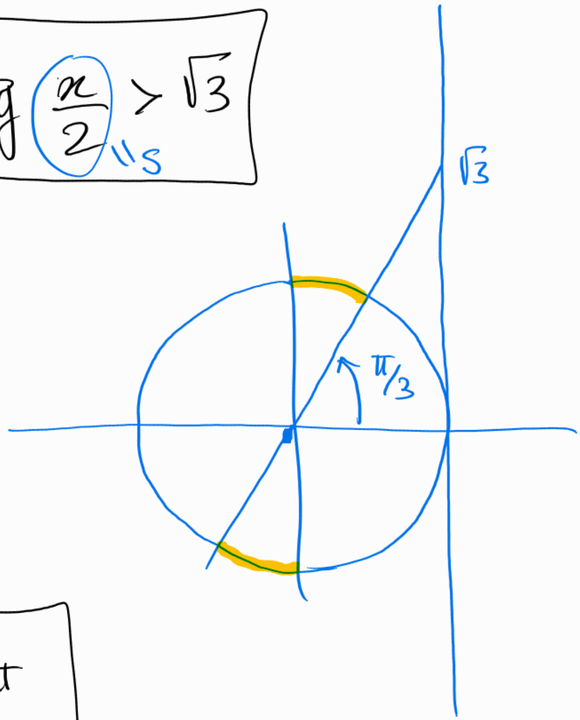
quindi

$$2t - \sqrt{3}\left(1-\cancel{t^2}\right) > \sqrt{3}\left(1+\cancel{t^2}\right)$$

$$2t - 2\sqrt{3} > 0$$

$$t > \sqrt{3}$$

$$\boxed{tg\left(\underbrace{\frac{x}{2}}_{s}\right) > \sqrt{3}}$$



$$tg\, s > \sqrt{3}$$

$$\Updownarrow$$

$$\frac{\pi}{3} + k\pi < \underbrace{s}_{x/2} < \frac{\pi}{2} + k\pi$$

$$\boxed{\frac{2\pi}{3} + 2k\pi < x < \pi + 2k\pi}$$

Cosa succede se i coeff^{ti} sono diversi

$$\sin x - \underbrace{\cancel{\sqrt{3}}}_{\frac{3}{2}} \cos x > \underbrace{\cancel{\sqrt{3}}}_{\frac{3}{2}}$$

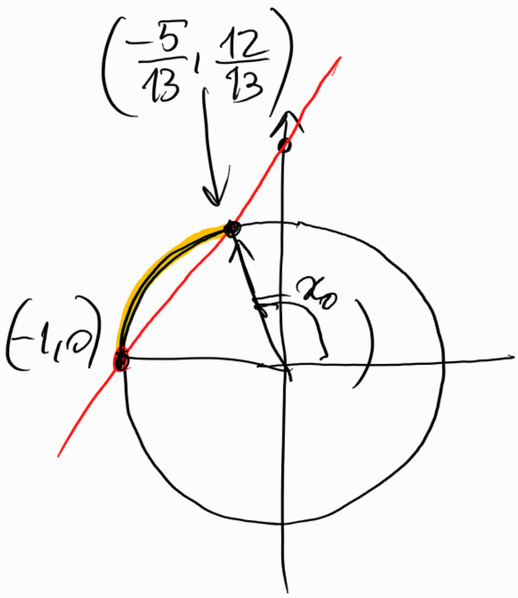
$$\boxed{2\sin x - 3\cos x > 3}$$

1° modo

$$\sin x = Y$$
$$\cos x = X$$

$$2Y - 3X > 3$$

$$\begin{cases} Y \geq \frac{3}{2}(X+1) \\ X^2 + Y^2 = 1 \end{cases}$$



$$X^2 + \frac{9}{4}(X+1)^2 = 1$$

$$4X^2+9(X+1)^2=4 \qquad Y=\frac{3}{2}(X+1)$$

$$4X^2+9X^2+18X+9=4$$

$$13X^2+18X+5=0$$

$$X=\frac{-9\pm\sqrt{81-65}}{13}=\frac{-9\pm 4}{13}=\begin{cases}-1\\ -\frac{5}{13}\end{cases}$$

$$X=-1\,,\ Y=0$$

$$X=-\frac{5}{13}\,,\ Y=\frac{3}{2}\,\frac{8}{13}=\frac{12}{13}$$

$$x_0+2k\pi<x<\pi+2k\pi\,.$$

Resta da vedere chi è x_0