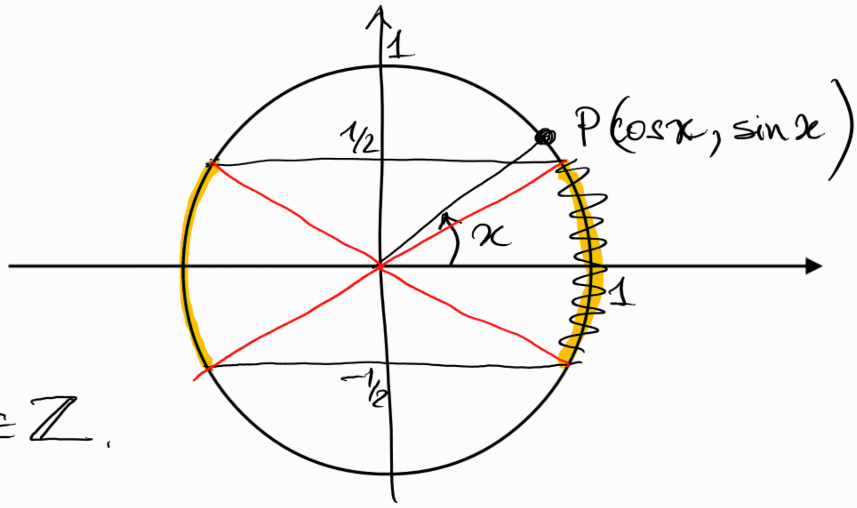
$$\sin^2 x \le \frac{1}{4}$$

$$-\frac{1}{2} \le \sin x \le \frac{1}{2}$$

$$-\frac{\pi}{6} + k\pi \le x \le \frac{\pi}{6} + k\pi$$

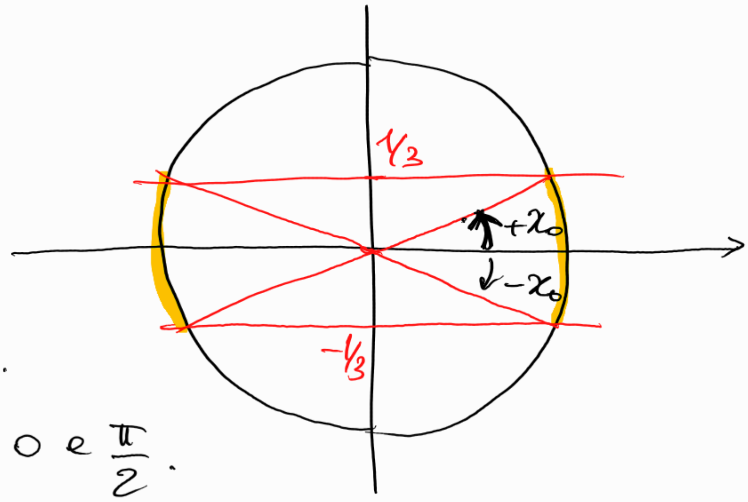
$k \in \mathbb{Z}$.



$$\sin^2 x \le \frac{1}{9}$$

$$-\frac{1}{3} \le \sin x \le \frac{1}{3}$$

$$-x_0 + k\pi \le x \le x_0 + k\pi$$

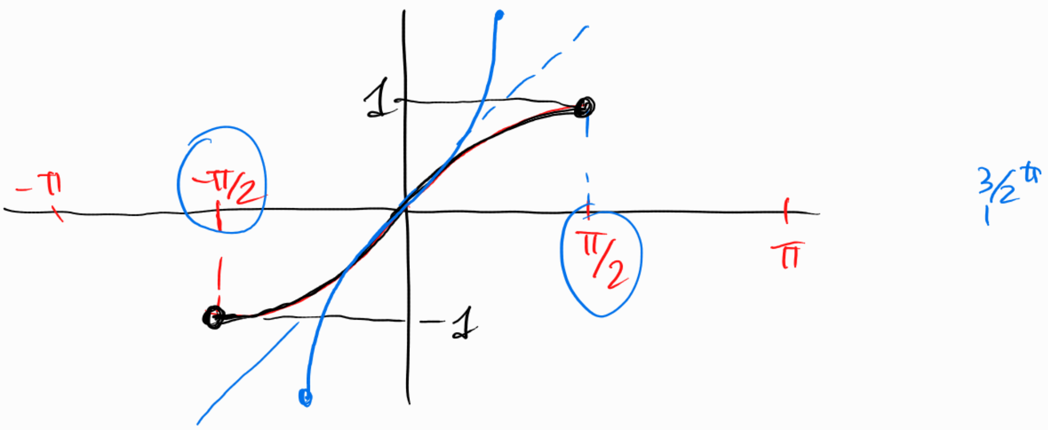


dove x_0 è l'angolo compreso tra 0 e $\frac{\pi}{2}$.
t.c. $\sin x_0 = 1/3$

Funzioni trigonometriche inverse

$\sin x : \mathbb{R} \longrightarrow \mathbb{R}$ non è iniettiva né suriettiva.

Per renderla suriettiva, rimpiazzo l'insieme di arrivo con $[-1,1]$



$$\sin x \Big|_{\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]} : \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \longrightarrow [-1,1].$$

è iniettiva? sì, perché è strett. crescente.

è suriettiva? sì, perché è continua e quindi (Teorema) assume tutti i valori compresi tra -1 e 1.

è biiettiva $\Leftrightarrow$

$$\forall y \in [-1,1] \quad \exists!\, x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ t.c. } \sin x = y.$$

Resta definita quindi l'inversa f^{-1} di $f(x) = \sin x \Big|_{[-\pi/2, \frac{\pi}{2}]}$

$$f^{-1}(y) = \arcsin y : [-1,1] \longrightarrow \left[-\frac{\pi}{2}, \pi/2\right]$$

$$y \longmapsto \text{l'unico } x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right] \text{ t.c. } \sin x = y.$$

se $x \in \left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, $y \in [-1,1]$.

$$\sin x = y \iff x = \arcsin y.$$

Per es.

$$\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6}$$

$$\cancel{\arcsin\left(-\frac{1}{2}\right) = \frac{7}{6}\pi}$$

$$\cancel{\arcsin\left(-\frac{1}{2}\right) = -\frac{\pi}{6} + 2k\pi}$$

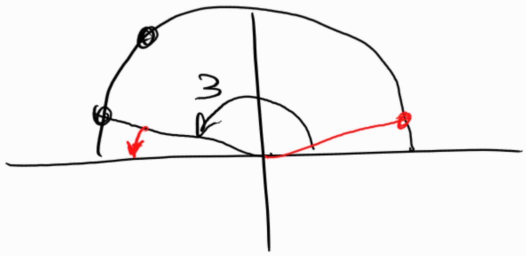
$$\arcsin 0 = 0$$

$$\arcsin \frac{\sqrt{3}}{2} = \frac{\pi}{3}$$

$$\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\frac{\pi}{4}$$

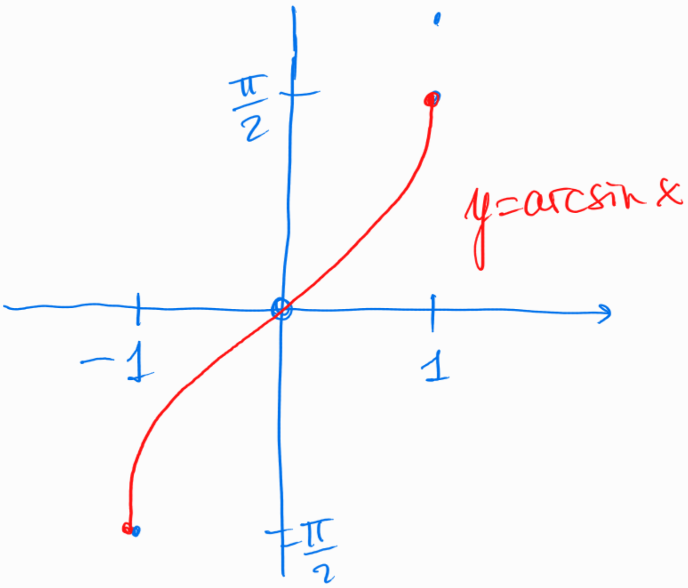
$$\arcsin(-1) = -\frac{\pi}{2}$$

$$\arcsin\left(\sin\frac{1}{4}\right) = \frac{1}{4}$$



$$\arcsin(\sin 3) = \pi - 3.$$

$$\arcsin\left(\sin\frac{3\pi}{4}\right) = \arcsin\left(\frac{\sqrt{2}}{2}\right) = \frac{\pi}{4}$$



$y = \arcsin x$

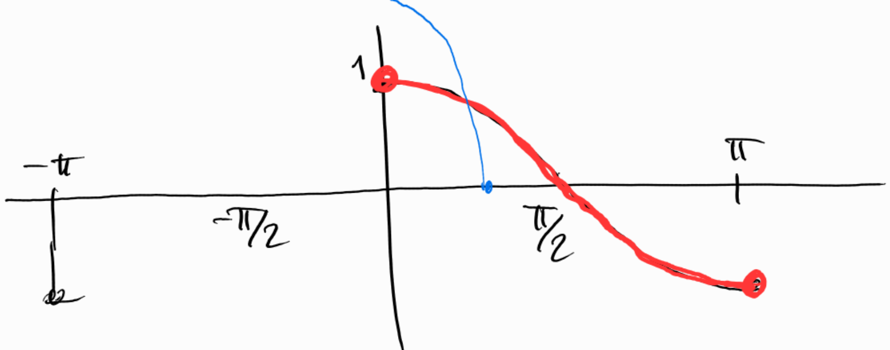
$$\sin^2 x \leq \frac{1}{9} \iff -\frac{1}{3} \leq \sin x \leq \frac{1}{3}$$

$$\Updownarrow$$

$$-\arcsin\frac{1}{3} + k\pi \leq x \leq \arcsin\frac{1}{3} + k\pi \quad (k \in \mathbb{Z}).$$

$f(x) = \cos x$ non è iniettiva né suriettiva.

$f(x) = \cos x\big|_{[0,\pi]} : [0,\pi] \longrightarrow [-1,1]$ è iniettiva
è suriettiva

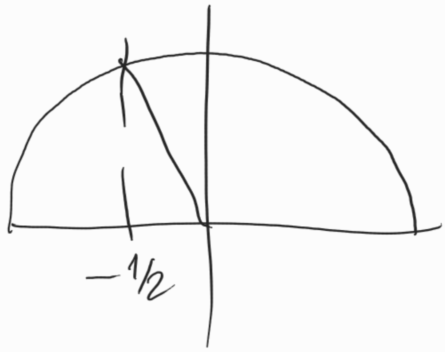


$\arccos y = f^{-1}(y) : [-1,1] \to [0,\pi]$

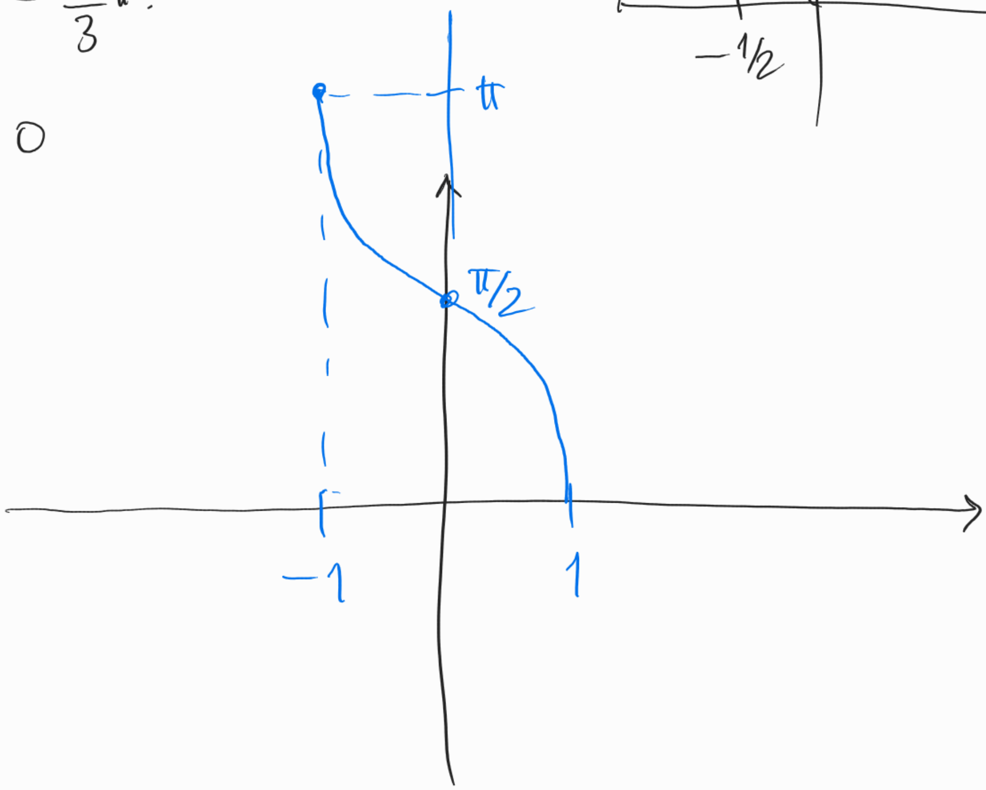
$y \longmapsto$ l'unico $x \in [0,\pi]$ t.c. $\cos x = y$.

$\arccos 0 = \frac{\pi}{2}$

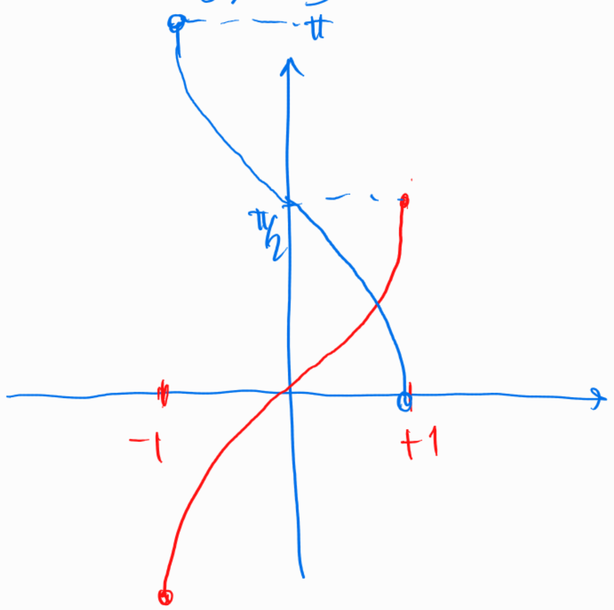
$\arccos\left(-\frac{1}{2}\right) = \frac{2}{3}\pi$.



$\arccos 1 = 0$



$\arccos\left(\cos \frac{\pi}{3}\right) = \frac{\pi}{3}$ $\qquad$ $\arccos\left(\cos \frac{7}{6}\pi\right) = \frac{5}{6}\pi$



$y = \arcsin x$

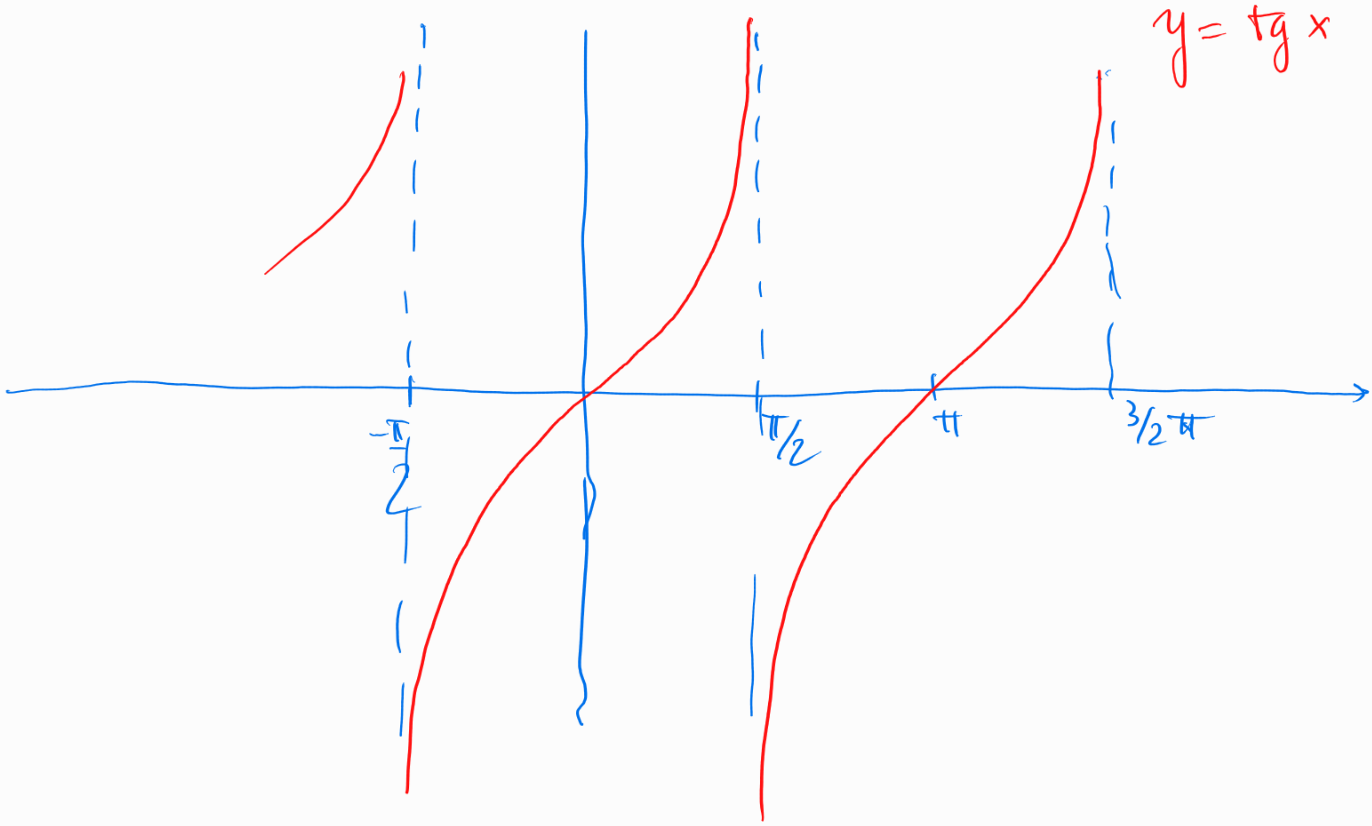
$y = \arccos x$

$\arccos x = \frac{\pi}{2} - \arcsin x$

$\forall x \in [-1,1]$.

$f(x) = \operatorname{tg} x = \frac{\sin x}{\cos x} : \mathbb{R} \setminus \{x = \frac{\pi}{2} + k\pi, k \in \mathbb{Z}\} \longrightarrow \mathbb{R}$.

non è iniettiva.



$$f(x) = \mathrm{tg}\big|_{\left(-\frac{\pi}{2},\frac{\pi}{2}\right)}(x) : \left(-\frac{\pi}{2},\frac{\pi}{2}\right) \to \mathbb{R}.$$
<u>biettiva.</u>

$$f^{-1}(y) = \mathrm{arctg}(y) : \mathbb{R} \to \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$$

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$y \longmapsto$ l'unico $x \in \left(-\frac{\pi}{2},\frac{\pi}{2}\right)$ t.c. $\mathrm{tg}\, x = y$.

$\mathrm{arctg}\, 1 = \frac{\pi}{4}$ $\qquad$ $\mathrm{arctg}(-\sqrt{3}) = -\frac{\pi}{3}$

$\qquad\qquad\qquad\qquad = -\mathrm{arctg}(\sqrt{3})$