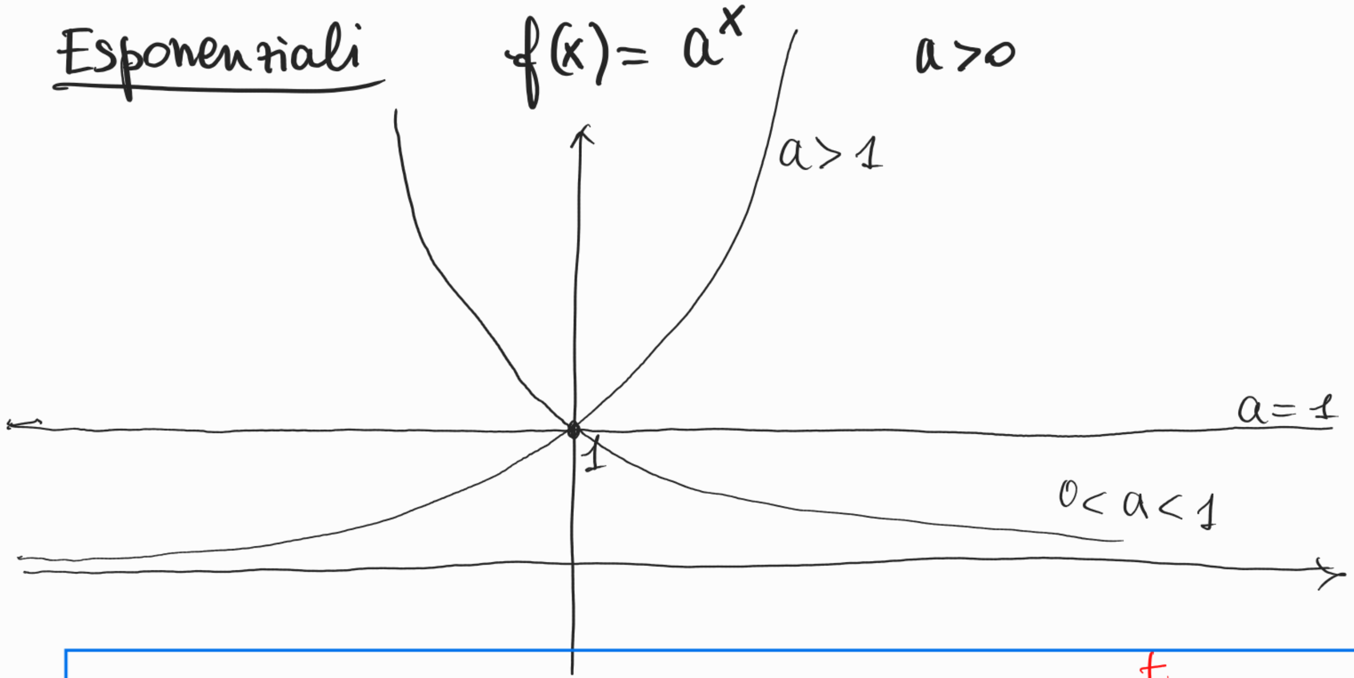
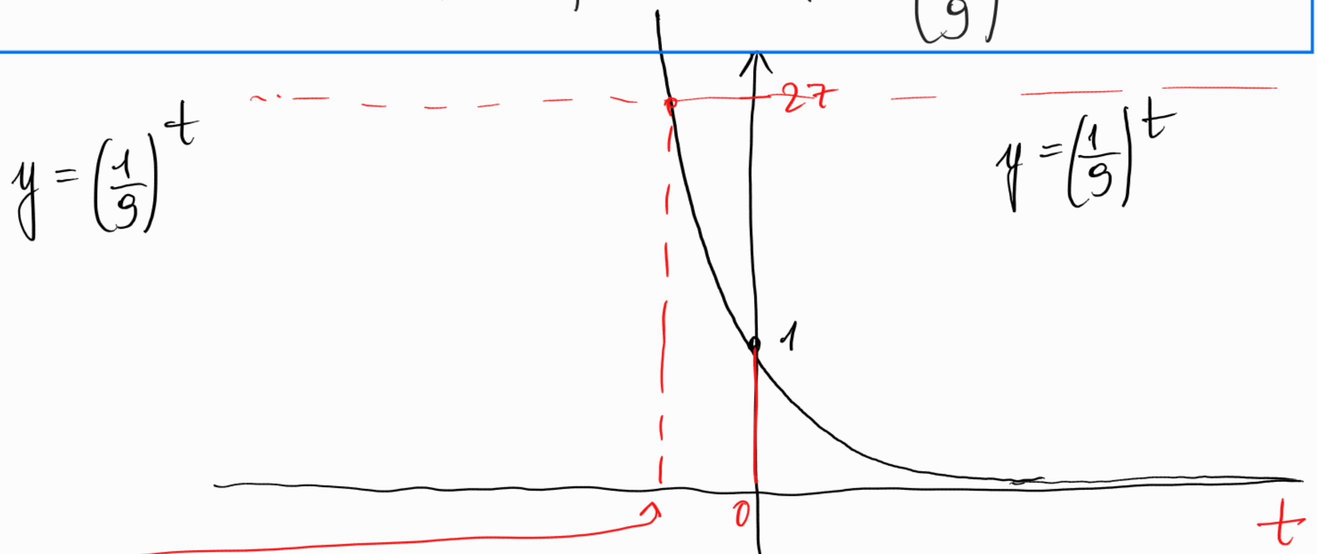
Esponenziali

$f(x) = a^x \qquad a > 0$



Risolvere la (doppia) diseq^{ne} $\quad 1 < \left(\frac{1}{9}\right)^{\overset{t}{2x-3}} < 27.$



Devo trovare t. t.c. $\left(\frac{1}{9}\right)^t = 27$. Basta prendere $t = -\frac{3}{2}$.

$$\left(\frac{1}{9}\right)^{-3/2} = 9^{3/2} = 27.$$

La diseq. $\quad 1 < \left(\frac{1}{9}\right)^t < 27 \quad$ corrisponde a $\quad \boxed{-\frac{3}{2} < \underset{2x-3}{\underset{\shortparallel}{t}} < 0}$

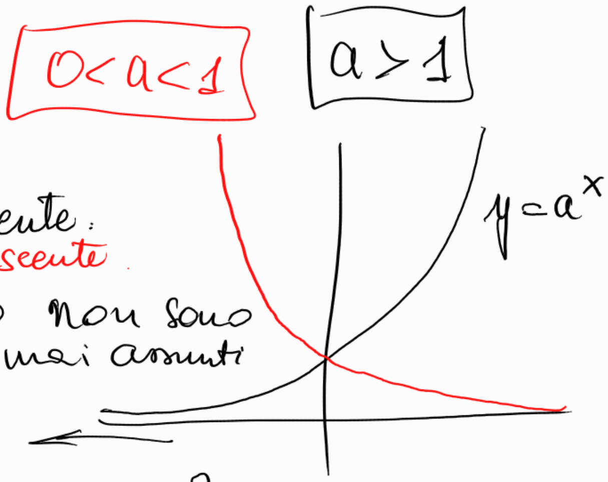
$$-\frac{3}{2} < 2x-3 < 0 \iff \frac{3}{2} < 2x < 3 \iff \boxed{\frac{3}{4} < x < \frac{3}{2}}$$

E se avessi avuto valori diversi?

$$\frac{1}{2} < \left(\frac{1}{9}\right)^t < 28 .$$

Occorre utilizzare i logaritmi (inverse degli esponenziali)

$$f(x) = a^x : \mathbb{R} \to \mathbb{R}$$
$$x \longmapsto a^x$$



Iniettiva? Sì perché strettamente crescente. decrescente

Suriettiva? NO perché i valori $y \le 0$ non sono mai assunti

$$a^x : \mathbb{R} \to (0,+\infty) \quad \text{è suriettiva?}$$
$$x \longmapsto a^x$$

Devo provare che $\forall y > 0 \ \exists x \in \mathbb{R}$ t.c. $f(x) = a^x = y$.

f è continua in un intervallo ($\mathbb{R}$).

$\Rightarrow f$ assume tutti i valori compresi tra $\underset{0}{\underset{\parallel}{\inf f}}$ e $\underset{+\infty}{\underset{\parallel}{\sup f}}$

$a^x : \mathbb{R} \to (0,+\infty)$ è biiettiva.

$\Rightarrow \exists$ l'inversa $f^{-1}(y) : (0,+\infty) \to \mathbb{R}$.

$y \longmapsto f^{-1}(y) = \log_a y =$ l'unico $x \in \mathbb{R}$ t.c. $a^x = y$

se $y \in (0,+\infty)$ $x \in \mathbb{R}$.

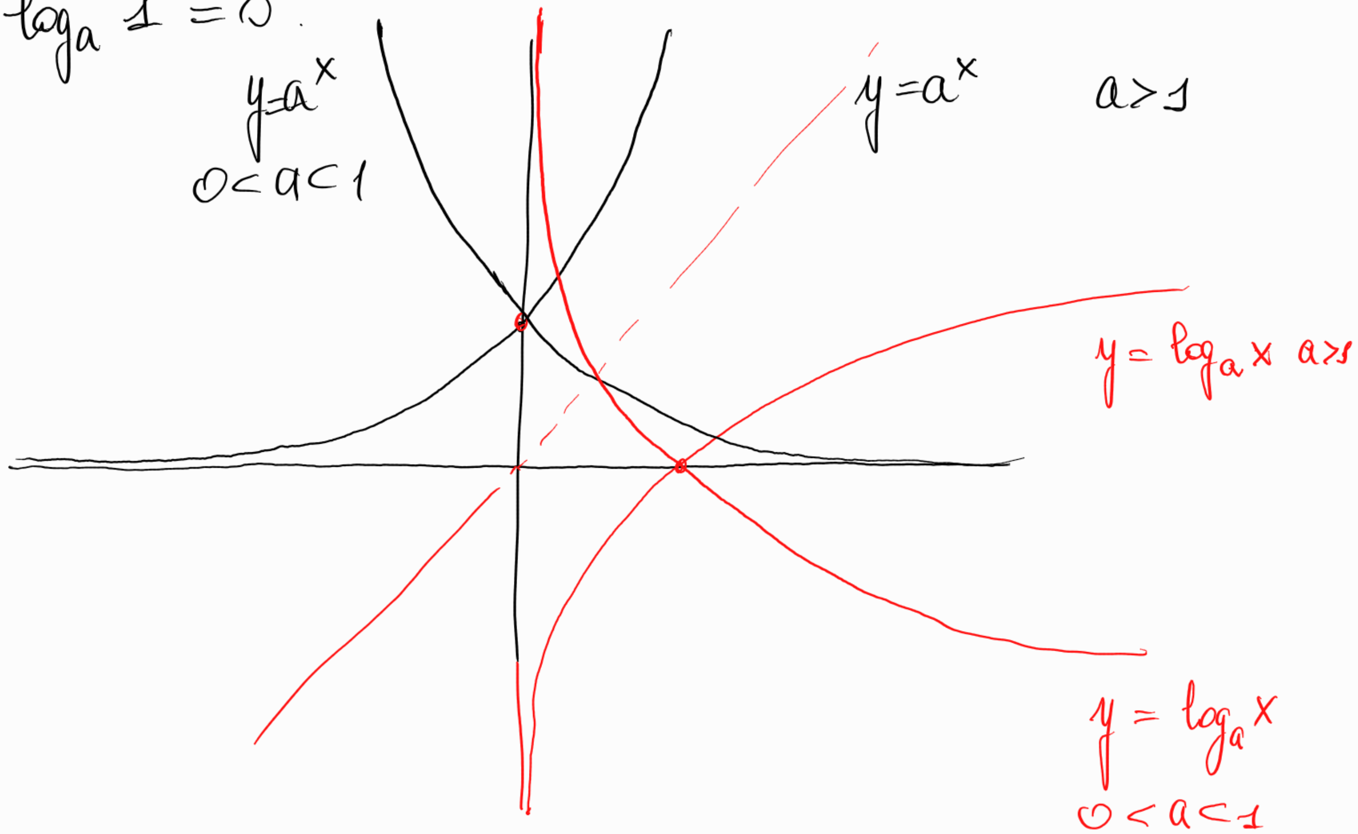
$$f(x) = y \iff x = f^{-1}(y)$$
$$a^x = y \iff x = \log_a y$$

$\log_a(x) : (0,+\infty) \to \mathbb{R}$ $\quad a>0 \ , \ a \neq 1.$

$\log_a(x) =$ l'unico $y \in \mathbb{R}$ t.c. $a^y = x.$

$\log_2 8 = 3 \qquad \log_{1/2} 8 = -3$

$\log_a 1 = 0.$



$$\boxed{a^{\log_a x} = x \quad \forall x>0} \qquad \boxed{\log_a a^x = x \quad \forall x \in \mathbb{R}}$$

$$\boxed{\log_a(xy) = \log_a x + \log_a y \qquad \boxed{\forall x,y>0.}}$$

$\Updownarrow$

$$a^{\log_a x + \log_a y} \overset{?}{=} xy$$

$$\underbrace{a^{\log_a x}}_{x} \cdot \underbrace{a^{\log_a y}}_{y}$$

se $a = e$ si parla di log naturale e si omette la base

$$\log_e x = \log x = \ln x$$

$$\underbrace{\log(x^2-x)}_{\text{dominio } = (-\infty,0)\cup(1,+\infty)} = \log(x(x-1)) = \underbrace{\log x + \log(x-1)}_{\text{dominio } = (1,+\infty)}$$

falso perché hanno domini diversi!

$$\log(x^2-x) = \begin{cases} \log x + \log(x-1) & \text{se } x>1 \\ \log(-x) + \log(1-x) & \text{se } x<0 \end{cases} =$$

$$\left[\, x^2 - x = x(x-1) = (-x)(1-x) \,\right]$$

$$= \underbrace{\log|x| + \log|x-1|}$$

Attenzione: vero se $(x<0) \vee (x>1)$

Attenzione, però: questo è definito anche in $(0,1)$

$$\boxed{\log_a\left(\frac{x}{y}\right) = \log_a x - \log_a y \qquad \forall x,y>0.}$$

dim

$$a^{\log_a x - \log_a y} \overset{?}{=} \frac{x}{y}$$

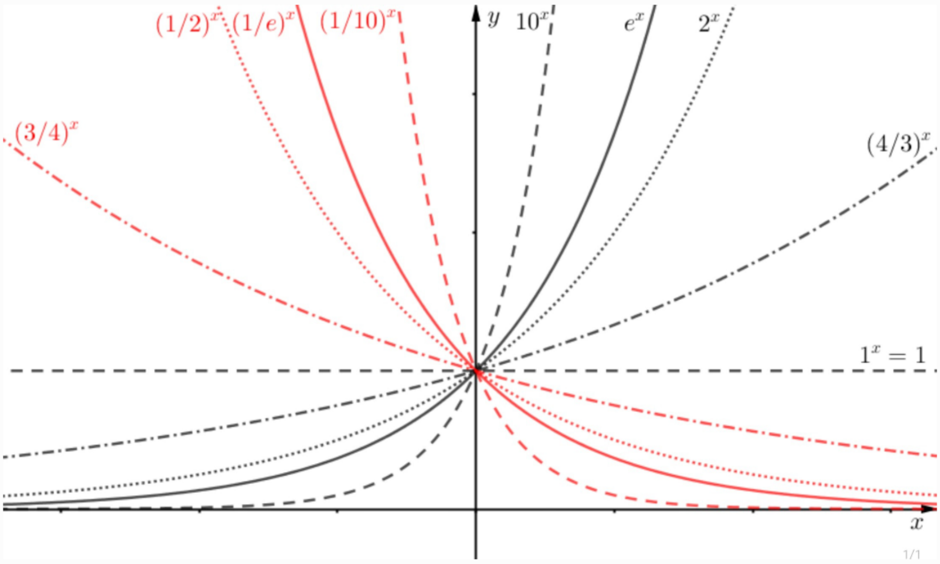
$$= \frac{\overbrace{a^{\log_a x}}^{x}}{\underbrace{a^{\log_a y}}_{y}}$$

$$\boxed{\log_a(x^r) = r\log_a x} \qquad \forall x>0 \quad \forall r\in\mathbb{R}.$$

devo provare $a^{r\log_a x} \overset{?}{=} x^r$

$$a^{r\log_a x} = \left(a^{\log_a x}\right)^r = x^r$$

Grafici degli esponenziali



Grafici dei logaritmi.