SAPIENZA
UNIVERSITÀ DI ROMA

Environmental Geophysics

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4. LFEM methods

Basic principles
Slingram method

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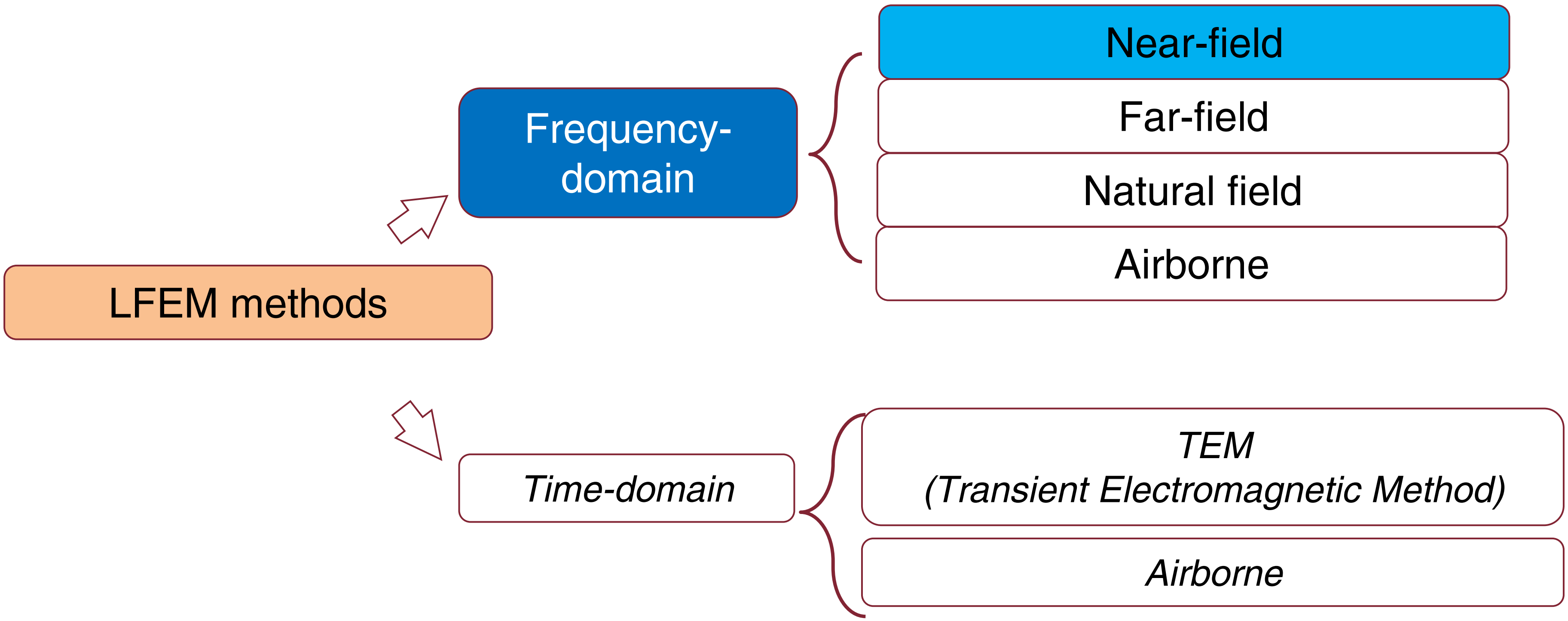
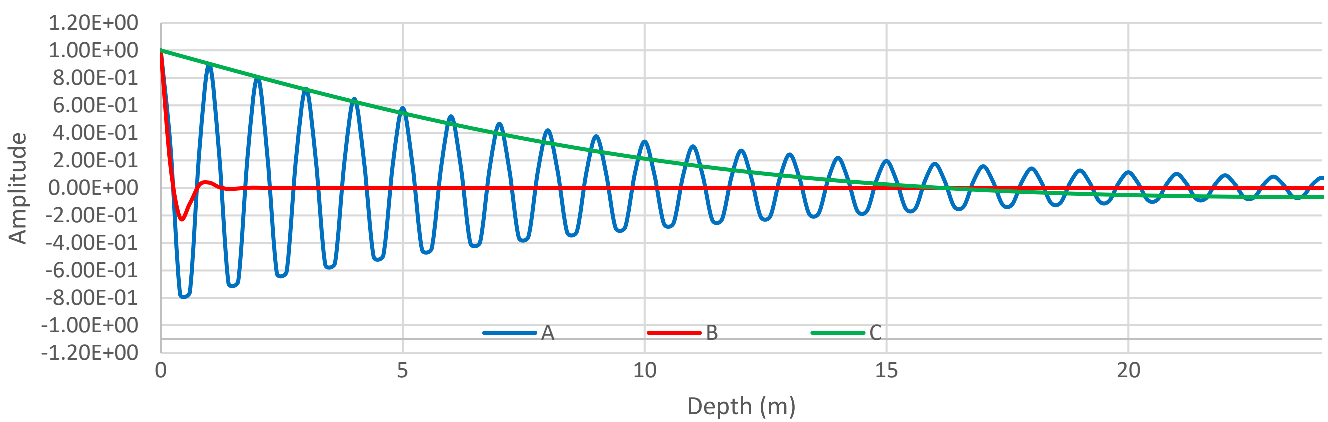
Low-frequency electromagnetic (LFEM) methods

Low-frequency electromagnetic (LFEM) methods are geophysical prospecting methods based on the EM induction theory (Faraday and Ampère's laws)



Comparing HFEM and LFEM

HFEM (GPR) $\boldsymbol{\omega \sim 10^7 - 3 \cdot 10^9 \mathrm{Hz}}$	A	Low σ	$\frac{\sigma}{\omega\varepsilon} \ll 1$	**poor attenuation** EM field oscillatory **Low losses** **Good DOI**	
	B	High σ	$\frac{\sigma}{\omega\varepsilon} \gg 1$	**strong attenuation** EM field diffusive **High losses** **Poor DOI**	**GPR is not effective**
LFEM $\boldsymbol{\omega \sim 10^3 - 10^5 \mathrm{Hz}}$	C	Regardless σ	$\frac{\sigma}{\omega\varepsilon} \gg 1$	EM field diffusive **Good DOI**	



LFEM method – EM induction principles

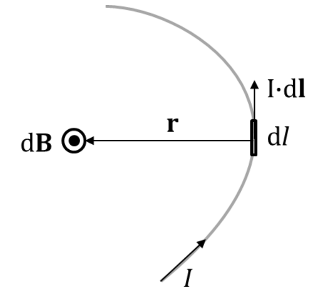
Ampère's law *neglecting polarization:* $\frac{\sigma}{\omega\varepsilon} \gg 1$ **DIFFUSION**

$$\nabla \times \mathbf{H} = \mathbf{J} + \cancel{\frac{\partial \mathbf{D}}{\partial t}}$$

For a circular coil having radius *r*:

$$B = \frac{\mu_0 I}{2r}$$

An **electric current *I* flowing** in a piece of wire will induce a **normally-directed magnetic field** (according to the Biot-Savart law). If the electric field is variable with time, also the magnetic field will be variable. Otherwise, it will be stationary.

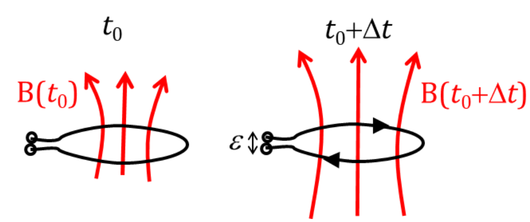


Faraday-Lenz's law

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

$$\varepsilon = -\frac{\partial \Phi_B}{\partial t}$$

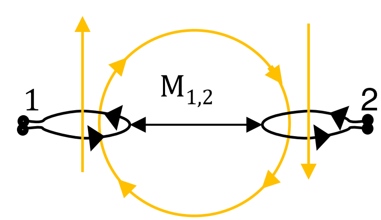
Only a temporal change in the magnetic field can induce an electric current on a coil thus exhibiting an **electromotive force (e.m.f. or *ε*)** at its ends



For two adjacent coils:

$$\varepsilon_2 = -M_{1,2}\frac{\partial I_1}{\partial t}$$

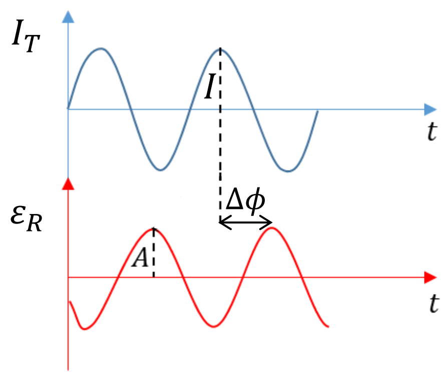
Mutual inductance
[Henry - H]=[$kg \cdot m^2/(s^2 \cdot A^2)$]



LFEM method – coil approximation

INPUT
AC current in the T coil

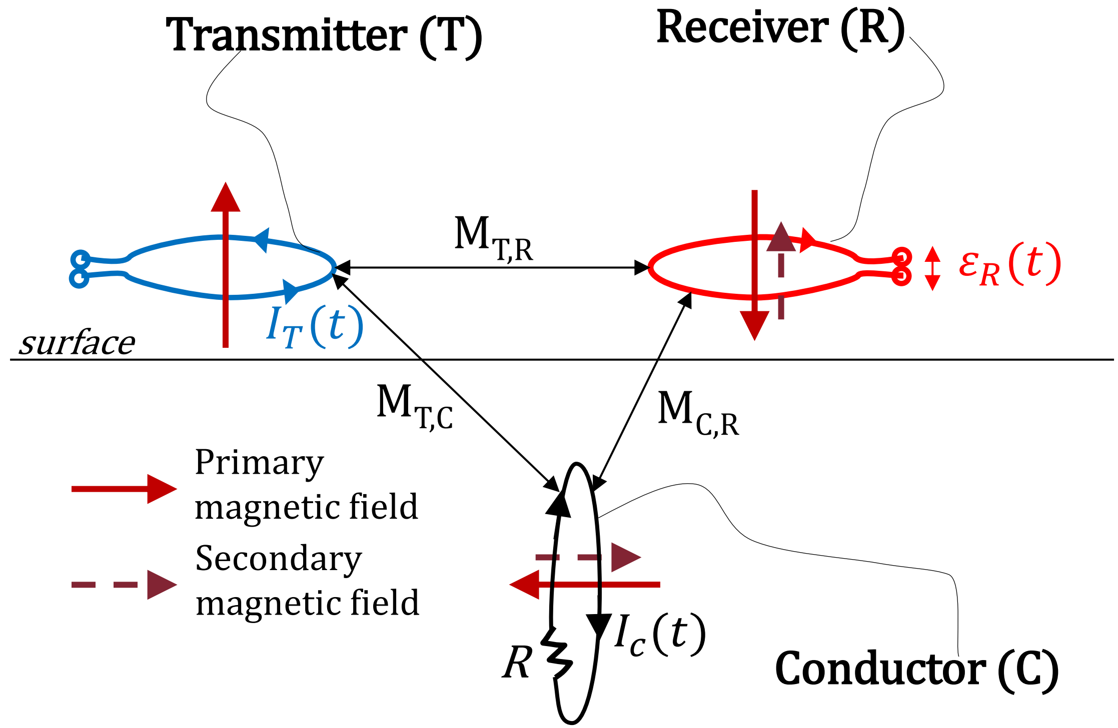
$$I_T(t) = I\sin(\omega t)$$



OUTPUT
Induced e.m.f. in the R coil (ε_R) due to the EM induction of both the **T** coil(ε_R^P) and the buried **conductor C** (ε_R^S)

$$\varepsilon_R(t) = \varepsilon_R^P(t) + \varepsilon_R^S(t)$$

$$\varepsilon_R(t) = A\cos(\omega t - \Delta\phi)$$

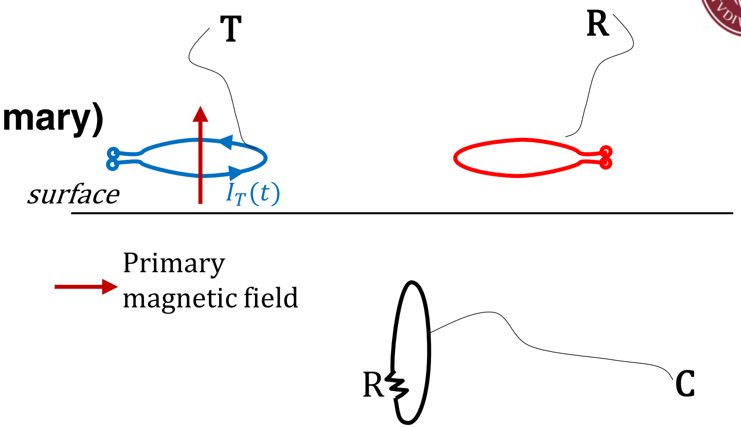


ASSUMPTION
The conductor can be approximated by a coil itself having a resistance R. We are negleting self-induction.

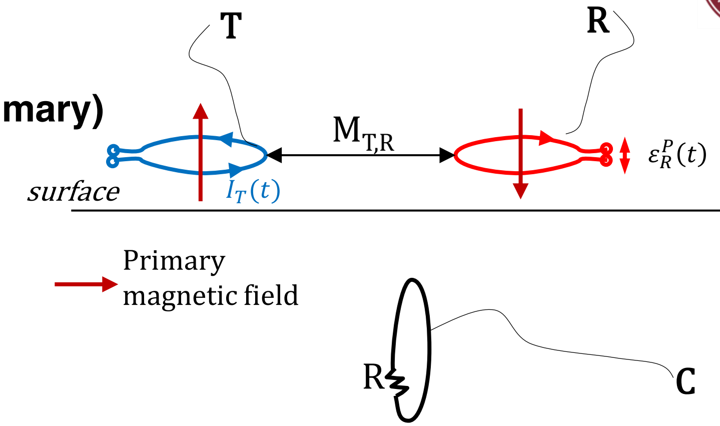
LFEM method – coil approximation

1. AC input current in the T coil→variable magnetic field (primary)

$$I_T(t) = I \sin \omega t$$



LFEM method – coil approximation



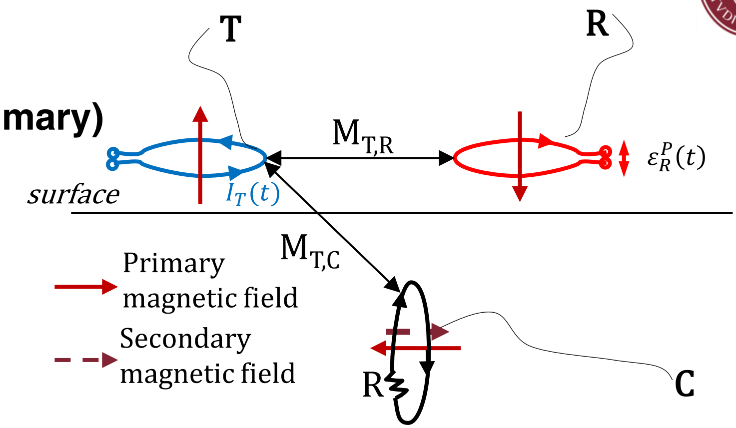
1. AC input current in the T coil→variable magnetic field (primary)

$$I_T(t) = I \sin \omega t$$

2. Induced e.m.f. in R due to the primary field

$$\varepsilon_R^P(t) = -M_{T,R} \frac{\partial I_T(t)}{\partial t} = \underbrace{-\omega M_{T,R} I}_{\left|\varepsilon_R^P\right|} \cos \omega t$$

LFEM method – coil approximation



1. AC input current in the T coil→variable magnetic field (primary)

$$I_T(t) = I \sin \omega t$$

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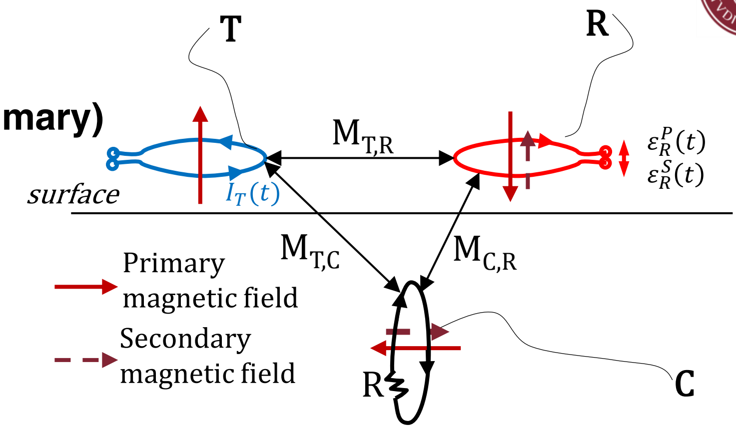
$$\varepsilon_R^P(t) = -M_{T,R}\frac{\partial I_T(t)}{\partial t} = \underbrace{-\omega M_{T,R} I}_{|\varepsilon_R^P|} \cos \omega t$$

3. AC eddy current in C due to the primary field → variable magnetic field (secondary)

$$\varepsilon_C^P(t) = RI_C(t) = -M_{T,C}\frac{\partial I_T(t)}{\partial t} = -\omega M_{T,C} I \cos \omega t \;\Rightarrow\; I_C(t) = -\omega \frac{M_{T,C}}{R} I \cos \omega t$$

LFEM method – coil approximation



1. AC input current in the T coil→variable magnetic field (primary)

$$I_T(t) = I \sin \omega t$$

2. Induced e.m.f. in R due to the primary field

$$\varepsilon_R^P(t) = -M_{T,R}\frac{\partial I_T(t)}{\partial t} = \underbrace{-\omega M_{T,R} I}_{|\varepsilon_R^P|} \cos \omega t$$

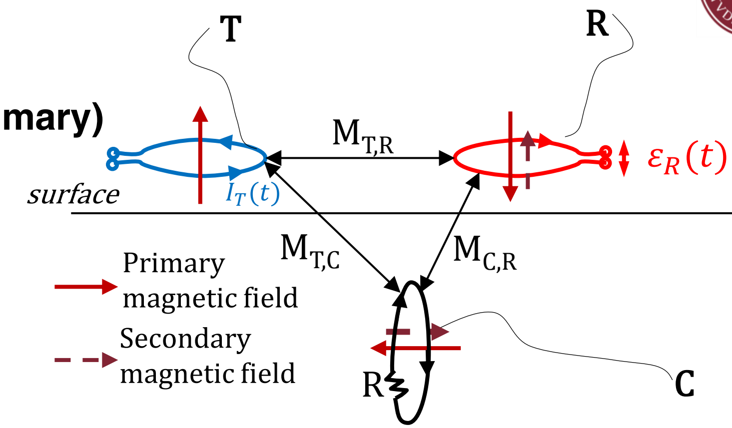
3. AC eddy current in C due to the primary field → variable magnetic field (secondary)

$$\varepsilon_C^P(t) = RI_C(t) = -M_{T,C}\frac{\partial I_T(t)}{\partial t} = -\omega M_{T,C} I \cos \omega t \quad \Rightarrow \quad I_C(t) = -\omega \frac{M_{T,C}}{R} I \cos \omega t$$

4. Induced e.m.f. in R due to the secondary field

$$\varepsilon_R^S(t) = -M_{C,R}\frac{\partial I_C(t)}{\partial t} = \underbrace{-\omega^2 \frac{M_{C,R} M_{T,C}}{R} I}_{|\varepsilon_R^S|} \sin(\omega t)$$

LFEM method – coil approximation



1. AC input current in the T coil→variable magnetic field (primary)

$$I_T(t) = I \sin \omega t$$

2. Induced e.m.f. in R due to the primary field

$$\varepsilon_R^P(t) = -M_{T,R}\frac{\partial I_T(t)}{\partial t} = \underbrace{-\omega M_{T,R} I}_{|\varepsilon_R^P|} \cos \omega t$$

3. AC eddy current in C due to the primary field → variable magnetic field (secondary)

$$\varepsilon_C^P(t) = R I_C(t) = -M_{T,C}\frac{\partial I_T(t)}{\partial t} = -\omega M_{T,C} I \cos \omega t \quad \Rightarrow \quad I_C(t) = -\omega \frac{M_{T,C}}{R} I \cos \omega t$$

4. Induced e.m.f. in R due to the secondary field

$$\varepsilon_R^S(t) = -M_{C,R}\frac{\partial I_C(t)}{\partial t} = \underbrace{-\omega^2 \frac{M_{C,R} M_{T,C}}{R} I}_{|\varepsilon_R^S|} \sin(\omega t)$$

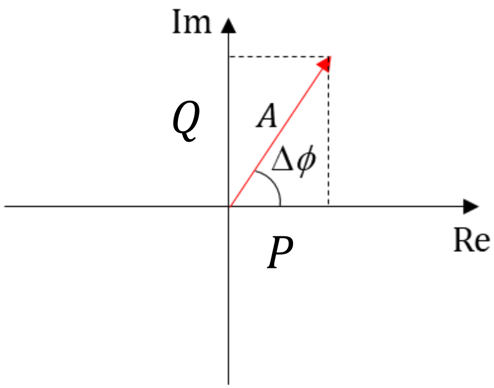
5. Observed e.m.f. (sum of the two contributions), scaled by the magnitude of known primary e.m.f.

$$\varepsilon_R(t) = \frac{\varepsilon_R^P(t) + \varepsilon_R^S(t)}{|\varepsilon_R^P|} = \cos(\omega t) + \frac{|\varepsilon_R^S|}{|\varepsilon_R^P|} \sin(\omega t) = A \cos(\omega t - \Delta\phi) \quad \begin{cases} A = \sqrt{1 + \left(\frac{|\varepsilon_R^S|}{|\varepsilon_R^P|}\right)^2} \\ \phi = \tan^{-1} \frac{|\varepsilon_R^S|}{|\varepsilon_R^P|} \end{cases}$$

OBSERVED E.M.F. - What we measure is amplitude and phase shift of the e.m.f. in the receiver (normalized by the primary e.m.f.)

$$\varepsilon_R^{OBS}(t) = \cos(\omega t) + \frac{|\varepsilon_R^S|}{|\varepsilon_R^P|}\sin(\omega t) = A\cos(\omega t - \Delta\phi) = \mathrm{A}\cos\Delta\phi\cos\omega t + \mathrm{A}\sin\Delta\phi\sin\omega t$$

$$\varepsilon_R^{OBS} = P + iQ$$

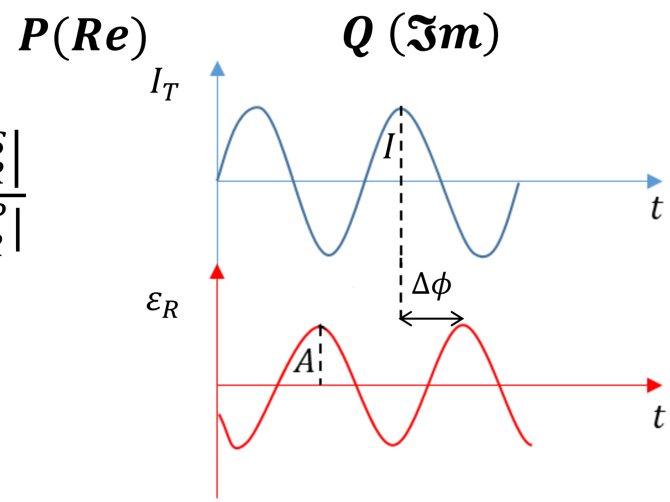


$\boldsymbol{P(Re)}$

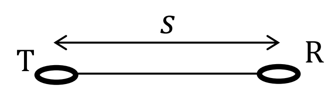
Phase component REAL

$$\boldsymbol{Q\ (\mathfrak{Im})} = \frac{|\varepsilon_R^S|}{|\varepsilon_R^P|}$$

Quadrature component (effect of the resistor) IMAGINARY



THEORETICAL E.M.F. – HOMOGENEOUS GROUND



For **homogeneous ground** having conductivity σ, we can derive the magnitude of the ratio $\frac{\varepsilon_S}{\varepsilon_P}$ directly from the Biot-Savart law for a fixed distance s between T and R coils:

$$|\varepsilon_R^{HOMOG}| = \frac{|\varepsilon^S|}{|\varepsilon^P|} = \frac{2}{\gamma^2 s^2}\left[9 - (9 + 9\gamma s + 4\gamma^2 s^2 + \gamma^3 s^3)\mathrm{e}^{-\gamma s}\right], \text{ where } \gamma^2 s^2 = i\sigma\mu_0\omega s^2$$

ASSUMPTION: low induction numbers *B*

Neglecting self-inductance of the ground

$$B = \Im m\left(\frac{\gamma^2 s^2}{2}\right) \ll 1$$

$$\frac{\omega\sigma\mu_0 s^2}{2} \ll 1 \rightarrow \omega \ll \frac{2}{\sigma\mu_0 s^2}$$

Example

$s = 1$ m, σ = 0.1 S/m

(ρ = 5-500 Ωm), $\mu_0 = 4\pi \cdot 10^{-7}$ H/m

$$\omega \ll \frac{2}{0.1 \cdot 4\pi \cdot 10^{-7} \cdot 1} \sim 8 - 800 \text{ MHz}$$

For a homogenous medium we get a simplified expression for low *B*:

$$\left|\varepsilon_R^{HOMOG}\right| \approx 1 + i\,\frac{\sigma\mu_{0\omega}s^2}{4}$$

⇒ **Measuring the quadrature component of the e.m.f.** ⇒

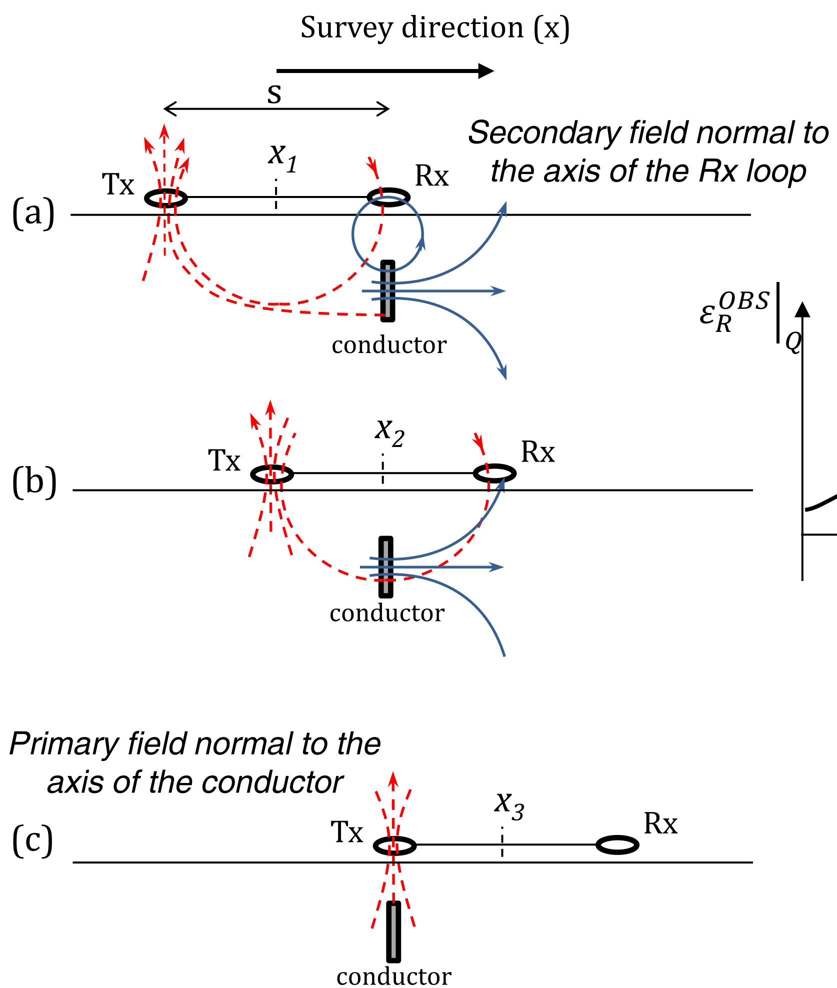
$$\sigma = \frac{4Q^{OBS}}{\mu_0\omega s^2}$$

For a real medium the expression for the ratio $\varepsilon_S/\varepsilon_P$ is much more complicated. Therefore the correct approach is to measure the quadrature component of the (scaled) e.m.f., thus yielding an **apparent conductivity value** σ_a^{OBS}

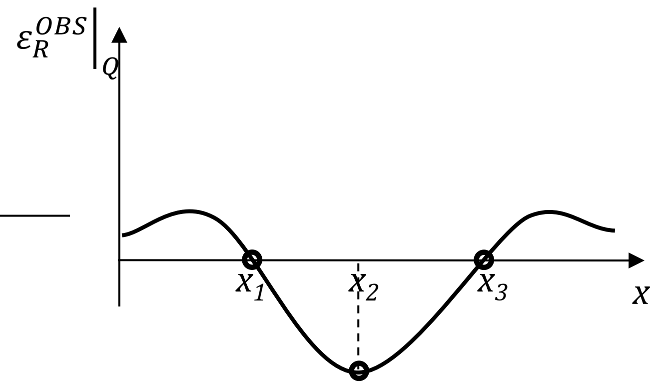
$$\sigma_a^{OBS} = \frac{4Q^{OBS}}{\mu_0\omega s^2}$$

LFEM Slingram method – observations

In the Slingram method T and R are located at a fixed distance s



$$\sigma_a^{OBS} = \frac{4Q^{OBS}}{\mu_0 \omega s^2}$$

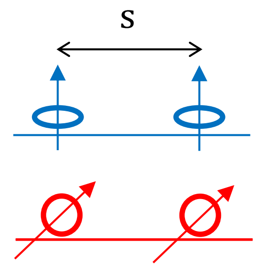


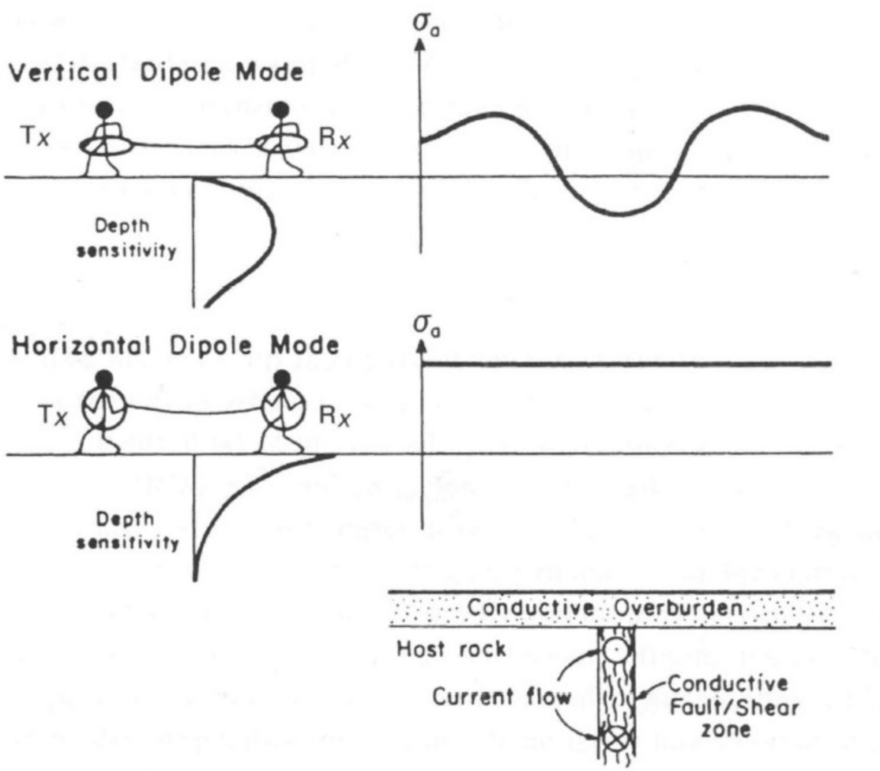
In this case we will have a negative apparent conductivity value where the anomaly is located

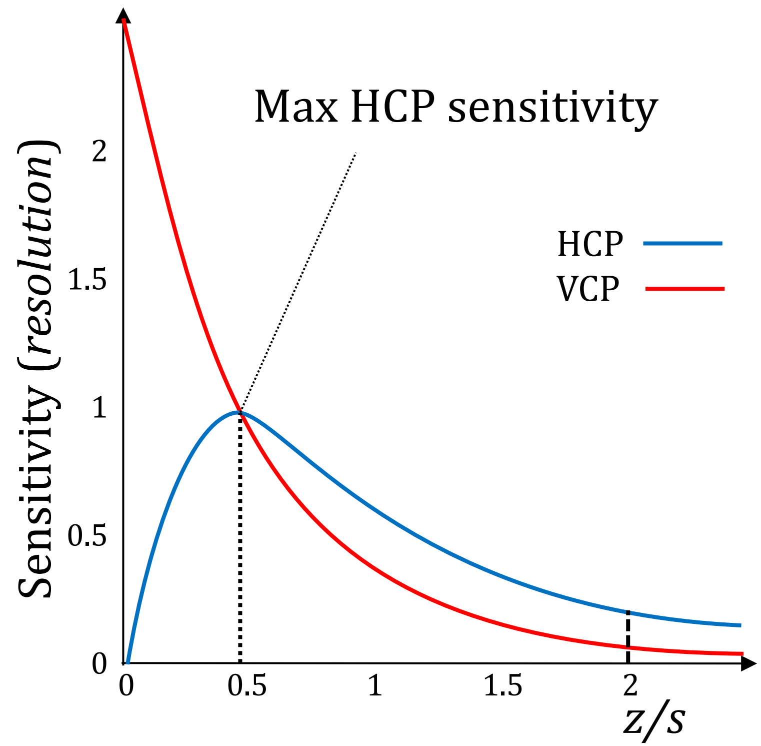
LFEM Slingram method – DOI

Acquisition mode

- **HCP**: Horizontal Co-Planar (vertical magnetic field)
- **VCP**: Vertical Co-Planar (horizontal magnetic field)



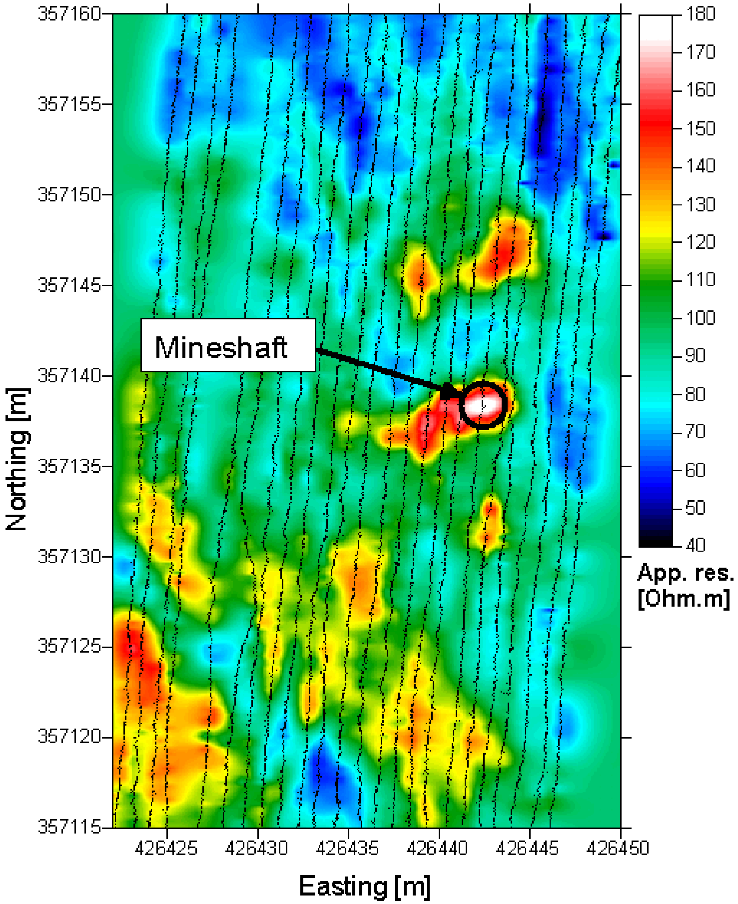




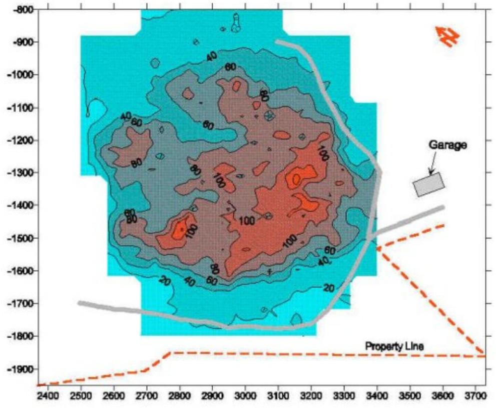
$$\mathrm{DOI}^{\mathrm{HCP}} \sim 2s$$

Apparent conductivity/resistivity map

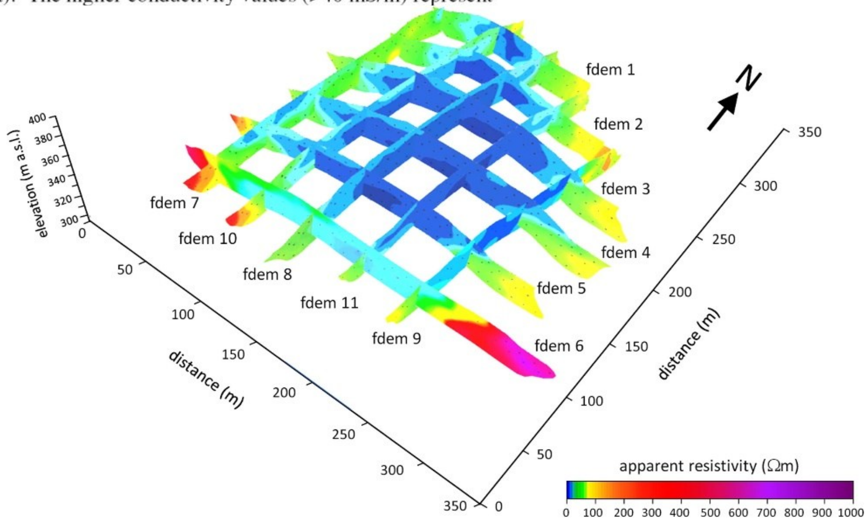
Mineshaft



Landfills

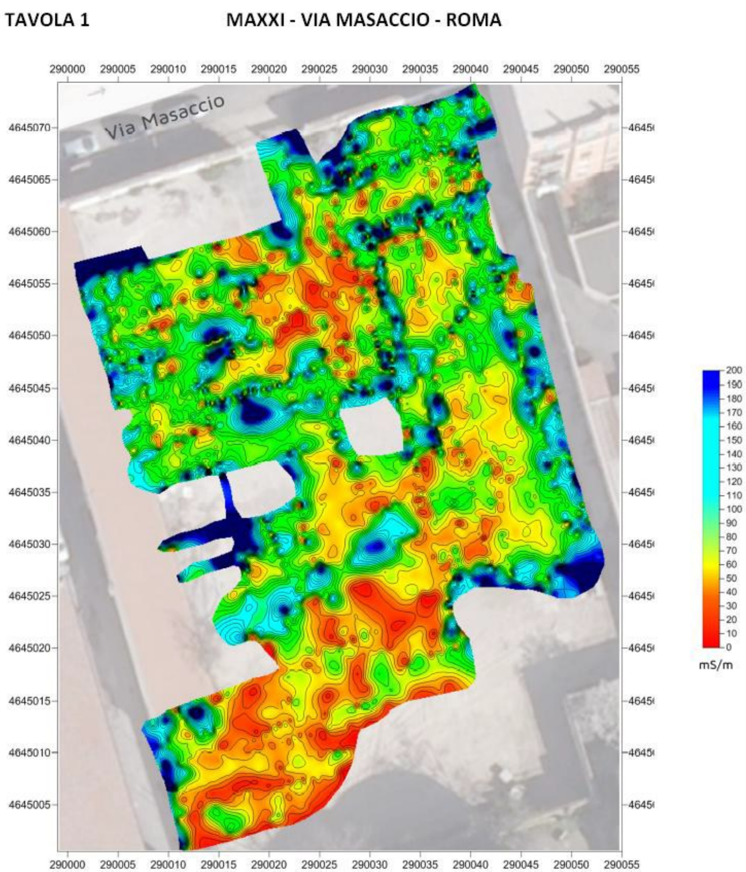


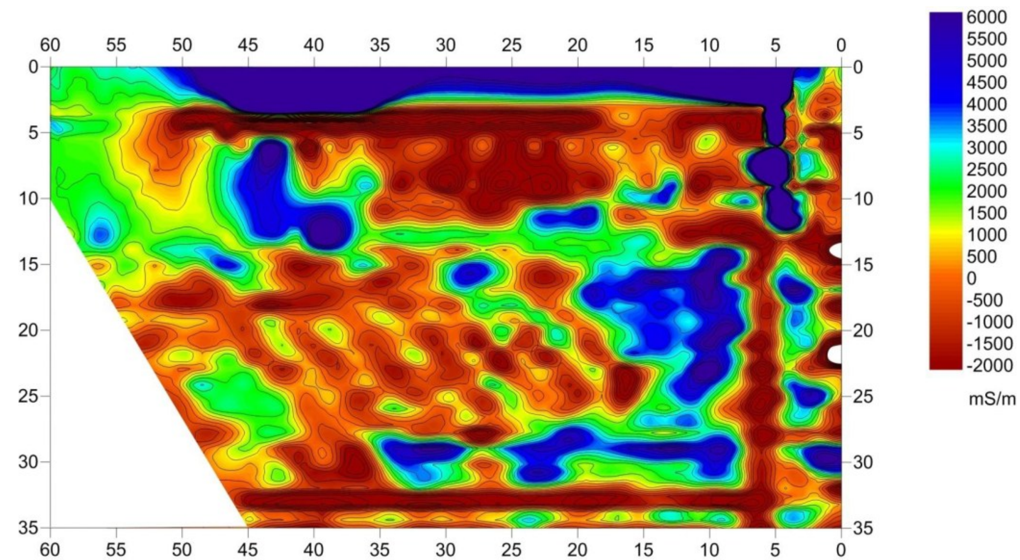
Electromagnetic terrain conductivity map (in mS/m) of Laurel Ridge Landfill, Lily, Kentucky (in feet). The higher conductivity values (>40 mS/m) represent areas of buried waste.



Apparent conductivity/resistivity map

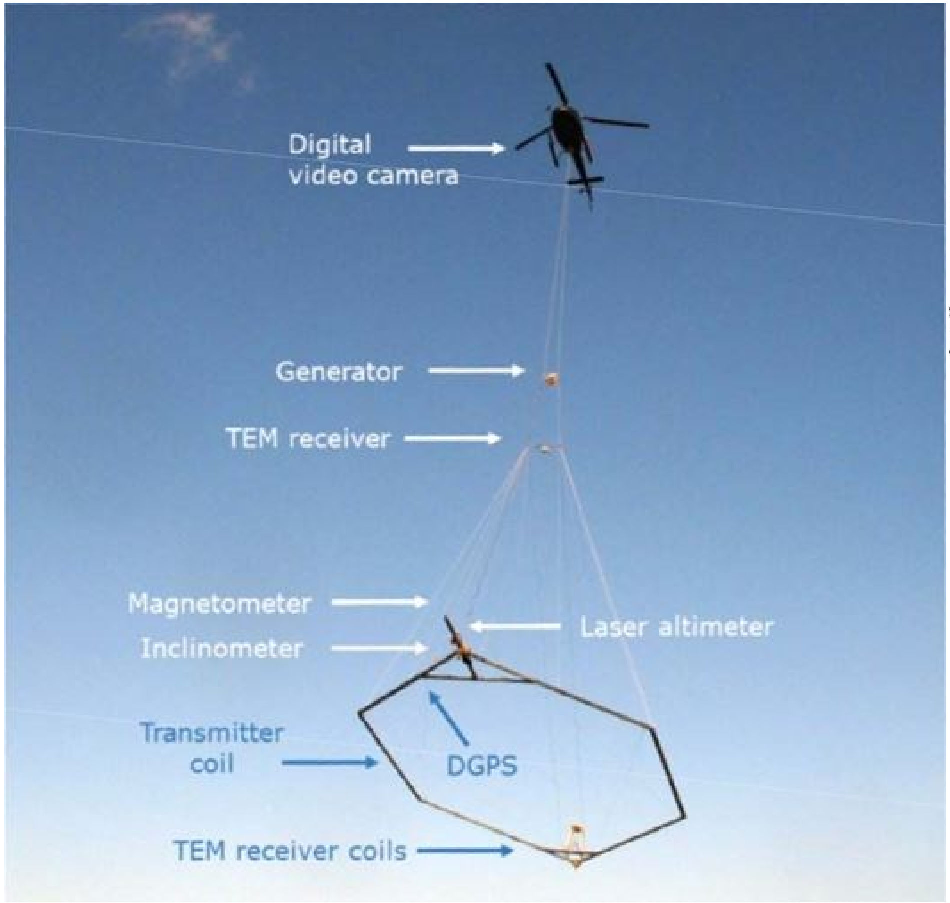
Utilities mapping: the absence of negative apparent conducivity values is related to over-filtering of good data



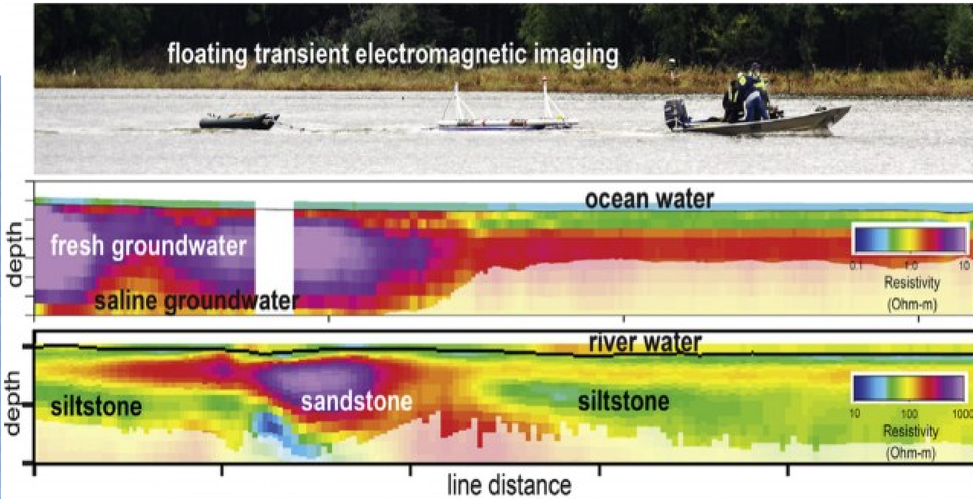


LFEM – Other EM systems

Airborne (helicopter-borne) EM SkyTEM



Waterborne EM



Land (backpack) EM

LFEM - Airborne EM

SkyTEM

Mapping the aquifers of the whole Denmark

Difference between borehole and geophysical LFEM data

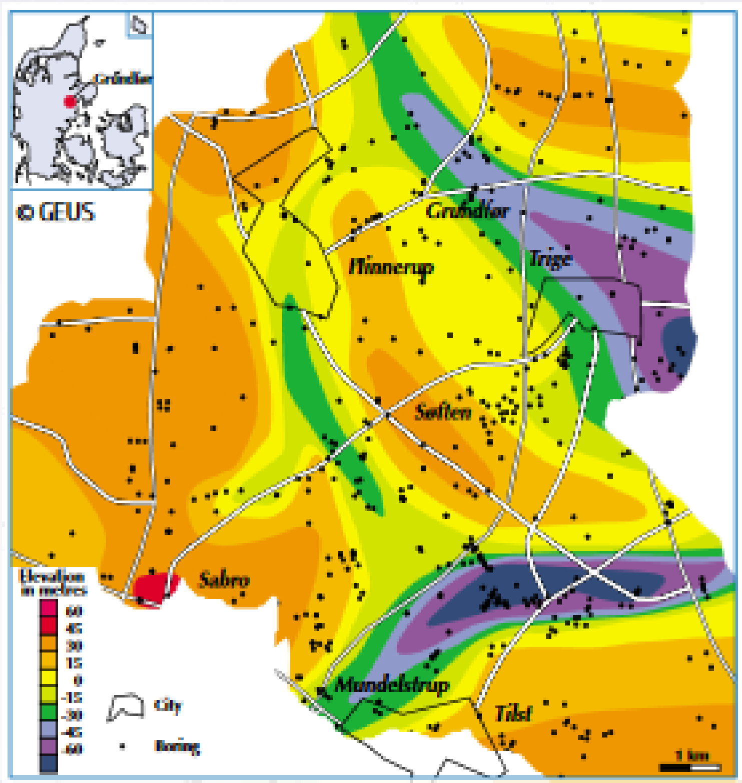


Figure 1 - Map based on 518 boreholes

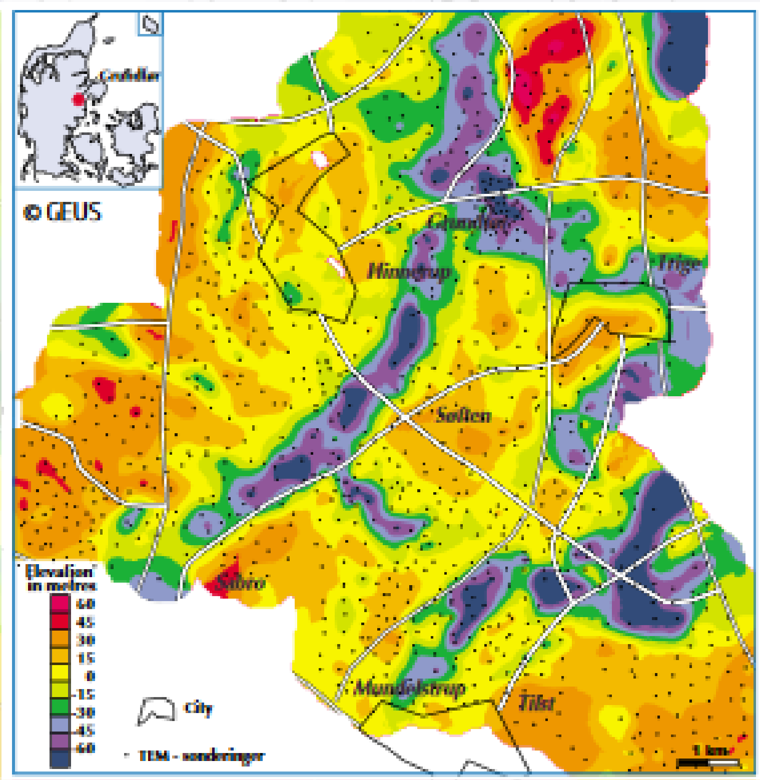
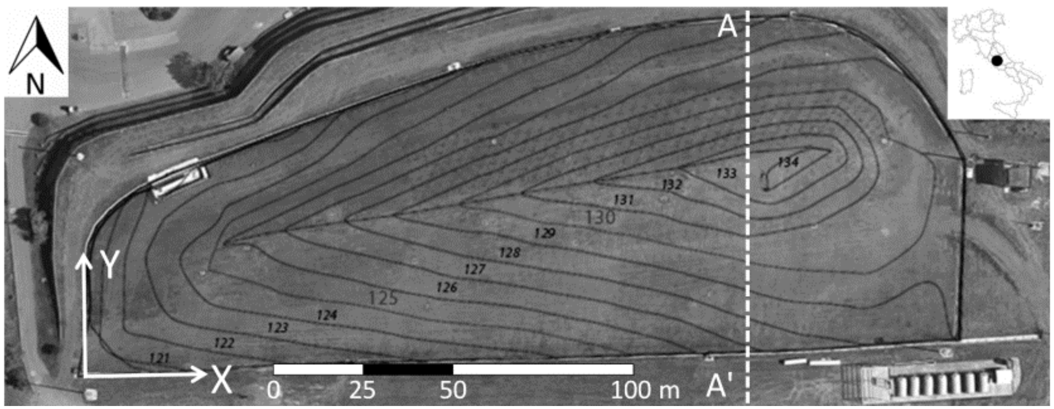


Figure 2 - Map based on 1,400 TEM soundings

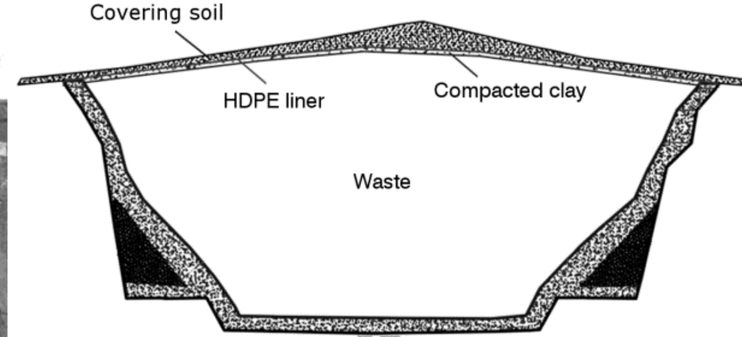
LFEM Slingram method – Example (exercise n.7)

Car-fluff landfill (central Italy)

Aerial view



A-A' cross-section

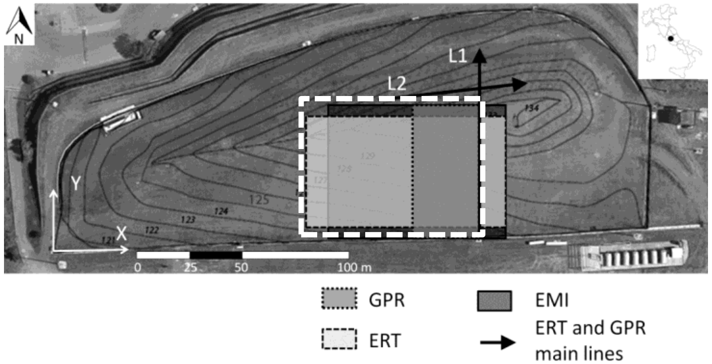


The investigated area (225 x 80 m) is a closed landfill, where both leachate and biogas are also collected: although low the organic fraction (and biogas production) is produced from the degradation compounds of car textiles.

Main goal: **detecting unwanted biogas upwelling flows outside the landfill**

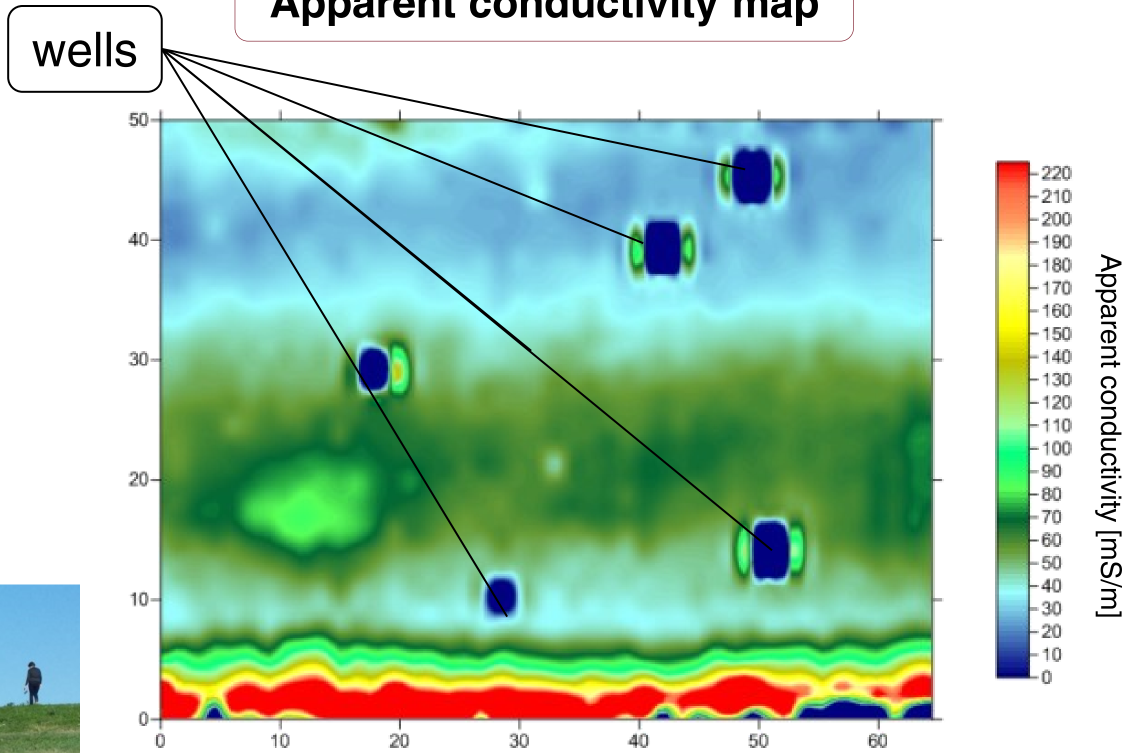
Car-fluff

LFEM Slingram method – Example

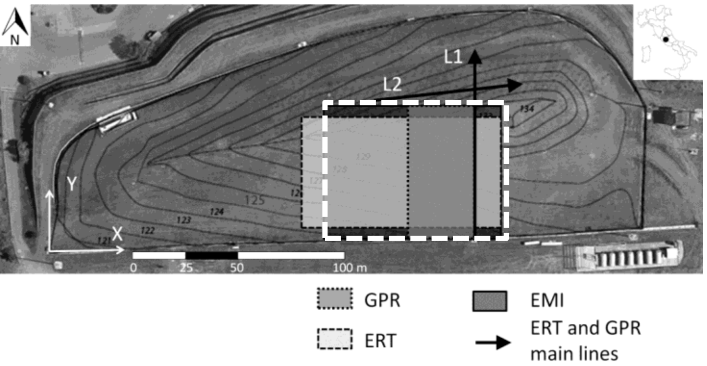


frequency=10 kHz

Apparent conductivity map



LFEM Slingram method – Example

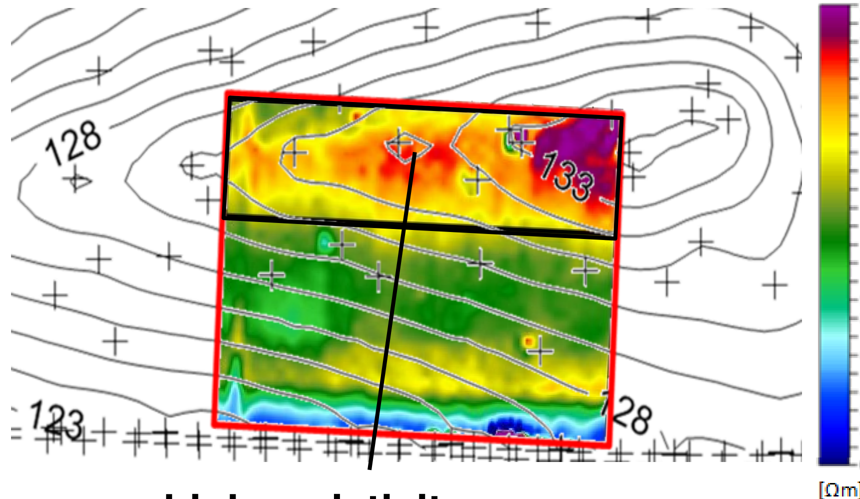


Monitoring seasonality effect

Both data-sets display a similar behavior in terms of resistivity: a high-resistive zone corresponding to the elevated area and some changes among the different seasons, owing to the different soil moisture conditions.

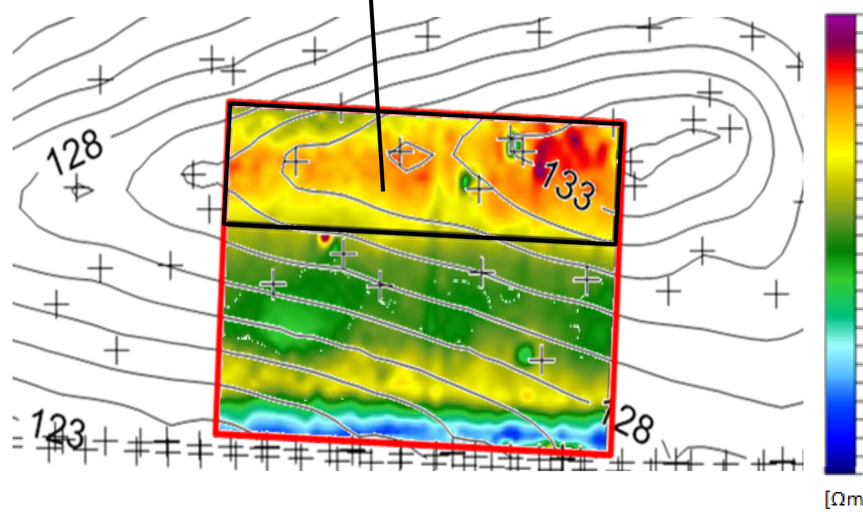
Apparent resistivity map

September 2015 (end of summer)



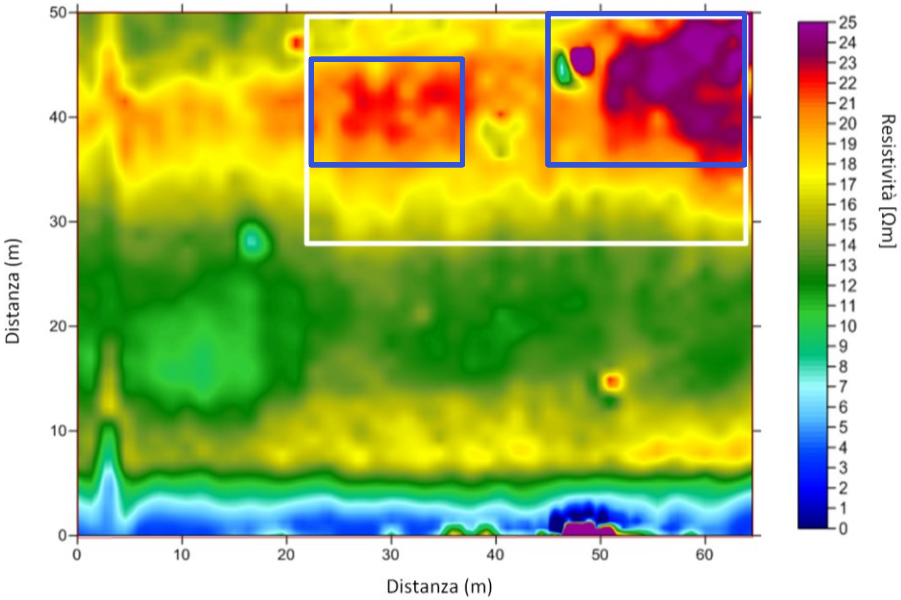
high-resistivity zone

January 2015 (winter)



LFEM Slingram method – Example

Apparent resistivity map



Aerial view

The high resistivity zone, included within the white rectangle (in detail in two blue rectangles) corresponds to the dry area in the aerial view, indicating the biogas effect on the covering soil