SAPIENZA
UNIVERSITÀ DI ROMA

Environmental Geophysics

Giorgio De Donno

5. DC electrical methods

Electrical Resistivity Tomography (ERT)

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Electrode classical arrays

Wenner

$$K = 2\pi\left(\frac{1}{\overline{AM}} - \frac{1}{\overline{MB}} - \frac{1}{\overline{AN}} + \frac{1}{\overline{NB}}\right)^{-1}$$

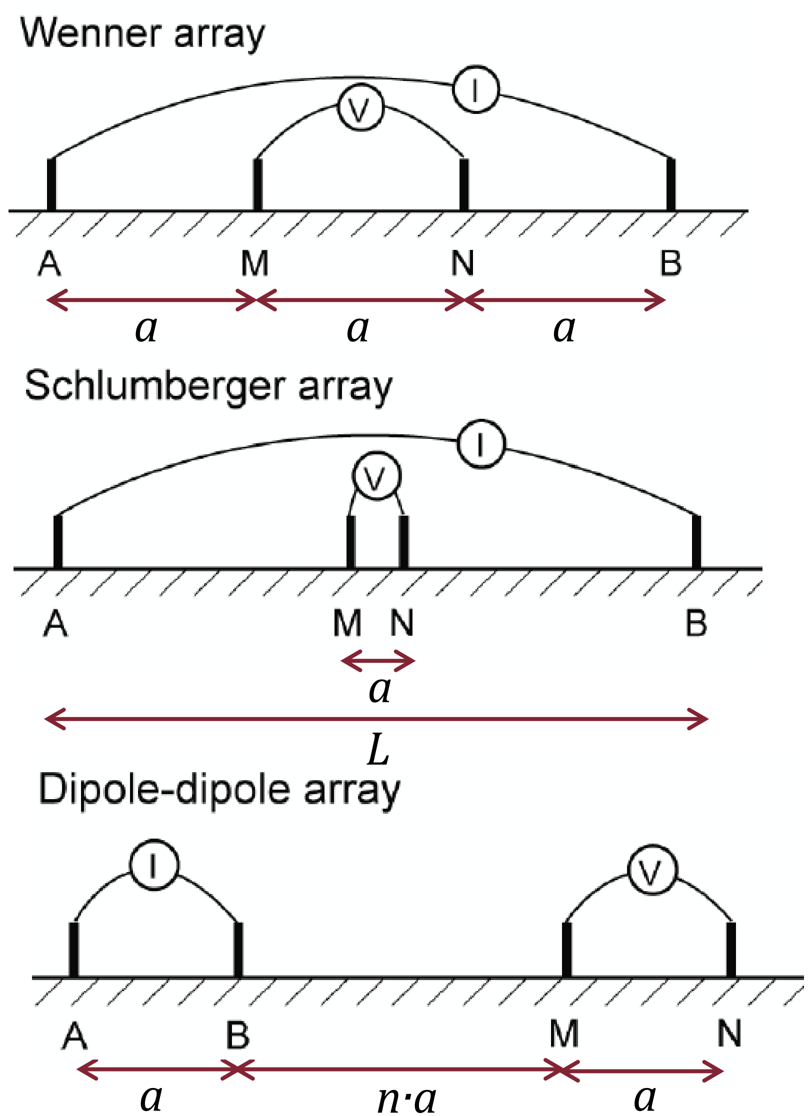
$$K = 2\pi\left(\frac{1}{a} - \frac{1}{2a} - \frac{1}{2a} + \frac{1}{a}\right)^{-1} = 2\pi a$$

Schlumberger (1-D VES)

$$K = \frac{\pi}{a}\left[\left(\frac{L}{2}\right)^2 - \left(\frac{a}{2}\right)^2\right]$$

Dipole-dipole

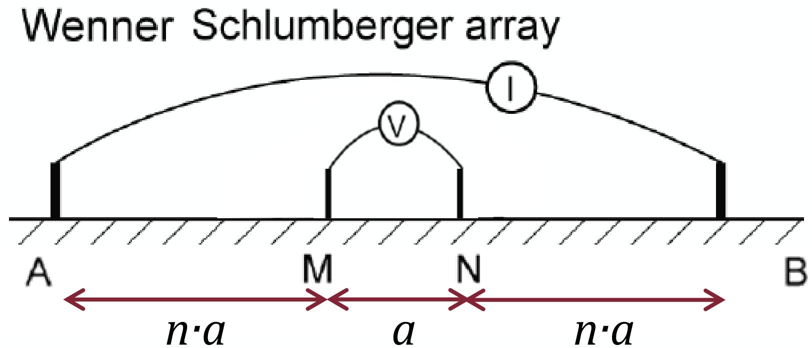
$$K = \pi n(n+1)(n+2)a$$



Electrode hybrid/advanced arrays

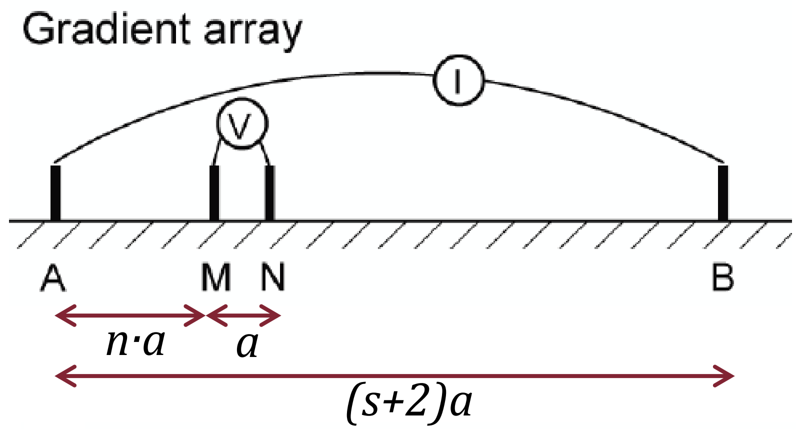
Wenner-Schlumberger

$$\boxed{K = \pi n(n+1)a}$$



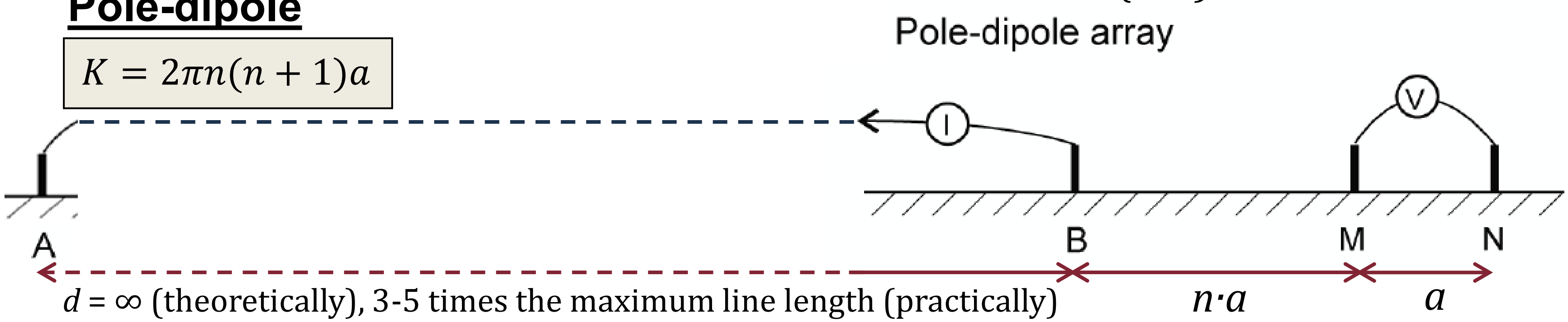
Multiple Gradient

$$\boxed{K = f(a,n,s)}$$

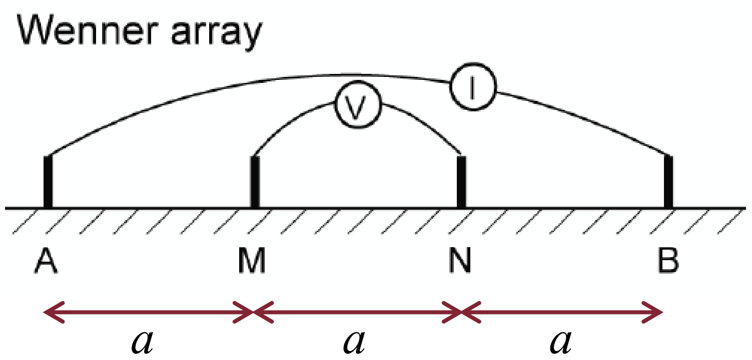


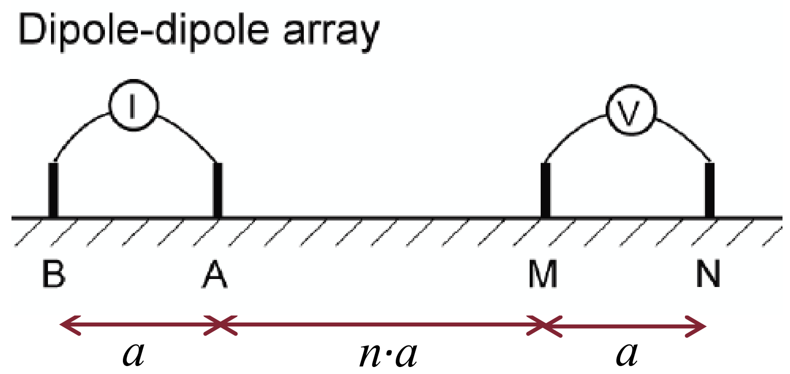
Pole-dipole

$$\boxed{K = 2\pi n(n+1)a}$$



Electrode arrays



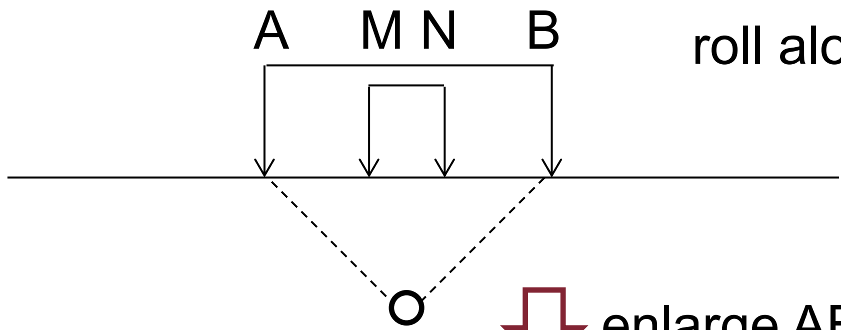


In practical cases we always need more measurements to map the desired target with a good degree of resolution both vertical and laterally

How can I deal with this new goal?

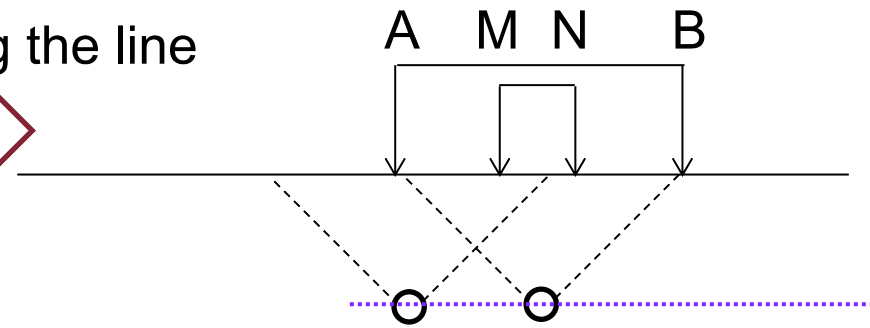
A very simple method is to ground more than 4 electrodes performing a number of measurements following a given sequence…

Multi-electrode acquisition – 2D acquisition

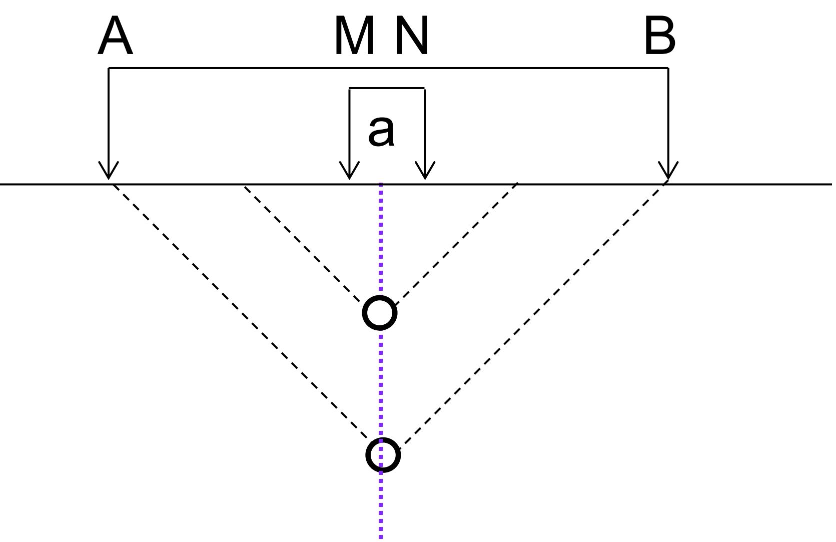


roll along the line

Horizontal Profiling (HP)

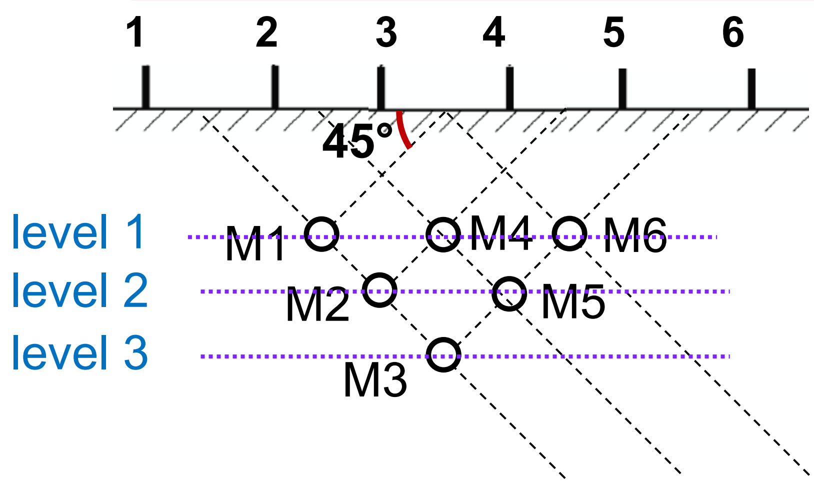


enlarge AB



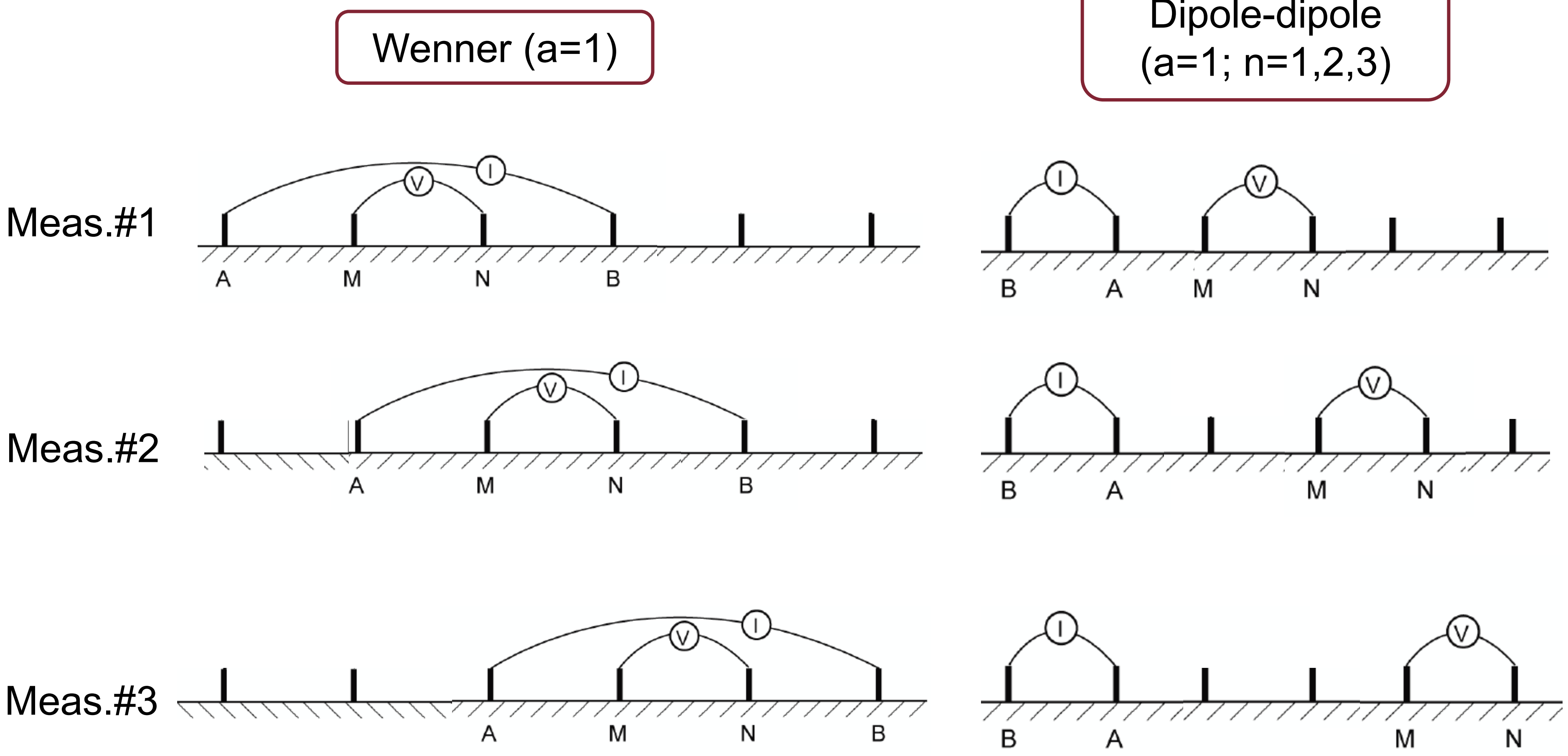
Vertical Electrical Soundings (VES)

2D Electrical Resistivity Tomography (ERT)
Merge together VES and HP



Multi-electrode acquisition

Ex. 1: Six electrode grounded

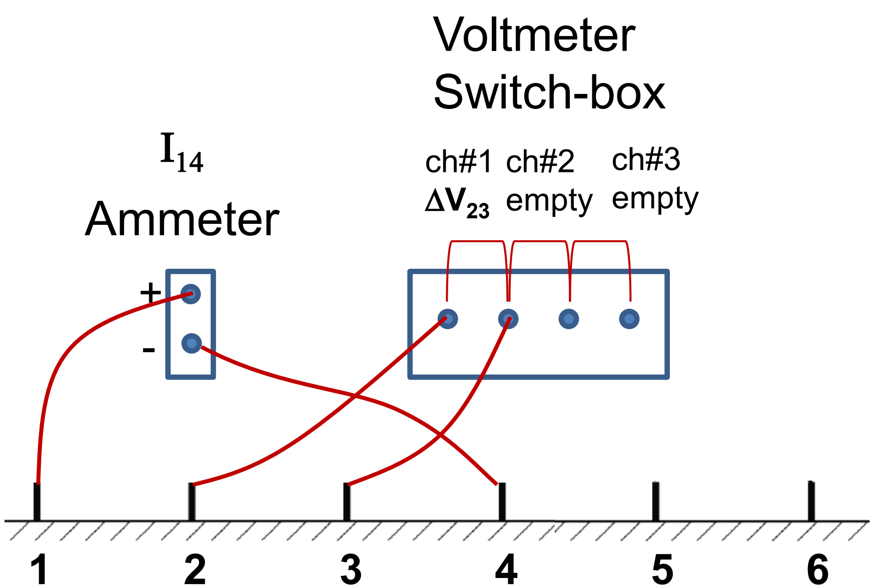


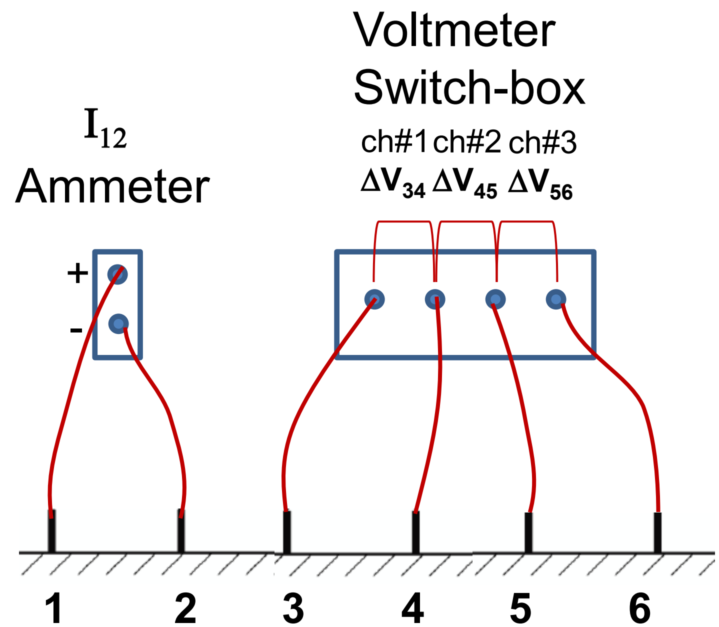
If a have a 3-channel switch box….

Wenner (a=1)

Dipole-dipole (a=1; n=1,2,3)

Cycle #1





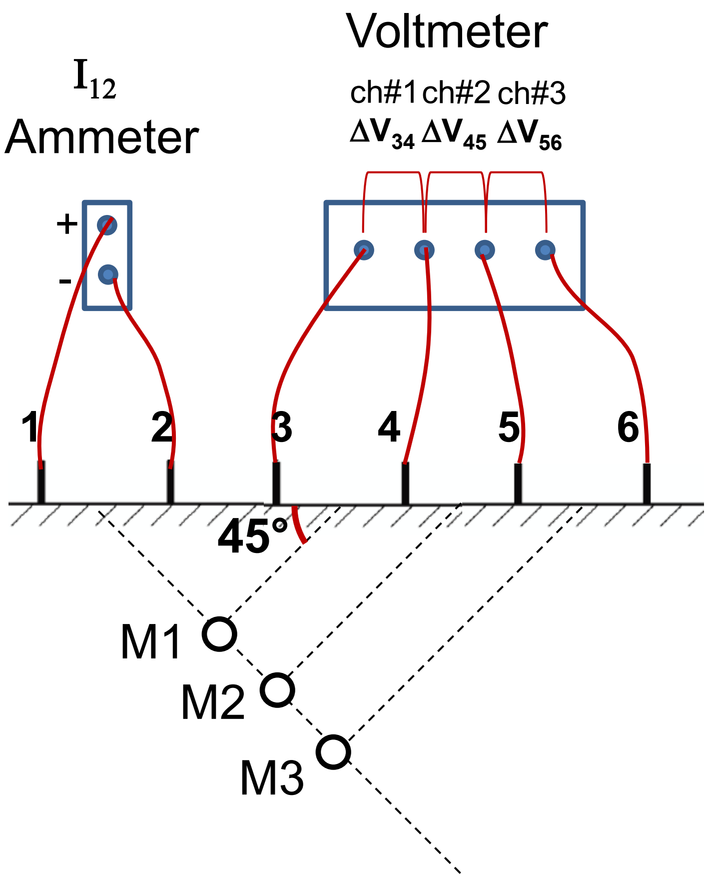
I can execute only 1 measurement for each cycle.
SINGLE-CHANNEL ACQUISITION

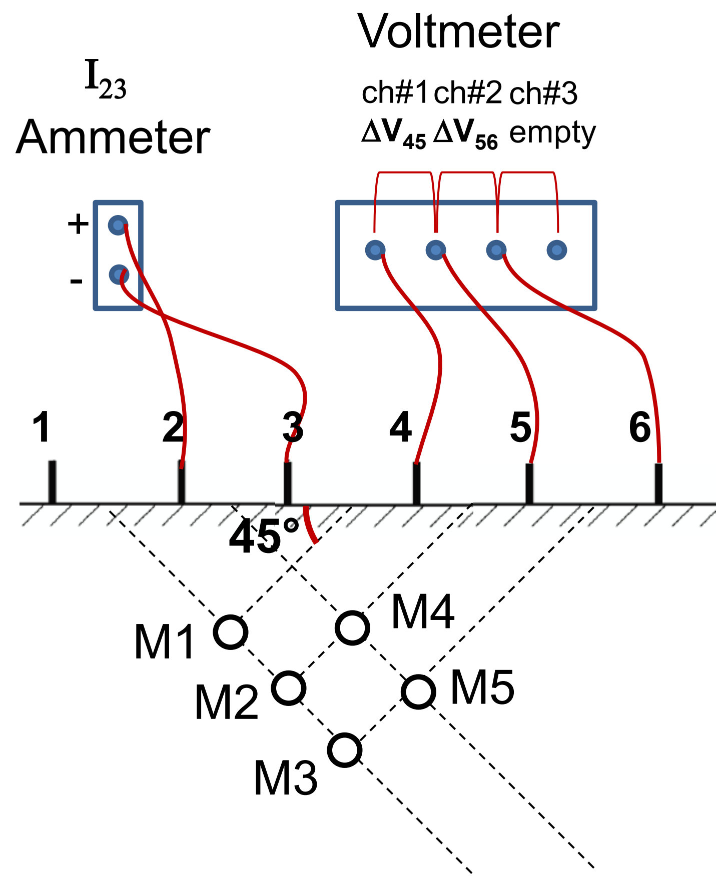
I can execute 3 measurements for each cycle:
66% reduction of acquisition time
MULTI-CHANNEL ACQUISITION

Conventional position of a measurement

Convention: Measured point is the cross-point of two straight lines with an angle of 45° starting from the axis of the two dipoles

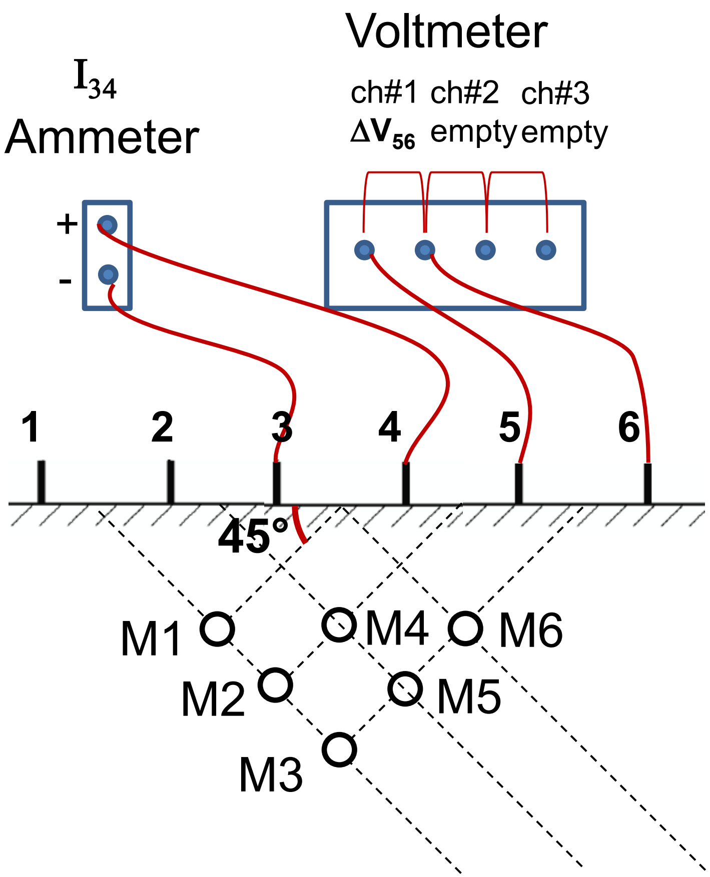
Moving the current electrode to el.2 and el.3 (a=1; n=1,2)



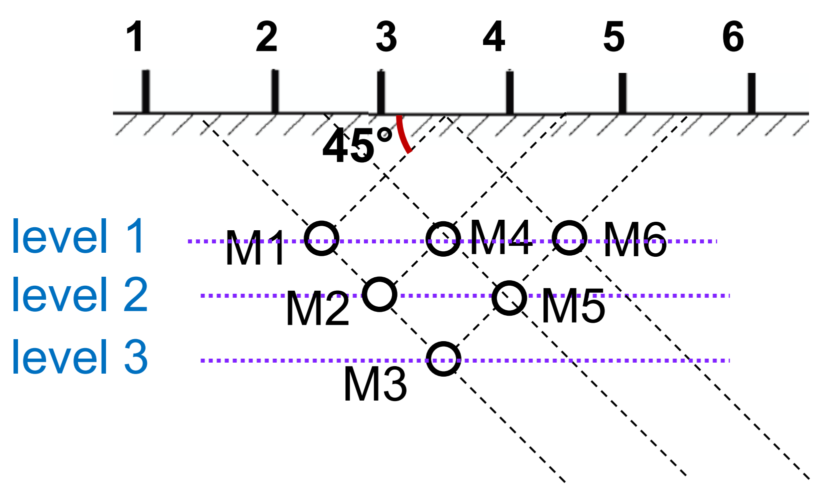


Conventional position of a measurement

Moving the current electrode to
el.3 and el. 4 (a=1; n=1)



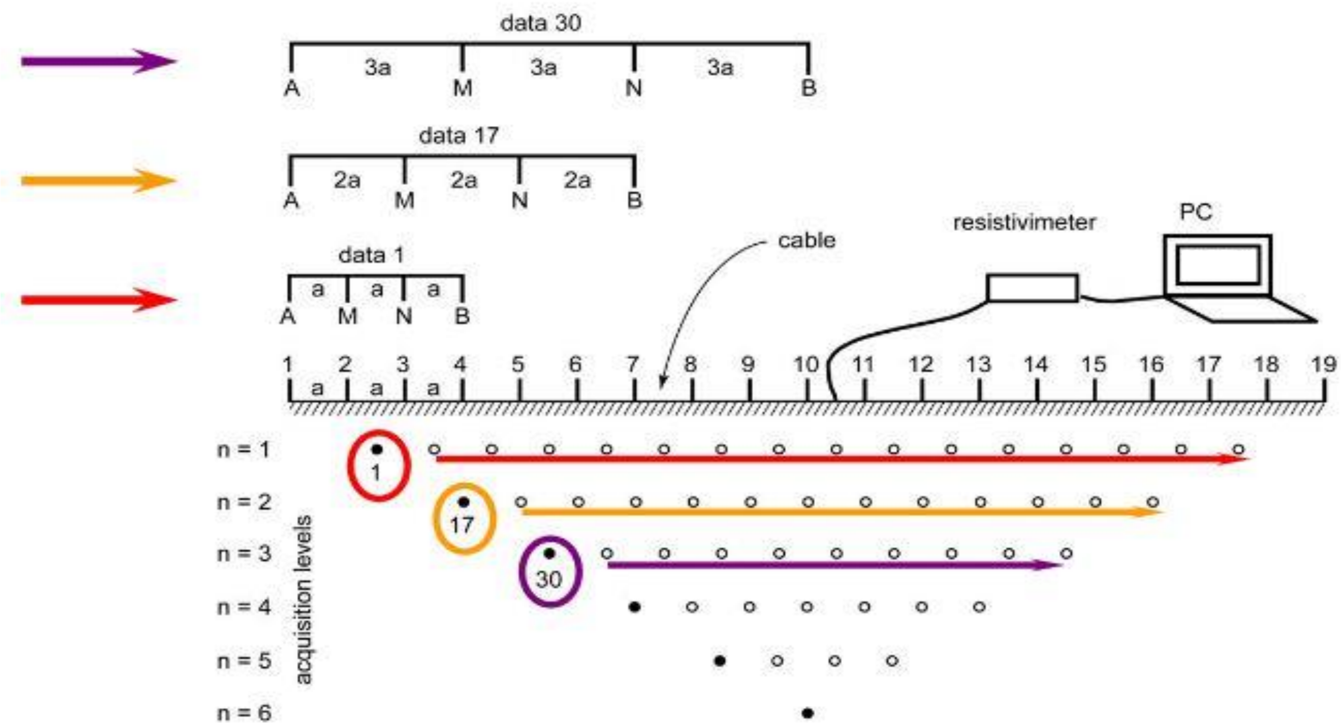
Final section



Multi-electrode acquisition

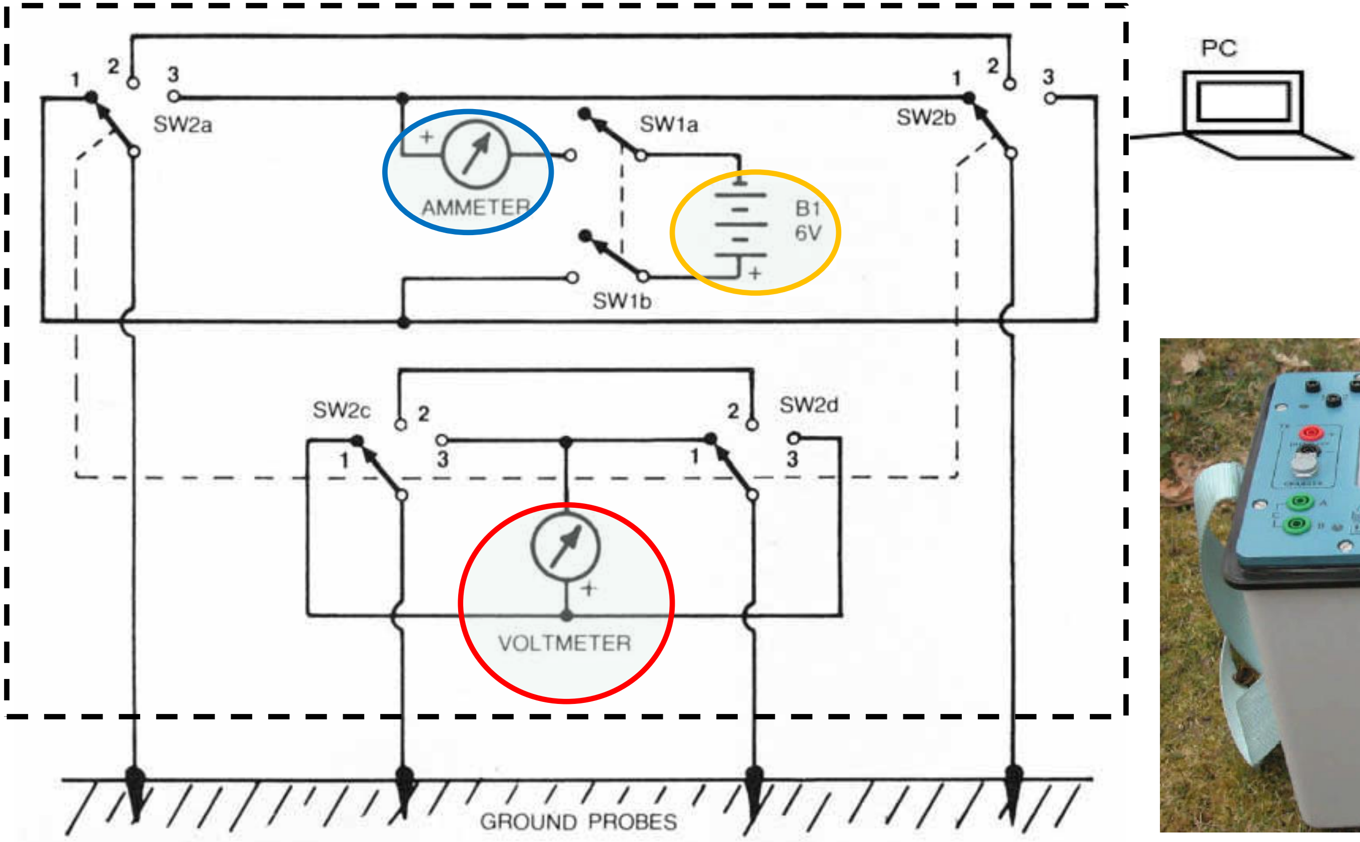
Ex. 2: Nineteen electrodes

Where number of electrodes grows there are multi-channel cables connected with all the electrodes. The **ammeter and voltmeter are combined into a single device called resistivity-meter** connected with the cables



Multi-electrode acquisition – Resistivimeter

Resistivity-meter: device able to measure both potential and current, to compute the geometric factor for the particular measurement and therefore to calculate the apparent resistivity

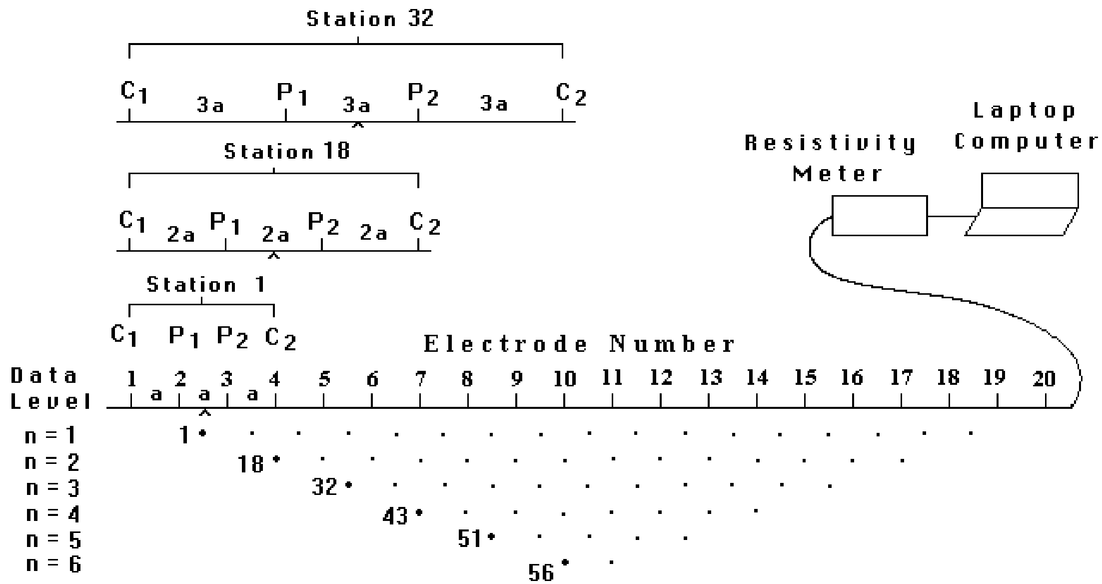


Acquistion software is able to calculate K and to compute the apparent resistivity for each quadrupole

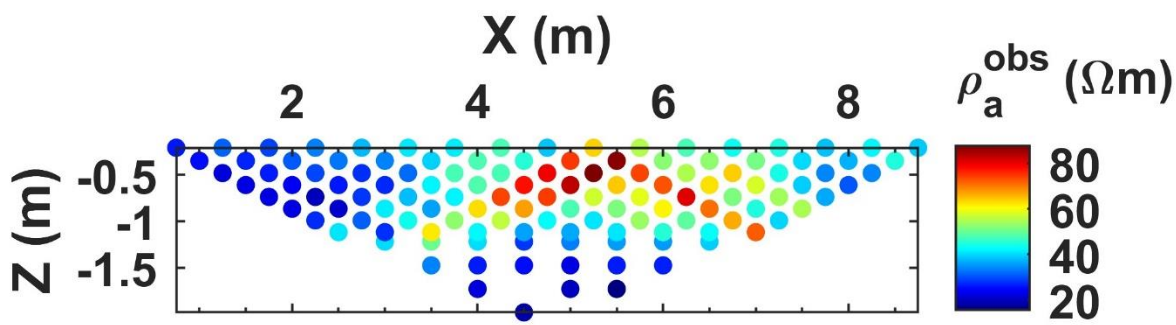
Multi-electrode acquisition – Observed dataset

OBSERVED DATA VECTOR
Apparent resistivities

$$\rho_{a,i}^{OBS} = K_i \frac{\Delta V_i}{I_i}, i = 1,2,\ldots,N_Q$$



Data can be plotted in the so-called apparent resistivity PSEUDO-SECTION



Multi-electrode acquisition – Observed dataset

Example of experimental dataset 48 electrodes
surface dipole-dipole array a=(1,2,3,4,5) n=(1,2,3,4,5,6)

Meas.	A B M N	ΔV (V)	I (mA)	σ(%)	$\rho_a(\Omega m)$
1	1 2 3 4	0.679910	101.45	1.20	42.72
2	1 2 4 5	0.266054	101.56	0.08	50.15
3	1 2 5 6	0.121435	101.43	2.02	45.78
4	1 3 4 6	0.172654	125.67	1.06	42.35
5	1 3 6 8	0.055216	125.14	0.82	50.05
6	1 3 8 10	0.032240	125.19	0.13	46.74
7	2 3 4 5	0.772654	115.67	3.06	45.65
8	2 3 5 6	0.355216	115.44	0.22	52.04
9	2 3 6 7	0.132240	115.09	0.03	42.54
…	…	…	…	…	…
945	45 46 47 48	0.545678	105.65	1.87	76.09

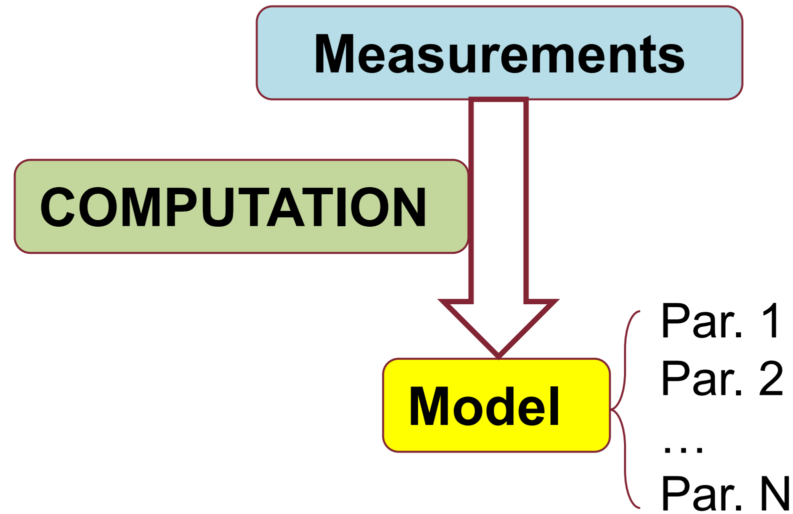
⇩

Which is the (true) resistivity model corresponding to this dataset?

Data inversion - ERT

Since we cannot measure resistivity directly, we have to infer it indirectly.

Therefore, we need to know how this computation process (called INVERSION) acts in order to switch from measurements to model

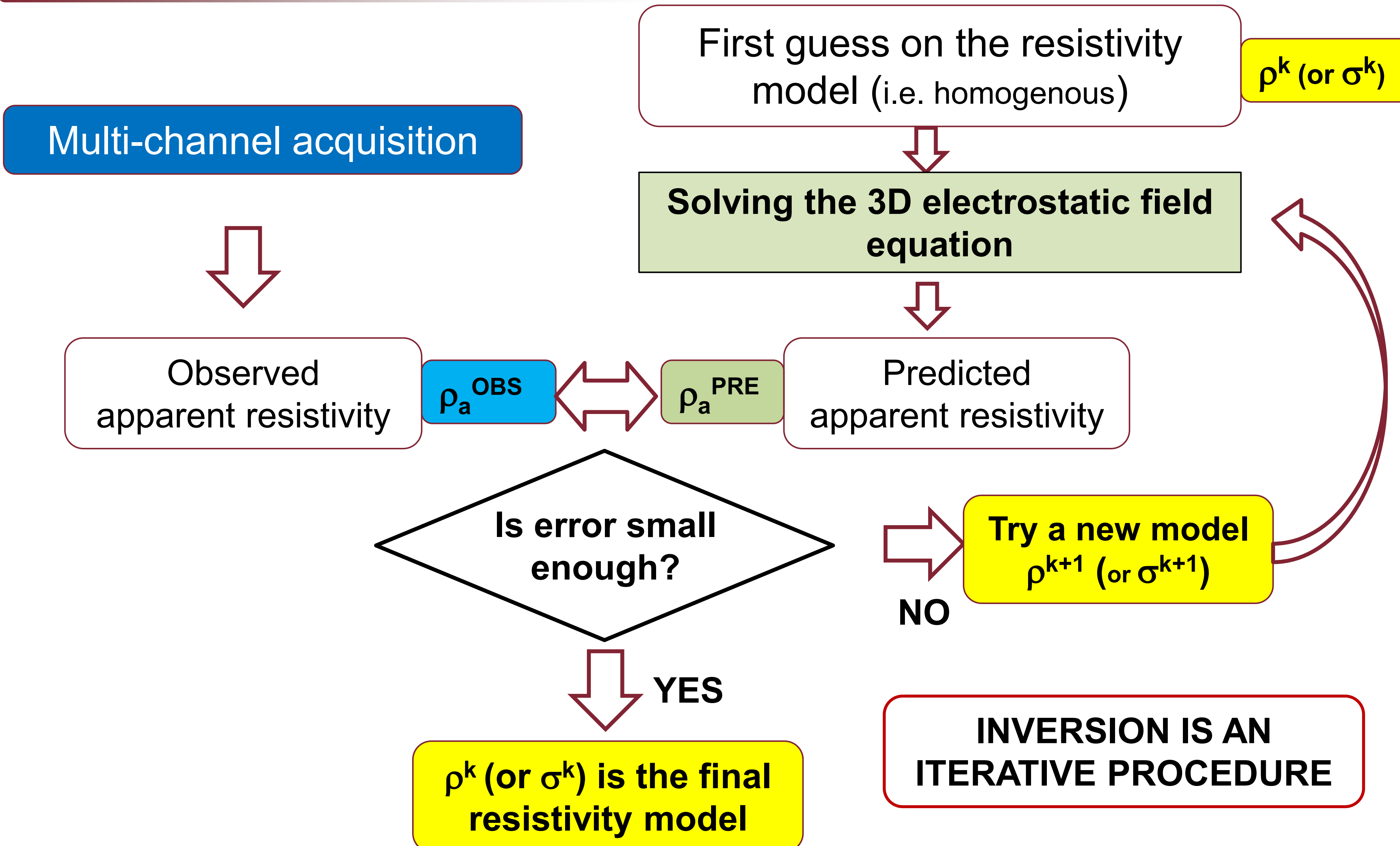


MEASUREMENTS = APPARENT RESISTIVITY

COMPUTATION = INVERSION

MODEL PARAMETER = RESISTIVITY or CONDUCTIVITY

Data inversion - ERT



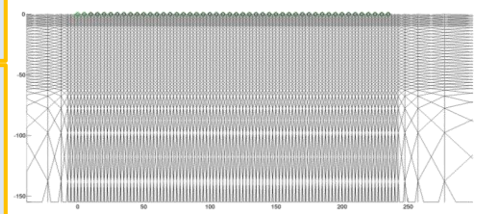
Solving the 3D electrostatic field equation

$$-\nabla \cdot (\sigma(\mathbf{r})\nabla V(\mathbf{r})) = I\delta(\mathbf{r})$$

We need to solve this equation by numerical methods, such as the finite-element method (FEM)

Q1. How can we solve this equation by numerical methods?

A1. By solving it only on a small number of points (nodes) on a bounded domain (not on the whole half-space) by subdividing the domain in a small set of elements (pixels).

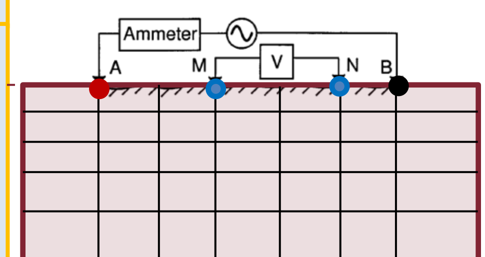


Q2. How can we evaluate the extension of the domain?

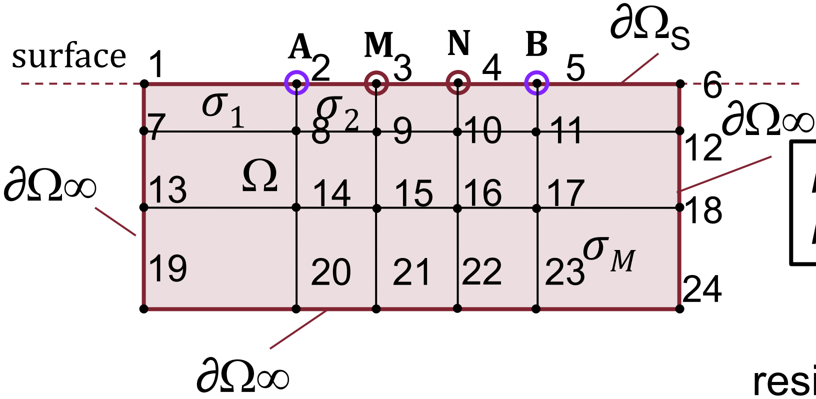
A2. Theoretically the potential vanishes at infinity. Practically it vanishes “far enough” from the source (e.g. 10 times the maximum spacing between the current electrodes).

Q3. How can we predict the apparent resistivity?

A3. Starting from the first guess of conductivity (or resistivity) model σ, we solve the equation for electric potential given a fixed current intensity injected at the nodes corresponding to A(+) and B(-). Then, we extract the potential only at the nodes corresponding to M and N, we make the difference ΔV_{MN} and we compute the predicted apparent resistivity (like for VES). We repeat the procedure to predict ρ_a for all the quadrupoles.



Solving the 3D electrostatic field equation



$$\boxed{-\nabla \cdot (\sigma(\mathbf{r})\nabla V(\mathbf{r})) - I\delta(\mathbf{r}) \neq 0 = R}$$

hp.1 conductivities are constant within each element
hp.2 the solution is achieved only at selected nodes

Using numerical methods the solution will generate a residual *R*. The **Finite Element Method** (FEM) minimize the residual on a bounded domain. Using FEM the electrostatic field equation becomes a linear system:

Stiffness (square) **matrix** (depends on **conductivities** and **mesh geometry**)

$$\boxed{[\mathbf{K}]\{\mathbf{V}\} = \{\mathbf{I}\}}$$ **linear system**

Potential

Current injection

$$\boxed{\{\mathbf{V}\} = [\mathbf{K}]^{-1}\{\mathbf{I}\}}$$ **solution**

nodal potential $\{\mathbf{V}\} = [V_1, V_2, \ldots, V_N]$

Boundary conditions

- **on $\partial\Omega_S$** $\sigma \dfrac{\partial V}{\partial n} = 0$

Neumann condition:
No current outward flow at the air-ground interface

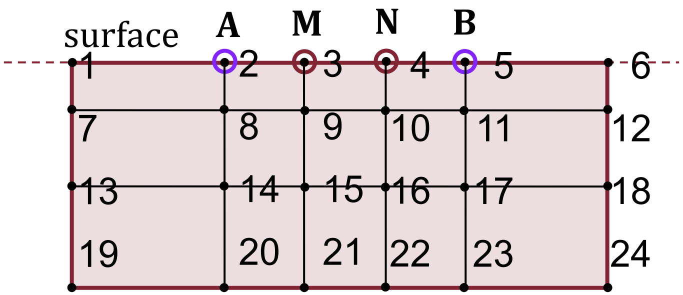
- **on $\partial\Omega_\infty$** $V=0$

Dirichlet condition:
Electric potential vanishes «far enough» from the current source

Q. Which are the nodes where I need the potential?
A. The nodes where M and N electrodes are located.

Solving the 3D electrostatic field equation

Ex.1 I=±1A at nodes no.2 and 5



$\{\mathbf{I}\} = [I_1, I_2, I_3, I_4, \ldots, I_{24}] = [0,1,0,0,-1,\ldots,0]$

solving the linear system

$\{\mathbf{V}\} = [V_1, V_2, V_3, V_4, \ldots, V_{24}]$

$V_M = V_3 \qquad V_N = V_4$

$\boxed{\Delta V_{MN}^{PRE} = V_M - V_N = V_3 - V_4}$

Predicted apparent resistivity

$$\rho_a^{PRE} = 2\pi\left(\frac{1}{\overline{AM}} - \frac{1}{\overline{MB}} - \frac{1}{\overline{AN}} + \frac{1}{\overline{NB}}\right)^{-1} \cdot \frac{\Delta V_{MN}^{PRE}}{I}$$

K

Predictions vs. observations

$$\rho_a{}^{OBS} = K \frac{\Delta V_{MN}{}^{OBS}}{I_{AB}}$$

Observed apparent resistivity

$$\rho_a{}^{PRE} = K \frac{\Delta V_{MN}{}^{PRE}}{I_{AB}}$$

Predicted apparent resistivity

We **measure both the injected current** in AB and **the potential difference** in MN. We know the geometric factor (it depends on the electrode array) and therefore we have the observations.

We **set the injected current** in AB to a fixed value (e.g. the same of the field experiment) and we **predict the potential difference** in MN **for the resistivity model at the current iteration**. We know the geometric factor (it depends on the electrode array) and therefore we have the predictions.

Q1. When does $\rho_a{}^{OBS}$ equal $\rho_a{}^{PRE}$?
A1. Theoretically when potential calculated from a resistivity model will be equal to the measured electric potential. Practically never due to the presence of measurement errors. We can only find a predicted value that “fits well” the observed one.

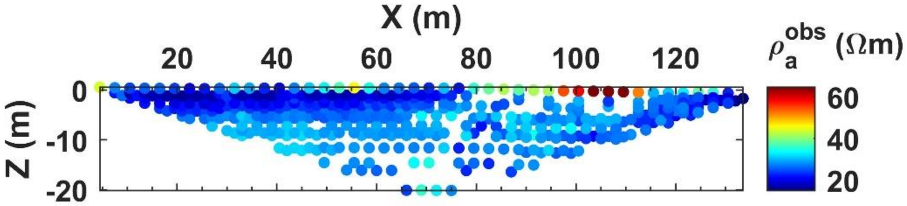
Q2. How can the final resistivity model?
A2. With inversion!

Data inversion – 2D ERT case

OBSERVED DATASET

48 elec. spaced 5 m apart – DD a_{max}=5 n_{max}=6

N.	A	B	M	N	ΔV (V)	I (mA)	σ(%)	ρ_a^{OBS}(Ωm)
1	1	2	3	4	0.679910	101.45	1.20	42.72
2	1	2	4	5	0.266054	101.56	0.08	50.15
3	1	2	5	6	0.121435	101.43	2.02	45.78
4	1	3	4	6	0.172654	125.67	1.06	42.35
5	1	3	6	8	0.055216	125.14	0.82	50.05
6	1	3	8	10	0.032240	125.19	0.13	46.74
7	2	3	4	5	0.772654	115.67	3.06	45.65
8	2	3	5	6	0.355216	115.44	0.22	52.04
9	2	3	6	7	0.132240	115.09	0.03	42.54
…								
945	45	46	47	48	0.545678	105.65	1.87	76.09



Data inversion – 2D ERT case

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N.	A	B	M	N	ΔV (V)	I (mA)	σ(%)	ρ_a^{OBS}(Ωm)
1	1	2	3	4	0.679910	101.45	1.20	42.72
2	1	2	4	5	0.266054	101.56	0.08	50.15
3	1	2	5	6	0.121435	101.43	2.02	45.78
4	1	3	4	6	0.172654	125.67	1.06	42.35
5	1	3	6	8	0.055216	125.14	0.82	50.05
6	1	3	8	10	0.032240	125.19	0.13	46.74
7	2	3	4	5	0.772654	115.67	3.06	45.65
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9	2	3	6	7	0.132240	115.09	0.03	42.54
…								
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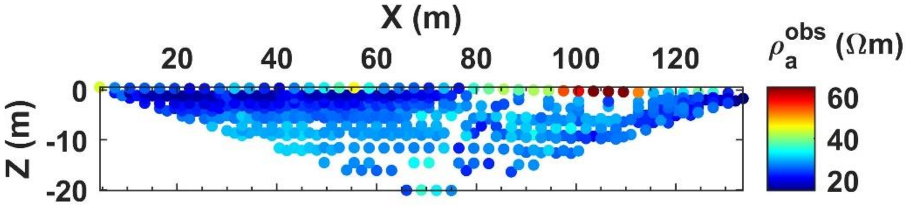
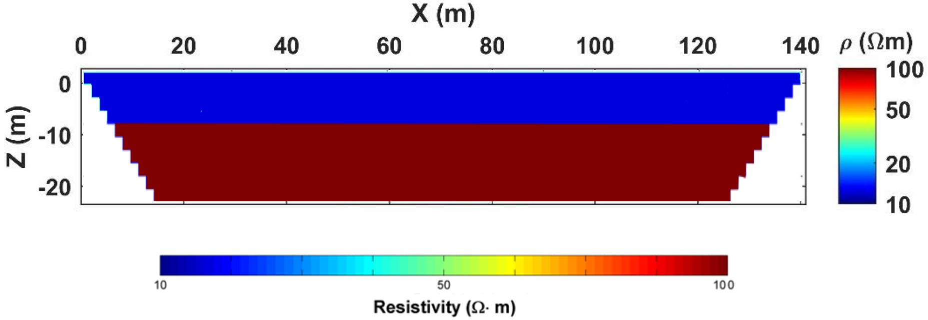


First trial model σ_1 (or ρ_1)



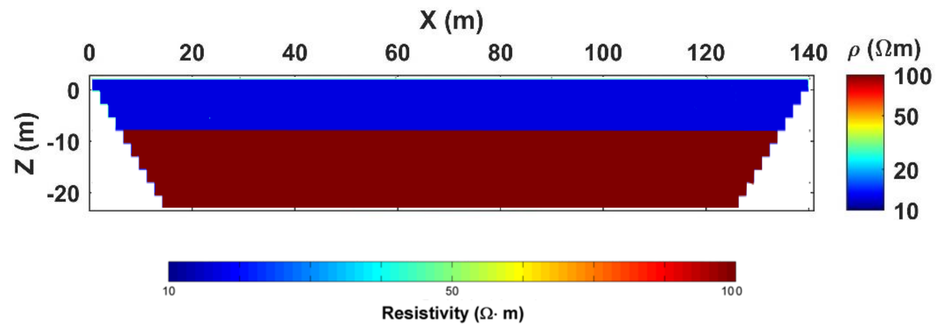
Data inversion – 2D ERT case

OBSERVED DATASET

48 elec. spaced 5 m apart – DD a_{max}=5 n_{max}=6

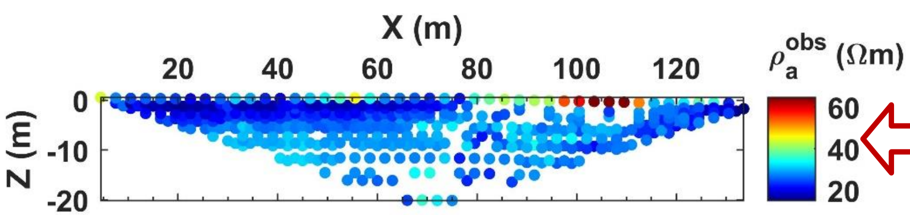
N.	A	B	M	N	ΔV (V)	I (mA)	σ(%)	ρ_a^{OBS}(Ωm)	ρ_a^{PRE}(Ωm)
1	1	2	3	4	0.679910	101.45	1.20	42.72	35.6
2	1	2	4	5	0.266054	101.56	0.08	50.15	44.1
3	1	2	5	6	0.121435	101.43	2.02	45.78	45.2
4	1	3	4	6	0.172654	125.67	1.06	42.35	75.2
5	1	3	6	8	0.055216	125.14	0.82	50.05	103.7
6	1	3	8	10	0.032240	125.19	0.13	46.74	60.0
7	2	3	4	5	0.772654	115.67	3.06	45.65	46.1
8	2	3	5	6	0.355216	115.44	0.22	52.04	53.2
9	2	3	6	7	0.132240	115.09	0.03	42.54	44.1
…									
94	45	46	47	48	0.545678	105.65	1.87	76.09	88.9

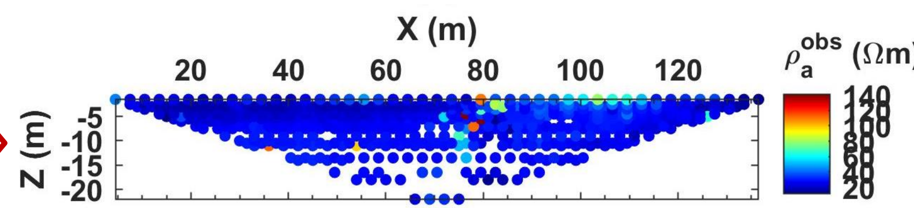
First trial model σ_1 (or ρ_1)



Solving the electrostatic field equation with FEM

PREDICTED DATASET





The fitting is not satisfying, we need to iterate the procedure…

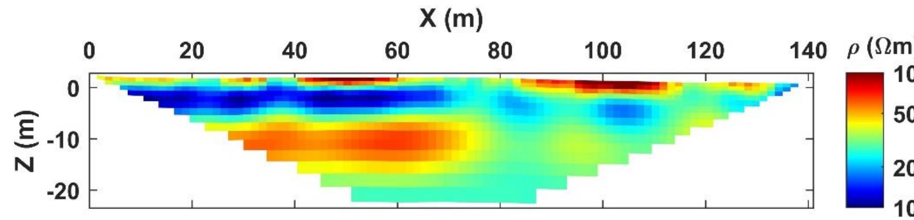
Data inversion – 2D ERT case

OBSERVED DATASET

48 elec. spaced 5 m apart – DD a_{max}=5 n_{max}=6

N.	A	B	M	N	ΔV (V)	I (mA)	σ(%)	ρ_a^{OBS}(Ωm)	ρ_a^{PRE}(Ωm)
1	1	2	3	4	0.679910	101.45	1.20	42.72	42.6
2	1	2	4	5	0.266054	101.56	0.08	50.15	49.1
3	1	2	5	6	0.121435	101.43	2.02	45.78	45.2
4	1	3	4	6	0.172654	125.67	1.06	42.35	45.2
5	1	3	6	8	0.055216	125.14	0.82	50.05	53.7
6	1	3	8	10	0.032240	125.19	0.13	46.74	46.0
7	2	3	4	5	0.772654	115.67	3.06	45.65	46.1
8	2	3	5	6	0.355216	115.44	0.22	52.04	52.2
9	2	3	6	7	0.132240	115.09	0.03	42.54	42.1
…									
94	45	46	47	48	0.545678	105.65	1.87	76.09	79.9

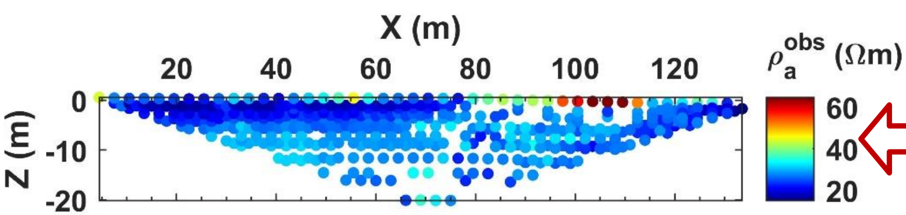
Final trial model σ_k (or ρ_k)

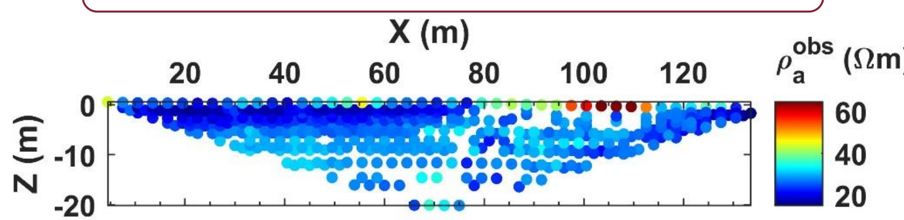


ITER: 5
RMSE (%): 6.9

Solving the electrostatic field equation with FEM

FINAL PREDICTED DATASET





The trial model is the final resistivity model

Data inversion - Goodness of fit

Q. When will the iterative process end?
A. If error at the current iteration is below a certain threshold.

Error is expressed at each iteration *k* as Root Mean Square Error (RMSE) for LS inversion:

$$RMSE_k(\%) = 100\sqrt{\frac{\sum_{i=1}^{N_Q}\left(\frac{\rho_{a,i}{}^{obs} - \rho_{a,i}{}^{pre}(\boldsymbol{\sigma}_k)}{\rho_{a,i}{}^{obs}}\right)^2}{N_Q}}$$

and Absolute Error (AE) for robust inversion:

$$AE_k(\%) = 100\,\frac{\sum_{i=1}^{N_Q}\left|\frac{\rho_{a,i}{}^{obs} - \rho_{a,i}{}^{pre}(\boldsymbol{\sigma}_k)}{\rho_{a,i}{}^{obs}}\right|}{N_Q}$$

When the difference between the RMSE (or AE) value of the iteration k and k-1 is below a certain acceptable value (i.e. 1%) the iterative procedure ends.