SAPIENZA
UNIVERSITÀ DI ROMA

Environmental Geophysics

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5. DC electrical methods

Basic principles

Vertical Electrical Soundings (VES)

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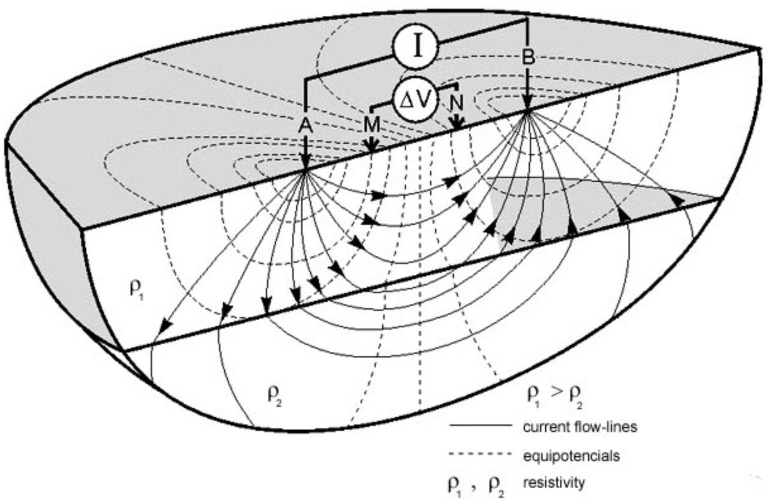
Electrostatic (or galvanic or geoelectrical) method

Geoelectrical methods are all methods that use the electric current (generally DC) flow within a medium.

The electric source can be **natural** (spontaneous potentials generated in the Earth) or **anthropogenic** by using an external source.

In this course we will study the latter method, i.e. the so-called **active** or **galvanic electrostatic** method.

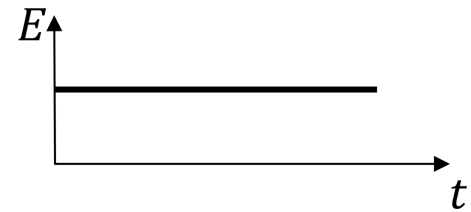


Electrostatic assumption

$$\nabla \times \mathbf{E}(\mathbf{r},t) = -\frac{\partial \mathbf{B}(\mathbf{r},t)}{\partial t}$$

Faraday's law
1st Maxwell's equation

E: electric field
B: magnetic induction
$\mathbf{r} = \mathbf{r}(x,y,z)$: position
t: time



1° assumption: electrostatic field - induction-free media (no time-varying magnetic field)

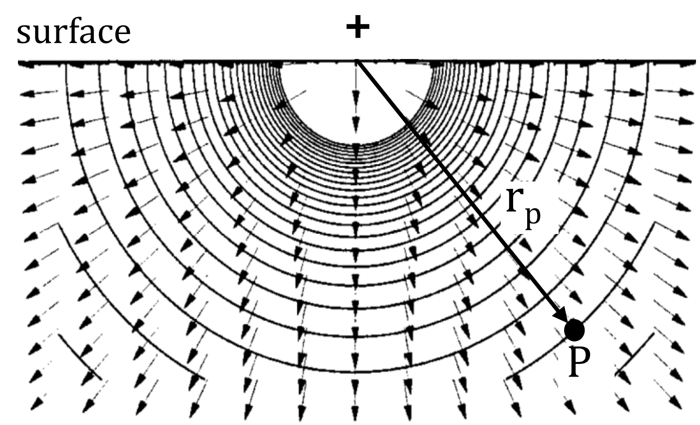
$$\frac{\partial \mathbf{B}}{\partial t} = 0 \rightarrow \nabla \times \mathbf{E} = 0$$

Electrostatic field is irrotational, conservative, that is described by a potential function V, such as:

$$\mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r}) \qquad \nabla V \equiv \left(\frac{\partial V}{\partial x}, \frac{\partial V}{\partial y}, \frac{\partial V}{\partial z}\right)$$

The minus sign indicates a potential decreasing in the direction of the electric field

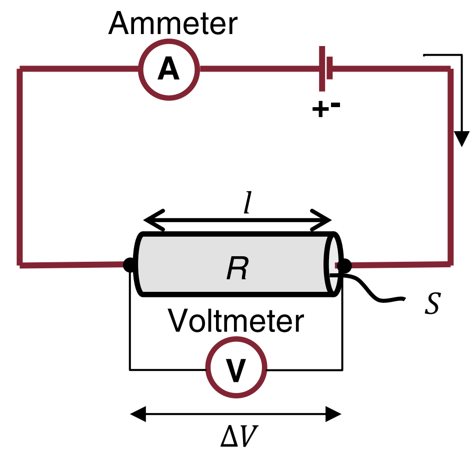
hp. **positive charge on surface and homogenous ground** → transformation to spherical coordinates → potential varies only as a function of the radial distance r_p between the observation point P and the charge



$$E = -\frac{\partial V}{\partial r} = -\frac{(V_P - V_0)}{r_P} = \frac{(V_0 - V_P)}{r_P} = \frac{\Delta V_P^+}{r_P}$$

ΔV_P^+ is **potential** (or voltage) **drop** at the observation point P

Ohm's law



Resistance
[Ohm - Ω]

$$R = \frac{\Delta V}{I}$$

Volt [V]

Ampere [A]

Resistivity
[Ohm·m - Ωm]

$$\rho = \frac{S}{l} R = \frac{S}{l}\frac{\Delta V}{I}$$

$$\sigma = \frac{1}{\rho}$$

Conductivity
[Siemens/m - S/m]

depends only on the geometry of the conductor

ΔV: voltage
I: current
S: cross-sectional area
l: length

$$\frac{\Delta V}{L} = \rho \frac{I}{S}$$

($\frac{\Delta V}{L}$: E; $\frac{I}{S}$: J)

- For a simple R-circuit the electrical field describes the variation of the electric potential by the distance l
- J is the current density, that is the amount of charge per unit time (current intensity I) that flows through a unit area of a chosen cross section S

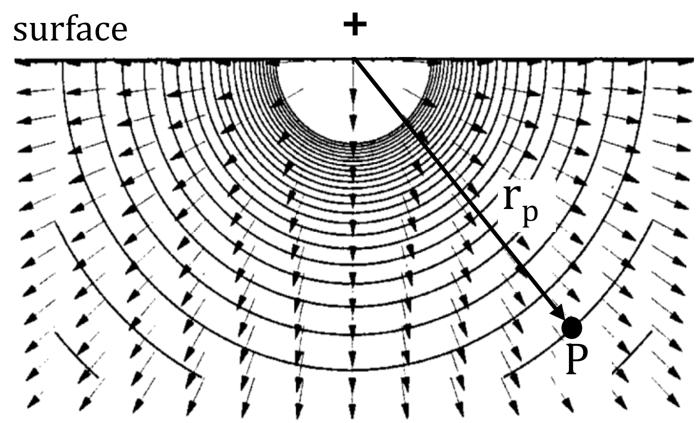
$$\mathbf{E} = \rho \mathbf{J}$$ or $$\mathbf{J} = \sigma \mathbf{E}$$

Ohm's law in differential form

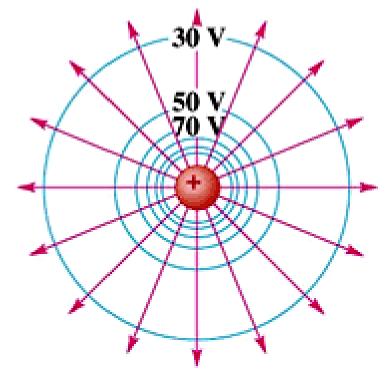
Electric field and electric potential – Single charge

- A positive point charge placed on the surface of a **homogeneous medium** induces a radially-directed current flow
- The voltage drop ΔV_P^+ will follow equipotential surfaces (hemispheres)
- The length of the conductor is the radial distance r_P while the surface to be considered is the equipotential surface (hemisphere) $2\pi {r_P}^2$

CROSS-SECTION



TOP VIEW



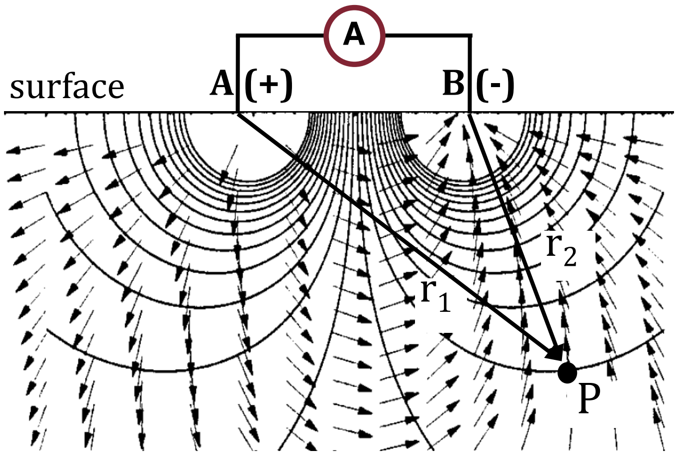
Potential drop as a function of resistivity:

$$\Delta V_P^+ = IR = I\left(\rho \frac{l}{S}\right) = I\left(\rho \frac{r_P}{2\pi {r_P}^2}\right) = I\left(\rho \frac{1}{2\pi r_P}\right) \Rightarrow \rho = 2\pi r_P \frac{\Delta V_P^+}{I}$$

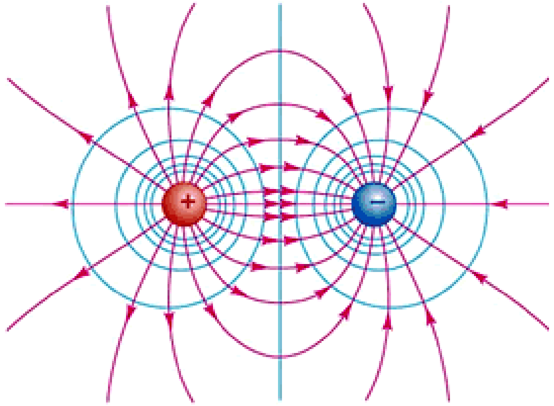
Electric field and electric potential - Dipole

- Inserting a negative charge at a certain distance from the positive one (A - positive and B - negative), thus forming the the so-called **dipole**, we can measure the current intensity through an Ammeter
- The potential drop at point P is the summation of effects due to both charges

CROSS-SECTION



TOP VIEW



field is inward directed in B

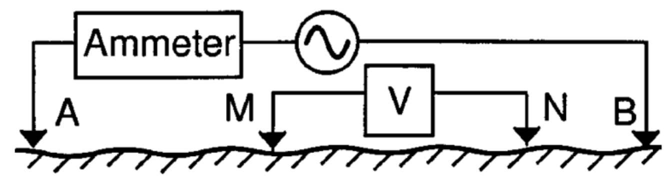
$$\Delta V_P^{(+,-)} = \Delta V_P^{+} + \Delta V_P^{-} = \frac{I\rho}{2\pi r_1} - \frac{I\rho}{2\pi r_2} = \frac{I\rho}{2\pi}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$\rho = 2\pi\left(\frac{1}{r_1} - \frac{1}{r_2}\right)\frac{\Delta V_P^{(+,-)}}{I}$$

Electric field and electric potential – Quadrupole

However, the electric **potential is generally measured by a couple of electrodes on surface** close to the source points, even though also borehole measurements can be performed. This particular geometry leads to a **quadrupole configuration** (A-B-M-N)

A and B: current electrodes
M and N: voltage electrodes



We measure the **voltage difference** ΔV_{MN} due to a dipolar source (A-positive, B-negative poles). Practically we only need to apply two times the previous formulation:

$$\Delta V_M^{(+,-)} = \frac{I\rho}{2\pi}\left(\frac{1}{\overline{AM}} - \frac{1}{\overline{MB}}\right)$$

$$\Delta V_N^{(+,-)} = \frac{I\rho}{2\pi}\left(\frac{1}{\overline{AN}} - \frac{1}{\overline{NB}}\right)$$

$$\Delta V_{MN} = \Delta V_M^{(+,-)} - \Delta V_N^{(+,-)} = \frac{I\rho}{2\pi}\left(\frac{1}{\overline{AM}} - \frac{1}{\overline{MB}} - \frac{1}{\overline{AN}} + \frac{1}{\overline{NB}}\right)$$

$$\rho = 2\pi\left(\frac{1}{\overline{AM}} - \frac{1}{\overline{MB}} - \frac{1}{\overline{AN}} + \frac{1}{\overline{NB}}\right)^{-1} \cdot \frac{\Delta V_M^{(+,-)}}{I}$$

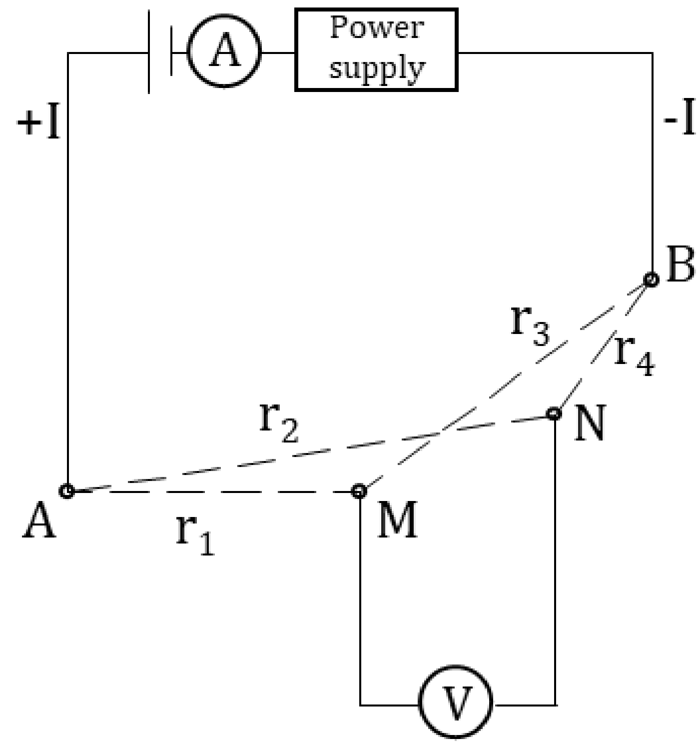
Resistivity of homogeneous ground

$$\rho = \frac{2\pi}{\left(\frac{1}{r_1} - \frac{1}{r_2} - \frac{1}{r_3} + \frac{1}{r_4}\right)} \cdot \frac{\Delta V_{MN}}{I_{AB}}$$

geometric factor *K*

$$\rho = K \frac{\Delta V_{MN}}{I}$$

Resistivity of homogenous ground



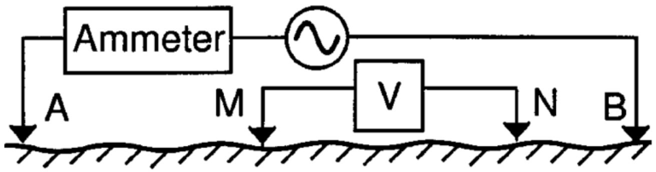
Do you remember the Ohm's law?
It is the same formulation

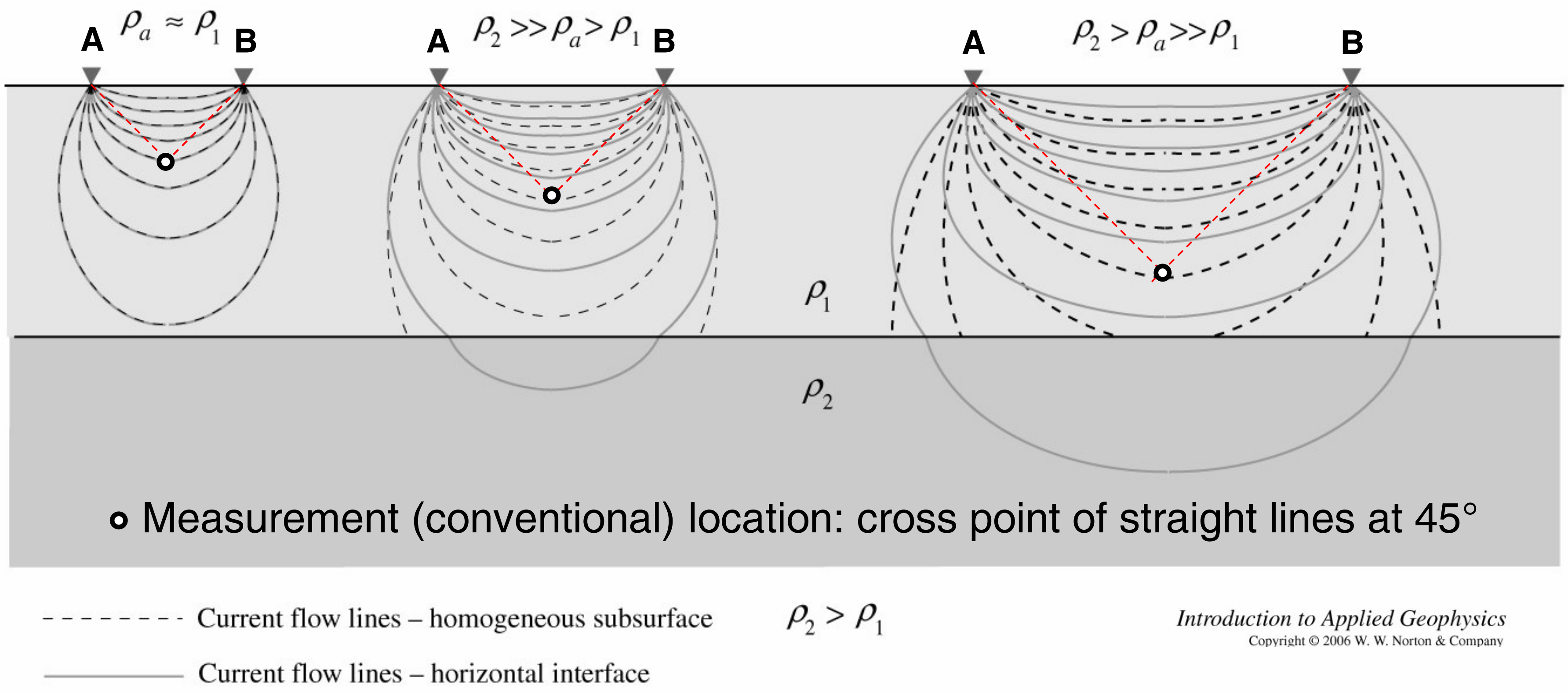
Q. What does it happen for a layered ground?
A. We cannot measure the true resistivity but only an "apparent" value"

Apparent resistivity - 1D case

$$\rho_a^{OBS} = K \frac{\Delta V_{MN}}{I_{AB}}$$

Apparent resistivity

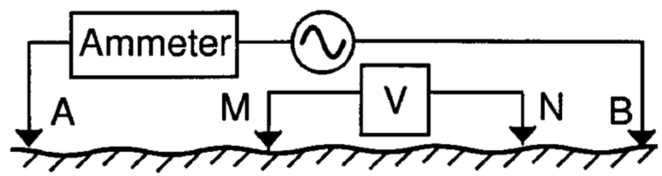




Apparent resistivity - 1D case

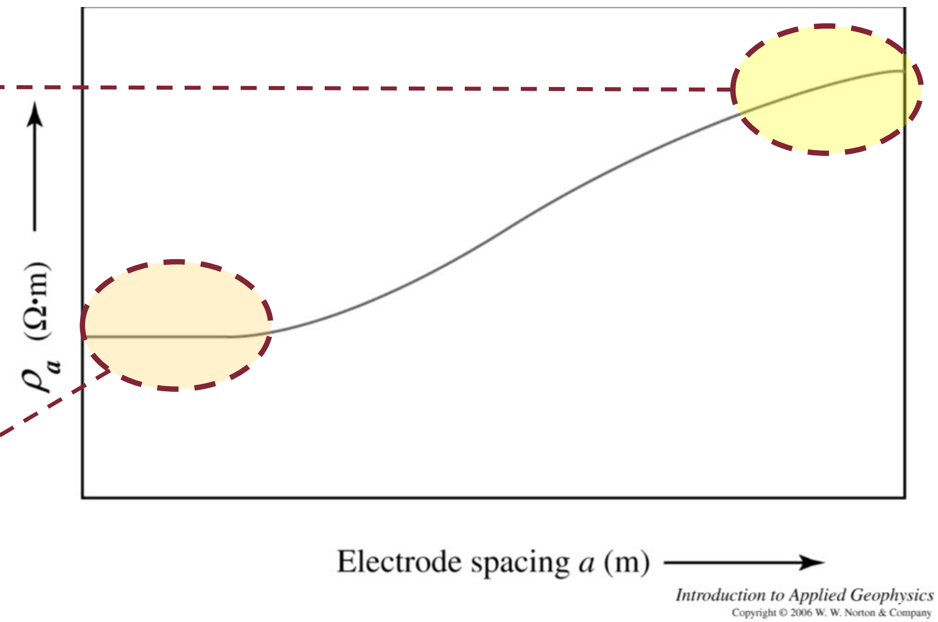
$$\rho_a^{OBS} = K \frac{\Delta V_{MN}}{I}$$

Apparent resistivity

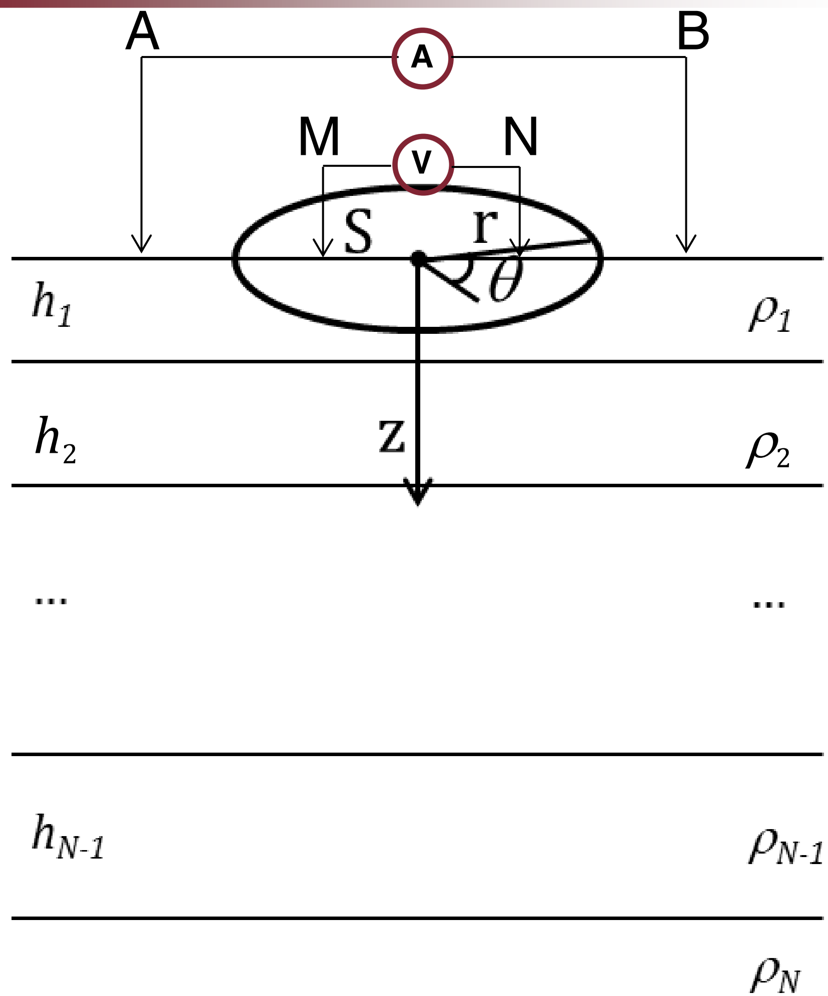


Electrode spacing **too large** to "feel" the shallow conductive layer ρ_1

Electrode spacing **too small** to "feel" the deep resistive layer ρ_2



Apparent resistivity - 1D case



The 1-D method is called
Vertical Electrical Sounding (VES)

Therefore, **we cannot measure directly on surface the "true" resistivity** of a "real" medium.
In case of inhomogeneities (always in practical cases) this value is only a "mean" resistivity value called **apparent resistivity** ρ_a

$$\rho_a{}^{OBS} = K \frac{\Delta V_{MN}{}^{OBS}}{I}$$

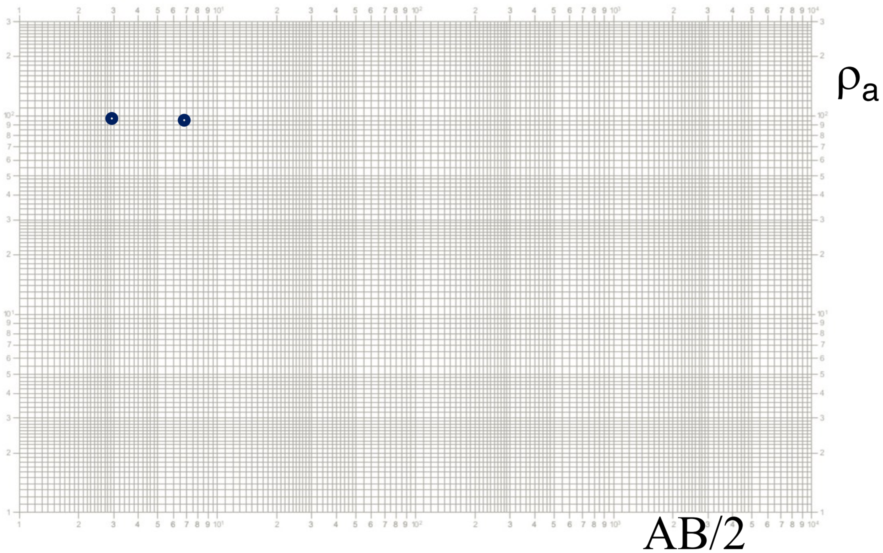
Observed apparent resistivity

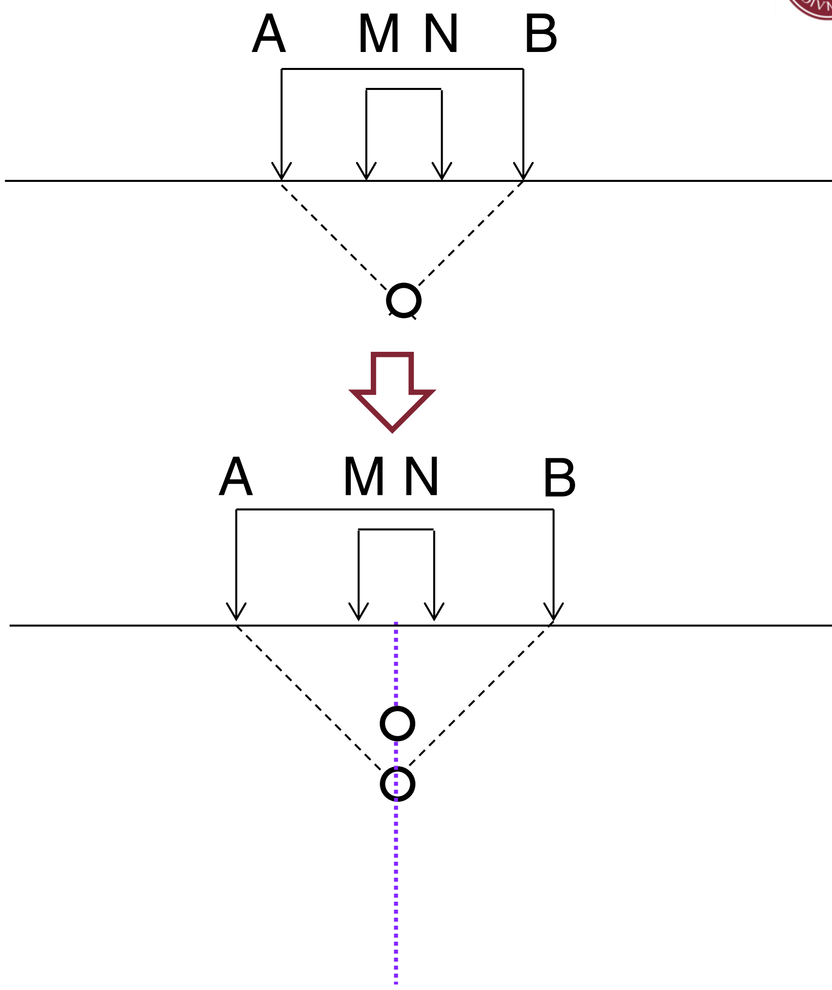
Since we cannot measure directly the "true" resistivity of a medium, the correct approach to retrieve a resistivity model is to perform DATA INVERSION

Data inversion – 1D VES case

OBSERVED DATASET

N.	AB/2	MN/2	ΔV (V)	I (mA)	σ(%)	ρ_a(Ωm)
1	1	0.5	0.679910	101.45	1.20	100
2	2	0.5	0.266054	101.56	0.08	99





Data inversion – 1D VES case

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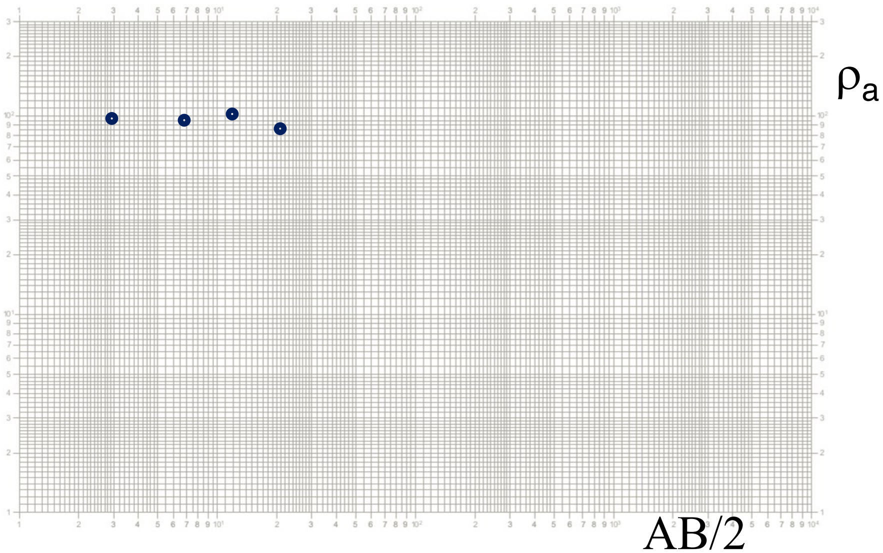
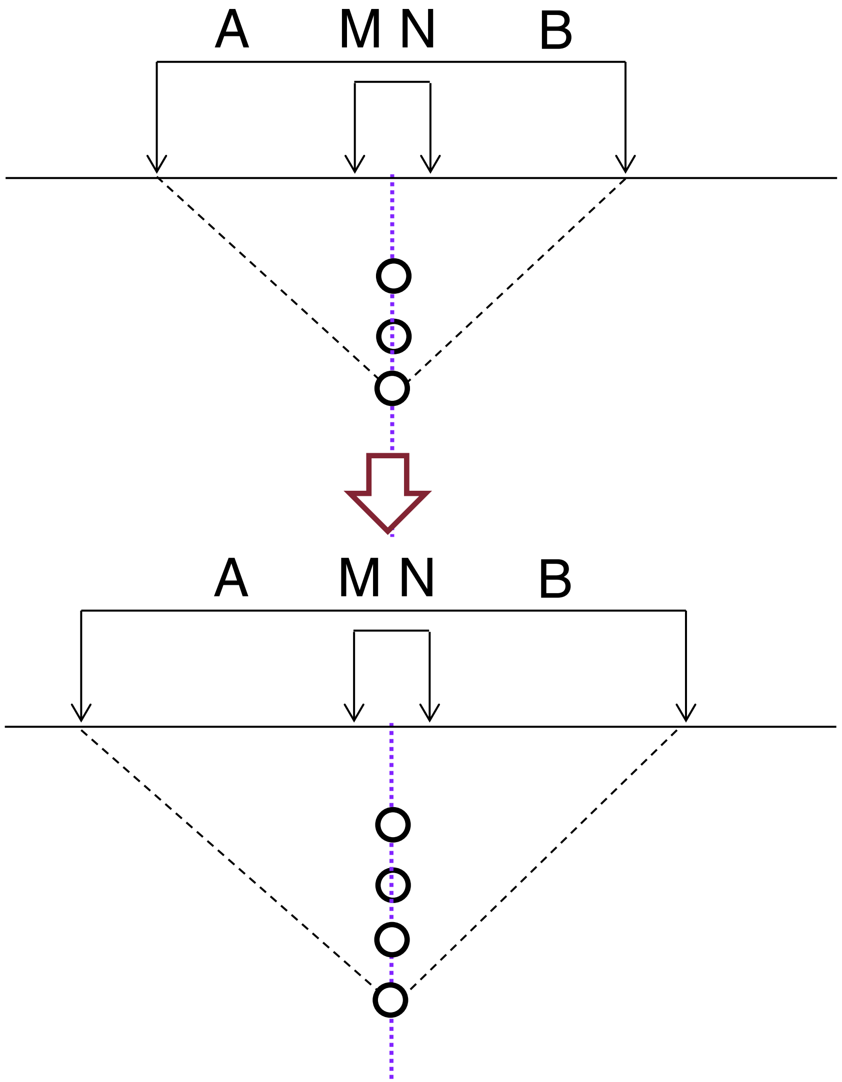
3	4	0.5	0.121435	101.43	2.02	102
4	8	0.5	0.172654	125.67	1.06	95





and so on…

Data inversion – 1D VES case

OBSERVED DATASET

N.	AB/2	MN/2	ΔV (V)	I (mA)	σ(%)	ρ_a(Ωm)
1	1	0.5	0.679910	101.45	1.20	100
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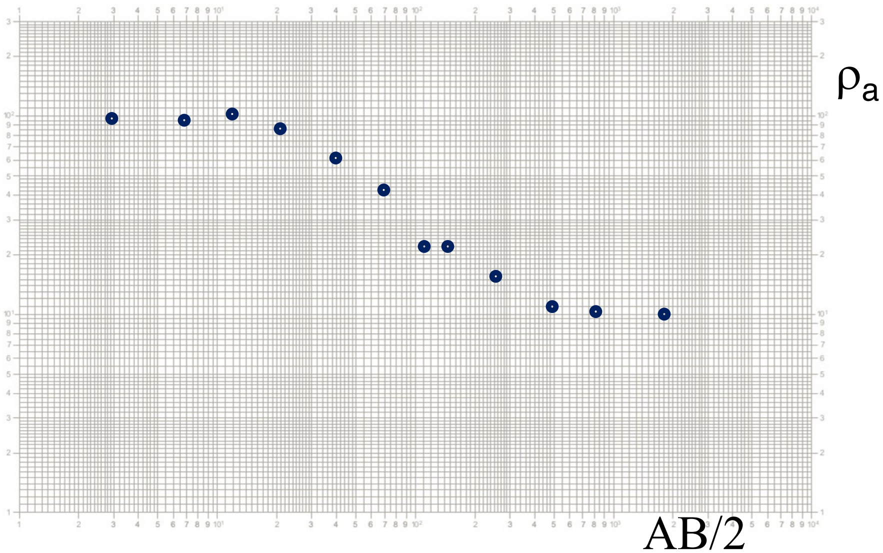
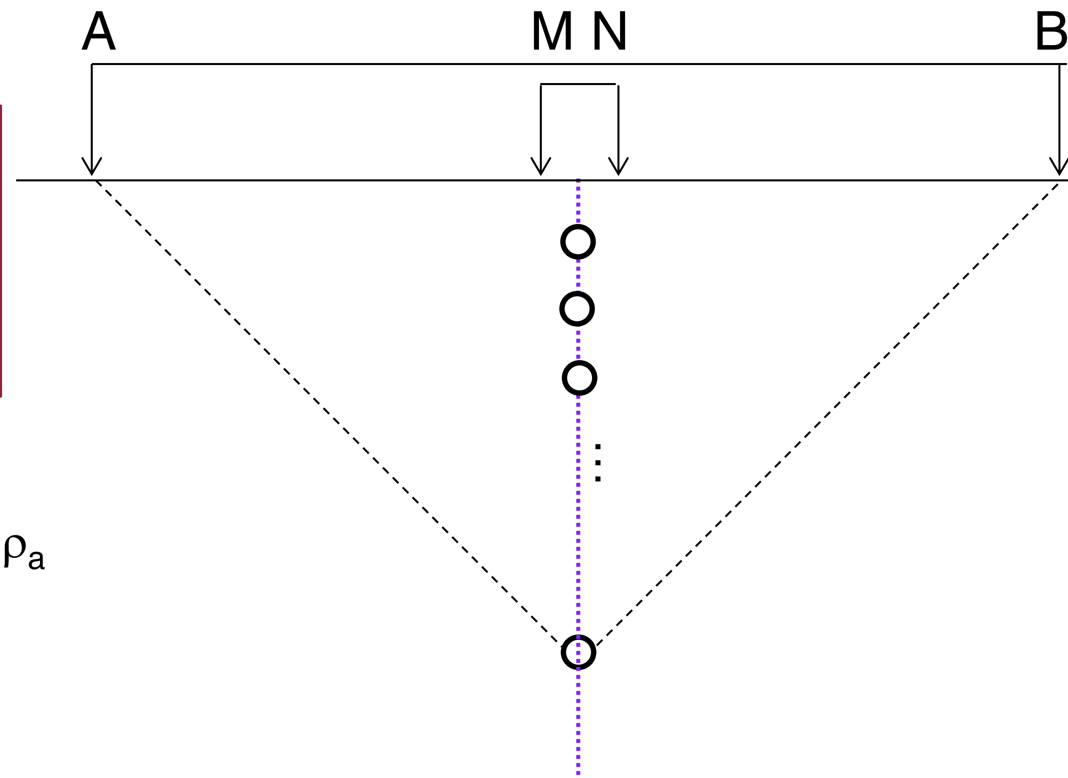
4	8	0.5	0.172654	125.67	1.06	95
…						
12	200	0.5	0.545678	105.65	1.87	10





OBSERVED DATASET

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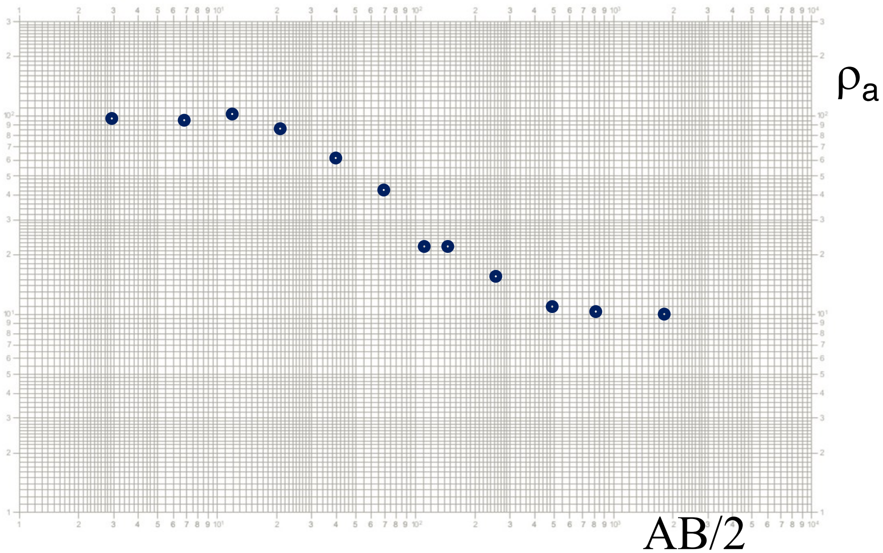
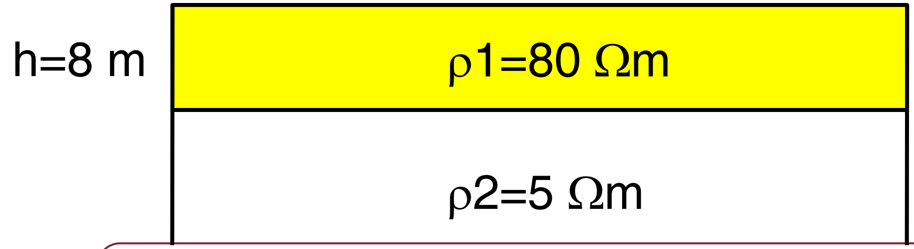
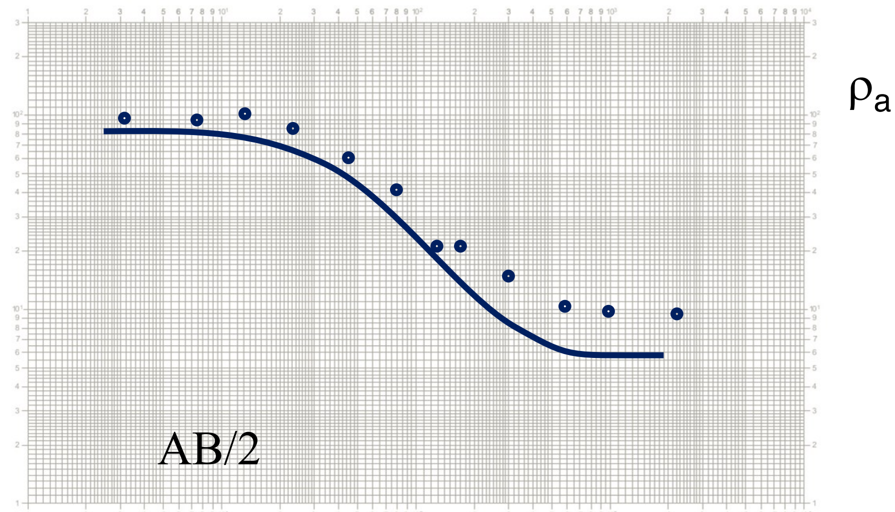


First trial model



PREDICTED DATASET
Solving the electrostatic field equation



TRY IF CALCULATED DATA FIT THE OBSERVED ONE

Data inversion – 1D VES case

OBSERVED DATASET

N.	AB/2	MN/2	ΔV (V)	I (mA)	σ(%)	ρ_a(Ωm)
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4	8	0.5	0.172654	125.67	1.06	95
…						
12	200	0.5	0.545678	105.65	1.87	10

Final model

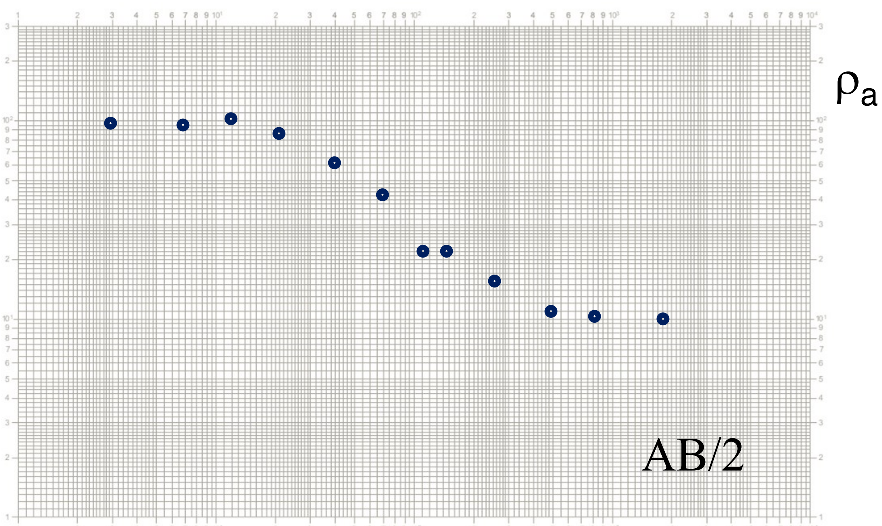
h=10 m

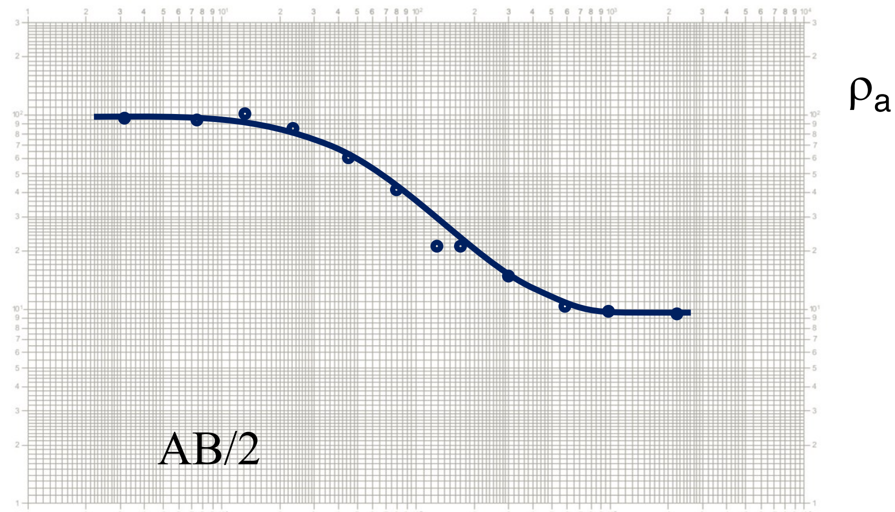
ρ1= 100 Ωm

ρ2= 10 Ωm

PREDICTED DATASET

Solving the electrostatic field equation





If the fit is satisfying, the trial model is the final resistivity model

Otherwise, we have to try an other model and repeat the procedure

Electrostatic field equation – Real media

Q. How we can solve the electrostatic field equation for inhomogneous media?

$$\begin{cases} \mathbf{E} = -\nabla V \\ \nabla \times \mathbf{H}(\mathbf{r},t) = \mathbf{J_T}(\mathbf{r}) + \cancel{\dfrac{\partial \mathbf{D}(\mathbf{r},t)}{\partial t}} \\ \mathbf{J_T} = \mathbf{J_C} + \mathbf{J_{ext}} = \sigma(\mathbf{r})\mathbf{E} + \mathbf{J_{ext}} \end{cases}$$

Electrostatic potential from Faraday's Law

Ampère-Maxwell's Law

Constitutive equation (CE) (generalized Ohm's law)

$\mathbf{D}$	electric displacement
$\mathbf{H}$	magnetic field
$\mathbf{J_T}$	total current density
$\mathbf{J_C}$	conduction current density
$\mathbf{J_{ext}}$	external current density (external source)
$\mathbf{r}(x,y,z)$	position
t	time

2° assumption: no capacitive effects (polarization) - pure resistive response

1. Applying the divergence to Ampère's Law:

$$\cancel{\nabla \cdot (\nabla \times \mathbf{H})} = \nabla \cdot \mathbf{J_T} \Rightarrow \boxed{\nabla \cdot \mathbf{J_T} = \mathbf{J_C} + \mathbf{J_{ext}} = 0}$$

Continuity equation

2. Substituting Ohm's law:

$$\nabla \cdot (\sigma \mathbf{E} + \mathbf{J}^{\mathbf{ext}}) = 0$$

3. Substituting the electrostatic potential:

$$\boxed{-\nabla \cdot (\sigma \nabla V) + \nabla \cdot \mathbf{J}^{\mathbf{ext}} = 0}$$

Electrostatic field equation – real media

Q. How can I express the external source term? ⇨ A. Please see this box

$$-\nabla \cdot (\sigma \nabla V) + \nabla \cdot \mathbf{J}^{\mathbf{ext}} = 0$$

$$-\nabla \cdot (\sigma \nabla V) - I\delta(\mathbf{r}) = 0$$

$$-\nabla \cdot (\sigma(\mathbf{r}) \nabla V(\mathbf{r})) = I\delta(\mathbf{r})$$

Electrostatic field equation

Differential equation describing the propagation of electric field within a non-homogenous half-space

For homogeneous ground:

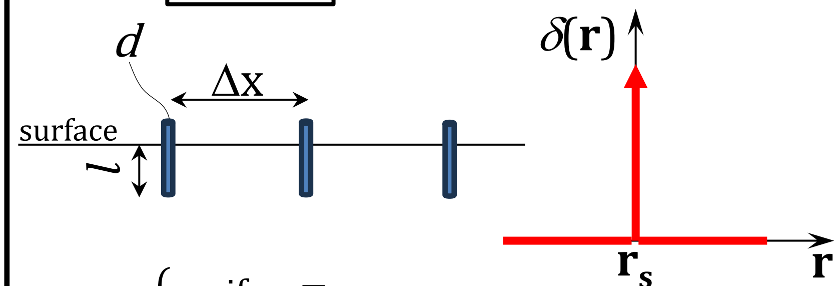
$$-\sigma \nabla \cdot \nabla V(r) = I\delta(r) \Rightarrow V(r) = \frac{\rho I}{2\pi r}$$

3°assumption: point source

Dirac's delta function $\delta(\mathbf{r})$

$\Delta x \gg l$
$\Delta x \gg d$

d: electrode diameter
l: electrode grounded length
Δx: electrode spacing



$$\delta(\mathbf{r}) = \begin{cases} \infty & \text{if } \mathbf{r} \equiv \mathbf{r_s} \\ 0 & \text{elsewhere} \end{cases}$$

$\mathbf{r_s} \equiv (x_s, y_s, z_s)$: source position

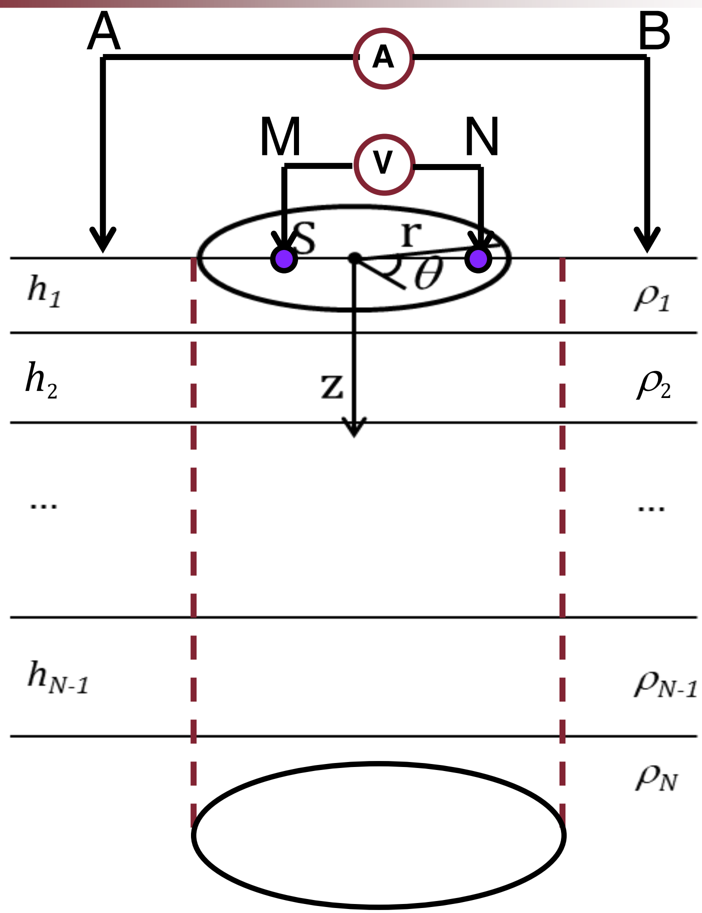
$$\nabla \cdot \mathbf{J}^{\mathbf{ext}} = \lim_{V \to 0} \frac{\int_S \mathbf{J}^{\mathbf{ext}} \cdot \mathbf{n}\, dS}{V} = -I\delta(\mathbf{r})$$

The minus sign indicates an inflow current opposite to the outflow conventionally expressed by the divergence

Q. And for inhomogeneous ground?

A. There is not a closed-form solution (analytical solution) for this equation, except for the 1-D case

1-D model: Vertical Electrical Soundings (VES)



Assumption: 1-D model

$$\rho(x,y,z) \rightarrow \rho(z)$$
$$\sigma(x,y,z) \rightarrow \sigma(z)$$

Cylindrical coordinates

$$-\nabla \cdot (\sigma(z)\nabla V(r,z)) = I\delta(r_S)\delta(z_S)$$

SOLVING FOR *V*

$$V(r,z)$$

I can **predict the potential function *V*** due to an arbitrary current *I* injected in A and B **everywhere in the space**.

Among all the (infinite) points, I need *V* only at the points M and N corresponding to the observation points (voltage electrodes) and I make the potential difference: $\Delta V_{MN} = V_M - V_N$

$$\rho_a^{PRE} = K\frac{\Delta V_{MN}{}^{PRE}}{I}$$

PREDICTED APPARENT RESISTIVITY

We compare predictions with observations and if the fitting is not satisfactory, we change iteratively the conducitivty model until finding the best fitting (SEE INVERSION)