SAPIENZA
UNIVERSITÀ DI ROMA

Environmental geophysics

Giorgio De Donno

3 . RADAR (high-frequency EM) methods

Basic principles

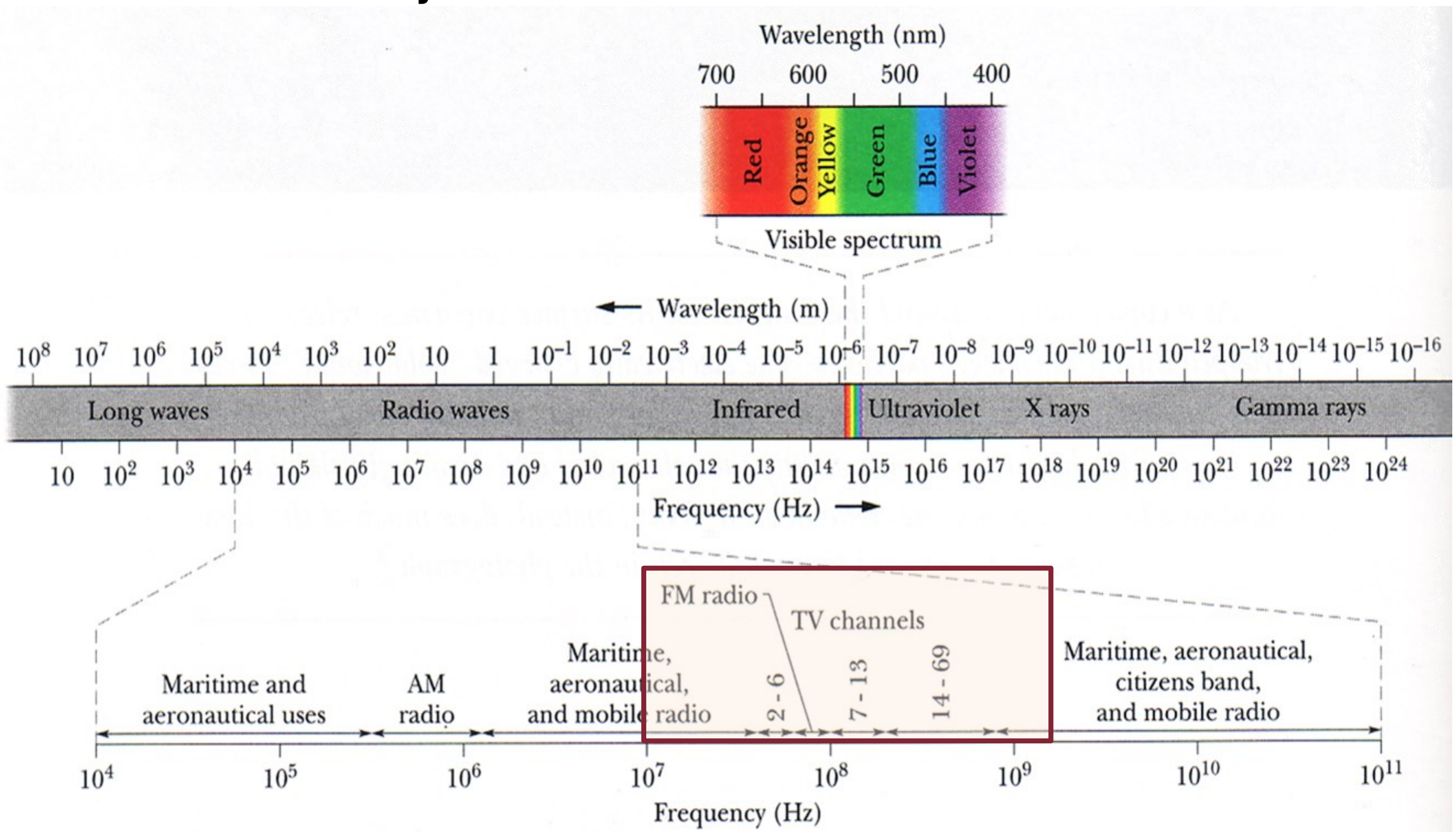
"Sapienza" University of Rome* - *DICEA Area Geofisica

phone: +39 06 44 585 078
email: giorgio.dedonno@uniroma1.it

Electromagnetic waves

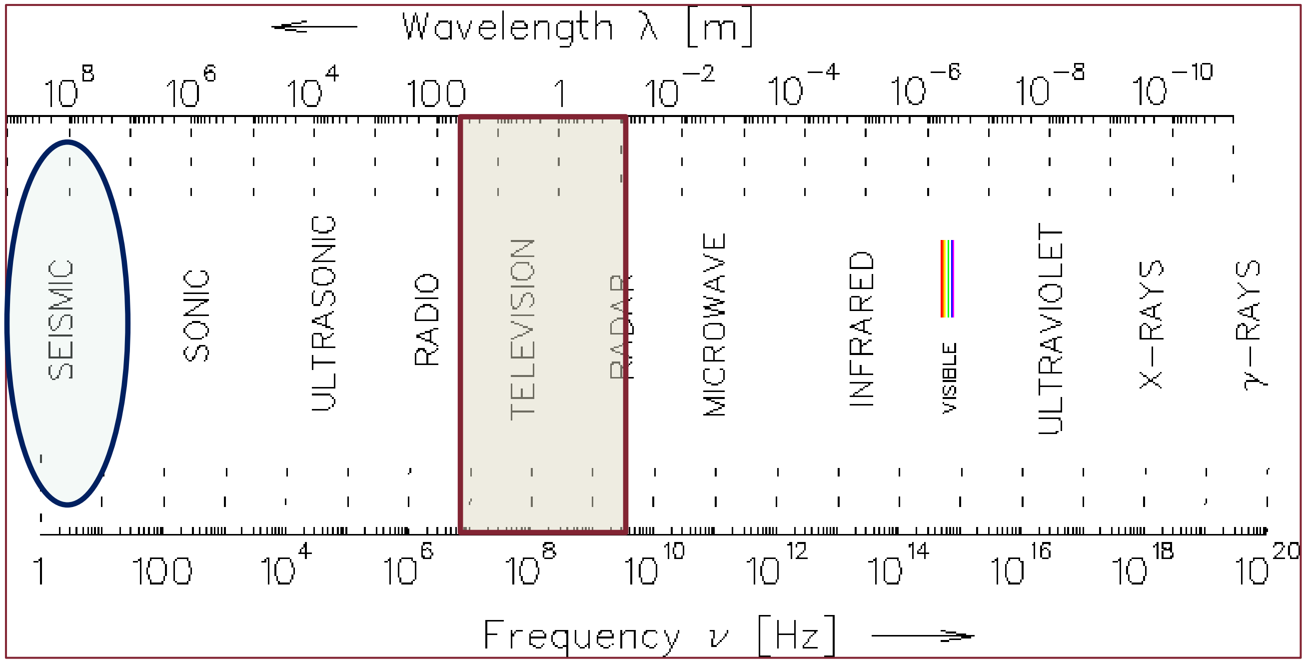
Electromagnetic (EM) waves are produced by the **motion of electrically charged particles**. These waves radiate from the electrically charged particles, travelling through free space as well as through air and other media.

We normally encounter a lot of EM fields in our lives



Electromagnetic waves

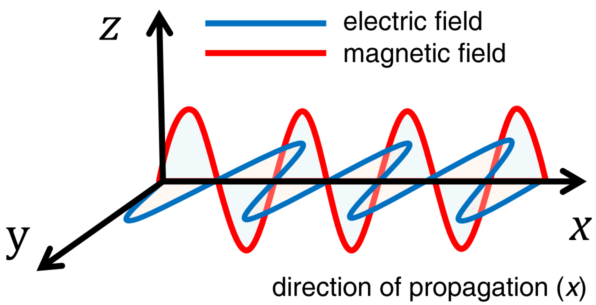
Electromagnetic (EM) waves are produced by the **motion of electrically charged particles**. These waves radiate from the electrically charged particles, travelling through free space as well as through air and other media.

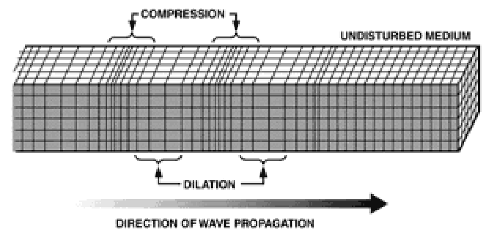


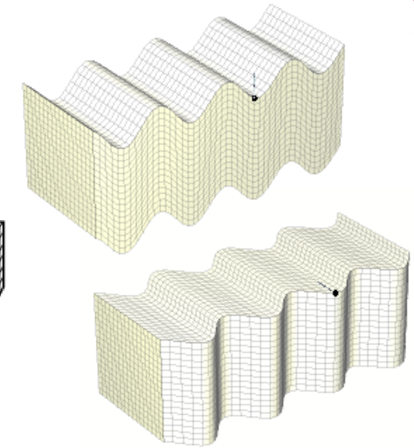
For geophysical purposes we deal with radio and radar frequency bandwidths:

Wave frequency *f*: 10^7-$3 \cdot 10^9$ Hz
Wavelength λ: 20-0.05 m

Electromagnetic vs. seismic waves







Electromagnetic Waves

- Microwaves / Radio Waves
- Velocity in air ≈ 3×10^8 m/s, but slower for lithotypes (0.02 – 0.2 m/ns – 2-20 cm/ns)
  - depends on **electromagnetic properties**
- Frequency ≈ 10-3000 MHz
  - depends on **antenna type**
- Wavelength ≈ 0.05- 20 m

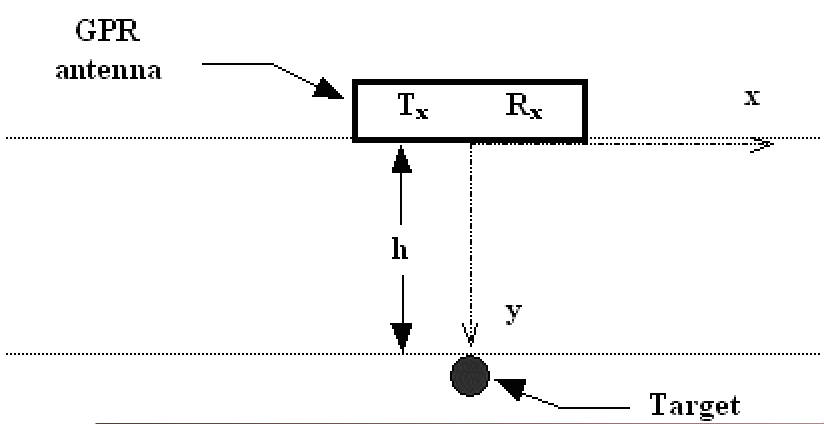
Seismic Waves

- Body and surface waves
- P-wave velocity in air ≈ 330 m/s, but higher for lithotypes (400-5000 m/s)
  - depends on **elastic properties**
- Frequency ≈ 10-500 Hz
  - depends on **source type**
- Wavelength ≈ 0.5-500 m

Ground Penetrating Radar (GPR)

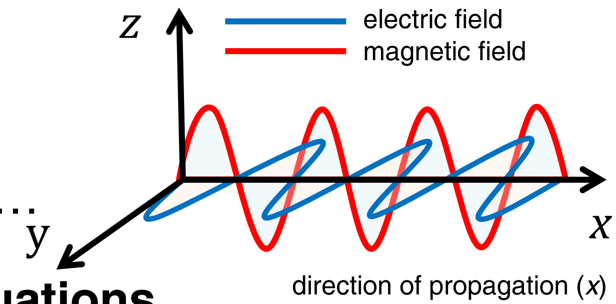
The most part of the EM fields radiate in air, but for geophysical purposes we generally needs to investigate the subsurface.
The idea is to establish an EM field throughout the Earth using wavelengths that are reasonably applicable for detection of hidden anomalies (few cm - few m)

Ground Penetrating Radar or georadar (GPR)



Why the EM field can be se as a wave propagation phenomenon?

Let's start from Maxwell's and Constitutive equations…



Maxwell's equations

$$\begin{cases} \nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} = \mathbf{J} + \dfrac{\partial \mathbf{D}}{\partial t} \\ \nabla \cdot \mathbf{D} = \rho_v \\ \nabla \cdot \mathbf{B} = 0 \end{cases}$$

Faraday's Law

Ampère-Maxwell's Law

Gauss's Laws (electric/magnetic)

Constitutive equations

$$\begin{cases} \mathbf{J} = \sigma \mathbf{E} \\ \mathbf{D} = \varepsilon \mathbf{E} = \varepsilon_0 \varepsilon_r \mathbf{E} \\ \mathbf{B} = \mu \mathbf{H} = \mu_0 \mu_r \mathbf{H} \end{cases}$$

E: electric field
H: magnetic field
D: electric displacement
B: magnetic induction
J: current density
ρ_v: volumetric charge density
σ: electrical conductivity
ε: electrical permittivity
μ: magnetic permeability

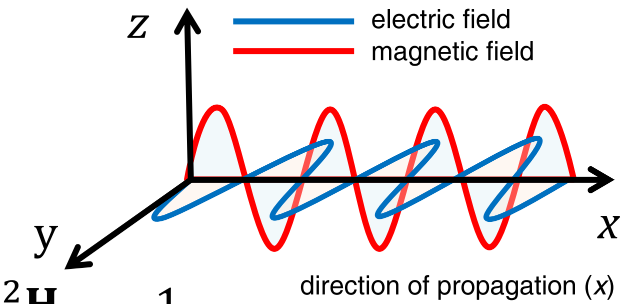
1. Propagation in free space

$$\begin{cases} \sigma = 0 \\ \rho_v = 0 \\ \varepsilon = \varepsilon_0 \\ \mu = \mu_0 \end{cases} \Rightarrow \begin{cases} \boxed{\nabla \times \mathbf{E} = -\mu_0 \dfrac{\partial \mathbf{H}}{\partial t}} \\ \boxed{\nabla \times \mathbf{H} = \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}} \\ \nabla \cdot \mathbf{E} = 0 \\ \nabla \cdot \mathbf{H} = 0 \end{cases}$$

We apply the curl to the Ampère-Maxwell's Law:

$$\boxed{\nabla \times (\nabla \times \mathbf{H}) = \varepsilon_0 \frac{\partial}{\partial t}}\boxed{(\nabla \times \mathbf{E})} = -\varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

1. Propagation in free space



$$\nabla \times (\nabla \times \mathbf{H}) = -\varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{H}}{\partial t^2}$$

Gauss's Law

$$\nabla(\cancel{\nabla \cdot \mathbf{H}}) - \nabla^2 \mathbf{H} = -\varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{H}}{\partial t^2} \rightarrow \nabla^2 \mathbf{H} = \varepsilon_0 \mu_0 \frac{\partial^2 \mathbf{H}}{\partial t^2} \rightarrow \frac{\partial^2 \mathbf{H}}{\partial t^2} = \frac{1}{\varepsilon_0 \mu_0} \nabla^2 \mathbf{H}$$

Similar equation can be achieved for **E** starting from the Faraday's Law...
Therefore, for an EM-wave travelling along the x-direction we have:

$$\begin{cases} \dfrac{\partial^2 \mathbf{E}(x,t)}{\partial t^2} = \dfrac{1}{\varepsilon_0 \mu_0} \nabla^2 \mathbf{E}(x,t) \\ \dfrac{\partial^2 \mathbf{H}(x,t)}{\partial t^2} = \dfrac{1}{\varepsilon_0 \mu_0} \nabla^2 \mathbf{H}(x,t) \end{cases} \Rightarrow \frac{\partial^2 \mathbf{f}}{\partial t^2} = v^2 \nabla^2 \mathbf{f} \Rightarrow \boxed{v_{EM}^{freespace} = \sqrt{\frac{1}{\varepsilon_0 \mu_0}} = c}$$

c: speed of light $\cong 3 \cdot 10^8 \mathrm{m/s}$
$c \cong 0.3\ \mathrm{m/ns} = 30\ \mathrm{cm/ns}$

Solution for a plane EM-wave propagating along the *x*-direction:

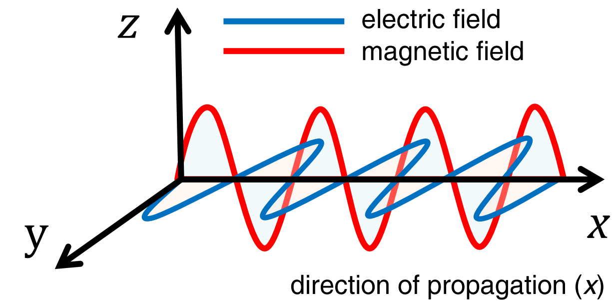
$$f(x,t) = A\mathrm{e}^{i(\omega t \pm kx)} \left\{ \boxed{\begin{array}{l} E_y(x,t) = E_0 \mathrm{e}^{i(\omega t \pm kx)} \\ H_z(x,t) = H_0 \mathrm{e}^{i(\omega t \pm kx)} \end{array}} \right.$$

NO INTRINSIC ATTENUATION (as per the elastic media)

2. Propagation through the air:

$$\begin{cases} \sigma = 0 \\ \rho_v = 0 \\ \varepsilon = \varepsilon_0 \varepsilon_r \\ \mu = \mu_0 \end{cases}$$

$$\boxed{v_{EM}^{air} = \sqrt{\frac{1}{\mu_0 \varepsilon_0 \varepsilon_r}} \sqrt{\frac{1}{\varepsilon_r}} = \frac{c}{\sqrt{\varepsilon_r}}}$$



$$\begin{array}{l} v_{EM}^{freespace} = c \\ v_{EM}^{air} \approx c \end{array}$$

NO INTRINSIC ATTENUATION

3. Propagation in the ground (hp. $\mu \approx \mu_0$ non-magnetic materials):

$$\begin{cases} \sigma > 0 \\ \rho_v = 0 \\ \varepsilon = \varepsilon_0 \varepsilon_r \\ \mu = \mu_0 \end{cases}$$

In this case the behaviour of the EM field can be diffusive or oscillatory as a function of the ratio $\sigma/_{\omega\varepsilon}$

$$\boxed{\frac{\sigma}{\omega\varepsilon} \gg 1}$$

strong attenuation
EM field diffusive
High losses

$$\boxed{\frac{\sigma}{\omega\varepsilon} \ll 1}$$

poor attenuation
EM field oscillatory
Low losses

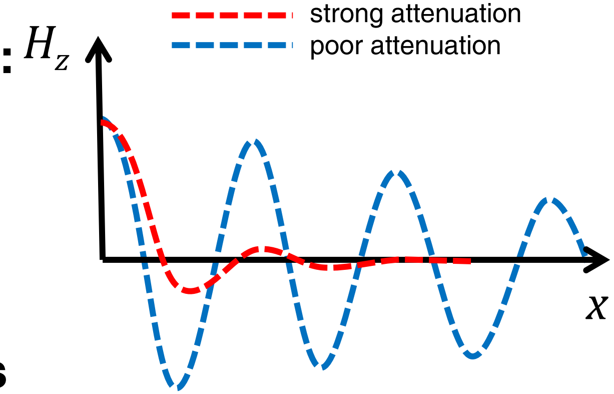
FOR BOTH CASES THERE WILL BE INTRINSIC ATTENUATION

Conductive media attenuate EM waves

Electromagnetic waves - Attenuation

As for seismic waves, EM waves lose energy due to:

- **geometrical spreading**: (see lecture 1.1)
- **intrinsic attenuation:** energy loss due to heating (ohmic losses)



3. Propagation in the ground ($\sigma > 0$), along the *x*-axis

$$\begin{cases} E_y(x,t) = E_0 \mathrm{e}^{-\gamma x} \mathrm{e}^{i(\omega t \pm kx)} \\ H_z(x,t) = H_0 \mathrm{e}^{-\gamma x} \mathrm{e}^{i(\omega t \pm kx)} \end{cases}$$

γ: attenuation coefficient [dB/m]

If $\boxed{\dfrac{\sigma}{\omega\varepsilon} \ll 1}$

Low-losses assumption

$$\boxed{v_{EM}^{ground} = \frac{c}{\sqrt{\varepsilon_r}}}$$

[m/s] or [m/ns]or [cm/ns]

$$\boxed{\gamma_{EM}^{ground} = \frac{\sigma}{2}\sqrt{\frac{\mu}{\varepsilon}} = \frac{\sigma}{2}\sqrt{\frac{\mu_0}{\varepsilon_0 \varepsilon_r}}}$$

$$\boxed{\gamma \approx 1690 \frac{\sigma}{\sqrt{\varepsilon_r}}}$$

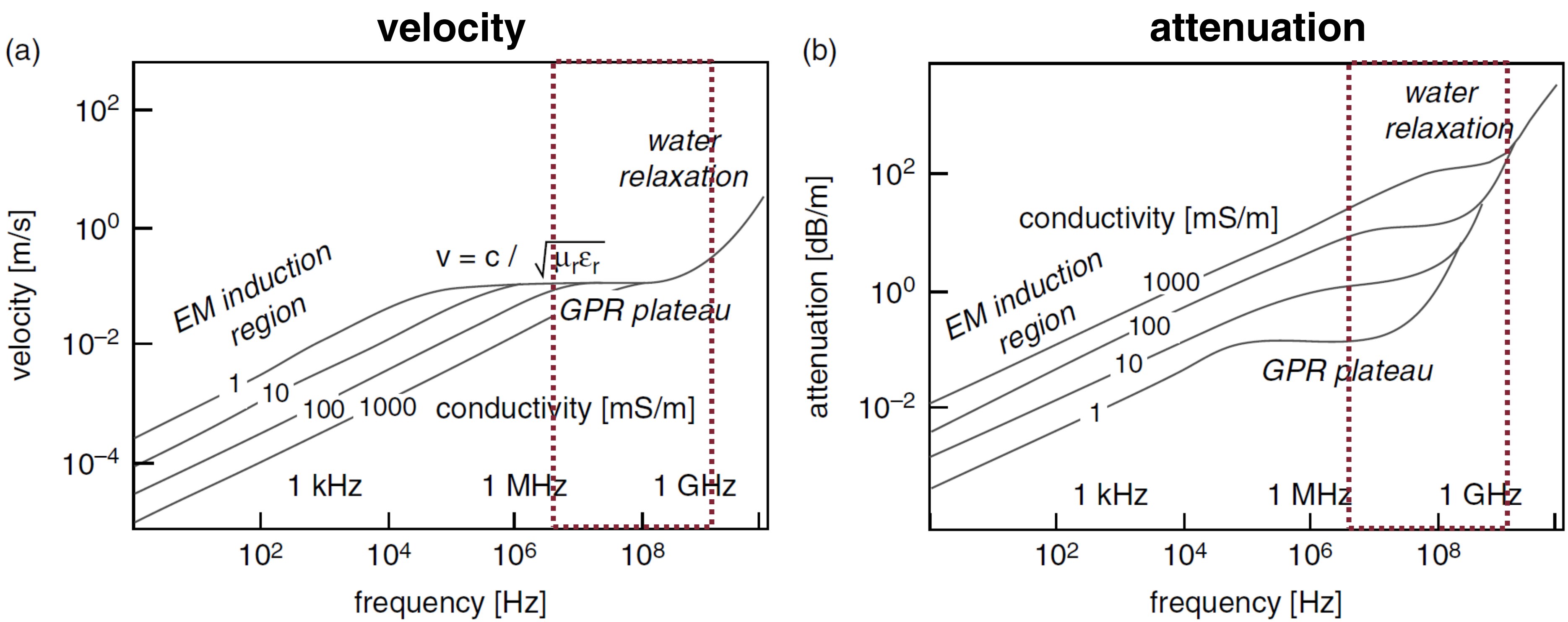
[dB/m]

Electromagnetic waves - Attenuation

- **intrinsic attenuation:** energy loss due to heating (ohmic losses)

Differently from the assumption of elastic media in seismic methods, the EM wave attenuation (intrinsic) cannot be neglected for a signal propagating in the subsurface:

3. Propagation in the ground ($\sigma > 0$), along the *x*-axis



Low-loss assumption

When the low-loss assumption is valid? $\boxed{\dfrac{\sigma}{\omega\varepsilon} \ll 1}$

Ex.1 **clay** investigated with GPR at f =300 MHz=$3 \cdot 10^8$ Hz → ω =$1.88 \cdot 10^9$ Hz, with:
σ= 0.2 S/m corresponding to ρ = 5 Ωm
ε_r= 16
$\varepsilon = \varepsilon_0 \varepsilon_r \approx 1.41 \cdot 10^{-10}$

$$\Rightarrow \boxed{\frac{\sigma}{\omega\varepsilon} = \frac{0.2}{1.88 \cdot 10^9 \cdot 1.41 \cdot 10^{-10}} = 0.75}$$

Ex.2 **sand** investigated with GPR at f =300 MHz=$3 \cdot 10^8$ Hz → ω =$1.88 \cdot 10^9$ Hz, with:
σ= 0.005 S/m corresponding to ρ = 200 Ωm
ε_r= 4
$\varepsilon = \varepsilon_0 \varepsilon_r \approx 3.5 \cdot 10^{-11}$

$$\Rightarrow \boxed{\frac{\sigma}{\omega\varepsilon} = \frac{0.005}{1.88 \cdot 10^9 \cdot 3.5 \cdot 10^{-11}} = 0.076}$$

Ex.3 **ice** investigated with GPR at f =300 MHz=$3 \cdot 10^8$ Hz → ω =$1.88 \cdot 10^9$ Hz, with:
σ= $1 \cdot 10^{-4}$ S/m corresponding to ρ = 10'000 Ωm
ε_r= 3.2
$\varepsilon = \varepsilon_0 \varepsilon_r \approx 2.8 \cdot 10^{-11}$

$$\Rightarrow \boxed{\frac{\sigma}{\omega\varepsilon} = \frac{1 \cdot 10^{-4}}{1.88 \cdot 10^9 \cdot 2.8 \cdot 10^{-11}} = 0.0019}$$

Velocity and attenuation

If the low-loss assumption is valid, the values of velocity and attenuation does not depend on frequency

$$\boxed{v_{EM} \approx \frac{c}{\sqrt{\varepsilon_r}}}$$ **velocity**

$$\boxed{\gamma_{EM} \approx 1690 \frac{\sigma}{\sqrt{\varepsilon_r}}}$$ **attenuation**

Velocity is inversely proportional to dielectric constant

Attenuation is directly proportional to conductivity and inversely to dielectric constant

Ex.1 **clay** σ= 0.2 S/m ε_r= 16

$$v \approx \frac{3 \cdot 10^8}{\sqrt{16}} \cong 7.5 \text{ cm/ns}$$

$$\gamma \approx 1690 \frac{0.2}{\sqrt{16}} \cong 84.5 \text{ dB/m}$$

Ex.2 **sand** σ= 0.005 S/m ε_r= 4

$$v \approx \frac{3 \cdot 10^8}{\sqrt{4}} \cong 15 \text{ cm/ns}$$

$$\gamma \approx 1690 \frac{0.005}{\sqrt{4}} \cong 4.22 \text{ dB/m}$$

Ex.3 **ice** σ= $1 \cdot 10^{-4}$ S/m ε_r= 3.2

$$v \approx \frac{3 \cdot 10^8}{\sqrt{3.2}} \cong 16.7 \text{ cm/ns}$$

$$\gamma \approx 1690 \frac{1 \cdot 10^{-4}}{\sqrt{3.2}} \cong 0.09 \text{ dB/m}$$

DOI - Skin depth

Skin depth δ: depth where the amplitude decays to 1/e(≅37%) of its original value (100%). It is a first-approximation index of depth of investigation (DOI):

Skin depth $\boxed{\delta = \frac{1}{\gamma}}$ $\boxed{\delta \approx 5.31 \frac{\sqrt{\varepsilon_r}}{\sigma} 10^{-3}}$ for non-metallic materials

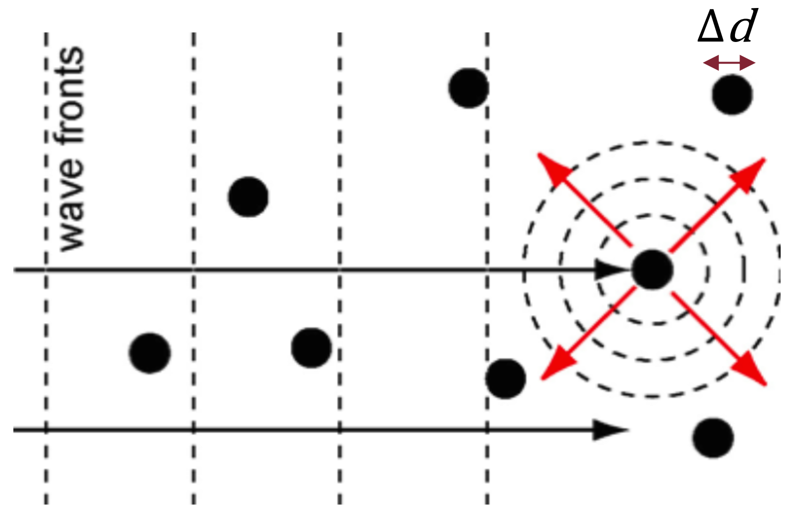
Ex.1 **clay** σ = 0.2 S/m ε_r= 16 ⇨ $\boxed{\delta \approx 5.31 \frac{\sqrt{16}}{0.2} 10^{-3} \approx 0.1 \text{ m}}$

Ex.2 **sand** σ= 0.005 S/m ε_r= 4 ⇨ $\boxed{\delta \approx 5.31 \frac{\sqrt{4}}{0.005} 10^{-3} \approx 2 \text{ m}}$

Ex.3 **ice** σ= $1{\cdot}10^{-4}$ S/m ε_r= 3.2 ⇨ $\boxed{\delta \approx 5.31 \frac{\sqrt{3.2}}{1{\cdot}10^{-4}} 10^{-3} \approx 95 \text{ m}}$

Scattering and resolution

• **scattering:** lithotypes can contain small heterogeneities (grains, mineral boundaries, pore edges, cracks, etc.) or there are antropogenic features (pipes, wells, buried bodies etc.)



In such cases some EM energy is scattered according to Huygens' principles. This phenomenon depends on the ratio of the heterogeneity size to the wavelength (it must be comparable).

For the GPR method scattering is "good" recordable signal, as for SONAR methods, because the wavelength λ is comparable to the dimension d of the target (inclusion) to be detected

$$\boxed{\lambda \sim \Delta d}$$

Resolution

Resolution

$$\Delta d \geq \frac{\lambda}{4} = \frac{v}{4f_c}$$

For low-loss media

$$\Delta d \geq \frac{c}{4f_c\sqrt{\varepsilon_r}}$$

***f* = 300 MHz = 3·10⁸ Hz**

Ex.1 **clay** σ = 0.2 S/m ε_r= 16

$$\Delta d \geq \frac{3\cdot 10^8}{4\cdot 3\cdot 10^8\cdot\sqrt{16}} = \frac{1}{16} \approx 0.06 \text{ m} \approx 6 \text{ cm}$$

Ex.2 **sand** σ= 0.005 S/m ε_r= 4

$$\Delta r \geq \frac{1}{8} \approx 0.12 \text{ m} \approx 12 \text{ cm}$$

Ex.3 **ice** σ= $1\cdot 10^{-4}$ S/m ε_r= 3.2

$$\Delta r \geq \frac{1}{7.15} \approx 0.12 \text{ m} \approx 14 \text{ cm}$$

***f* = 3000 MHz = 3 GHz = 3·10⁹ Hz**

Ex.1 **clay** σ = 0.2 S/m ε_r= 16

$$\Delta d \geq \frac{3\cdot 10^8}{4\cdot 3\cdot 10^9\cdot\sqrt{16}} = \frac{1}{160} \approx 0.006 \text{ m} \approx 6 \text{ mm}$$

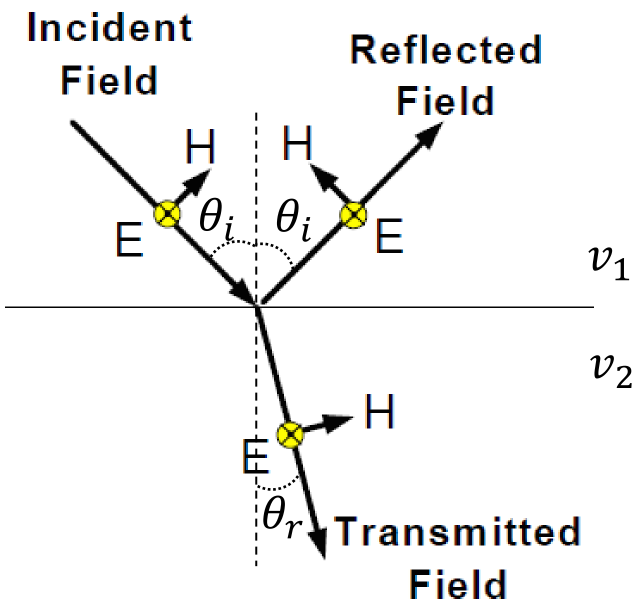
Ex.2 **sand** σ= 0.005 S/m ε_r= 4

$$\Delta r \geq \frac{1}{80} \approx 1.2 \text{ cm}$$

Ex.3 **ice** σ= $1\cdot 10^{-4}$ S/m ε_r= 3.2

$$\Delta r \geq \frac{1}{71.5} \approx 1.4 \text{ cm}$$

When an EM-wave impacts on an interface, there will be reflected and transmitted (refracted) rays according to the **Snell's Law, as for seismic waves.**



Snell's Law

$$\boxed{\frac{\sin\theta_i}{\sin\theta_r} = \frac{v_1}{v_2}}$$

v – velocity of EM wave

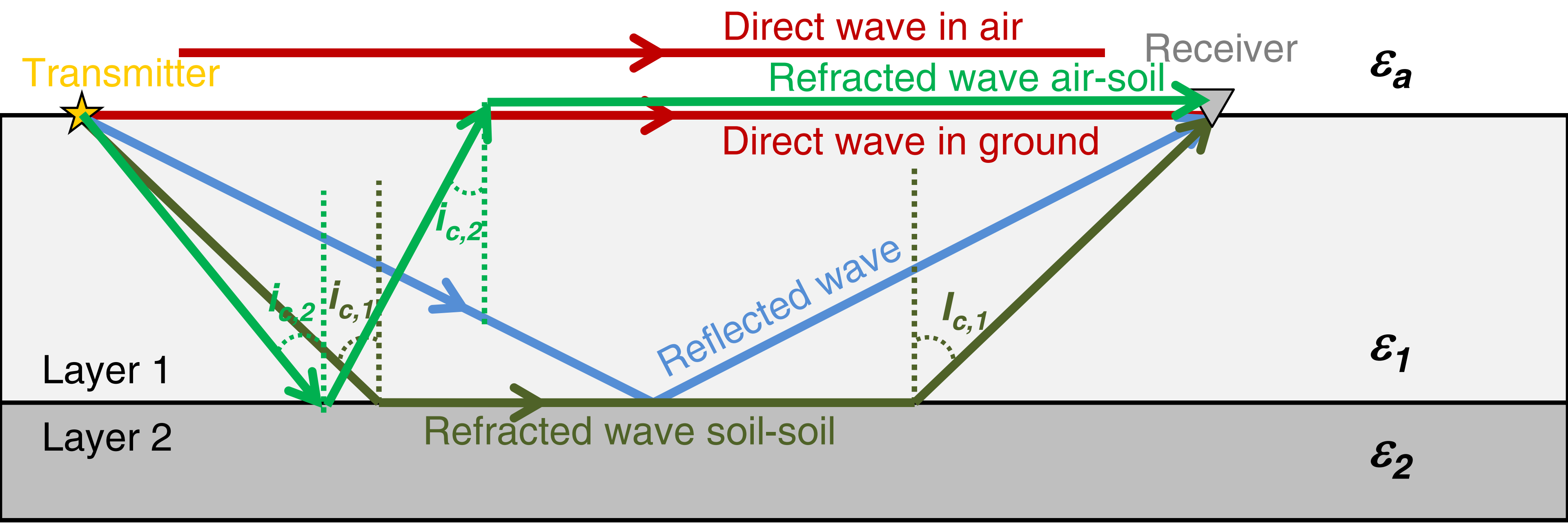
Q. Is there also for EM-waves the critical angle?

A. Yes, potentially we have two critical angles…

EM signal

When the EM signal is received at a certain distance from the transmission point, I have basically 5 main contributions on the recorded signal:

- ✓ **Direct wave in air** and in **ground**
- ✓ **Reflected wave**
- ✓ **Refracted wave** at **air-soil and soil-soil** discontinuities



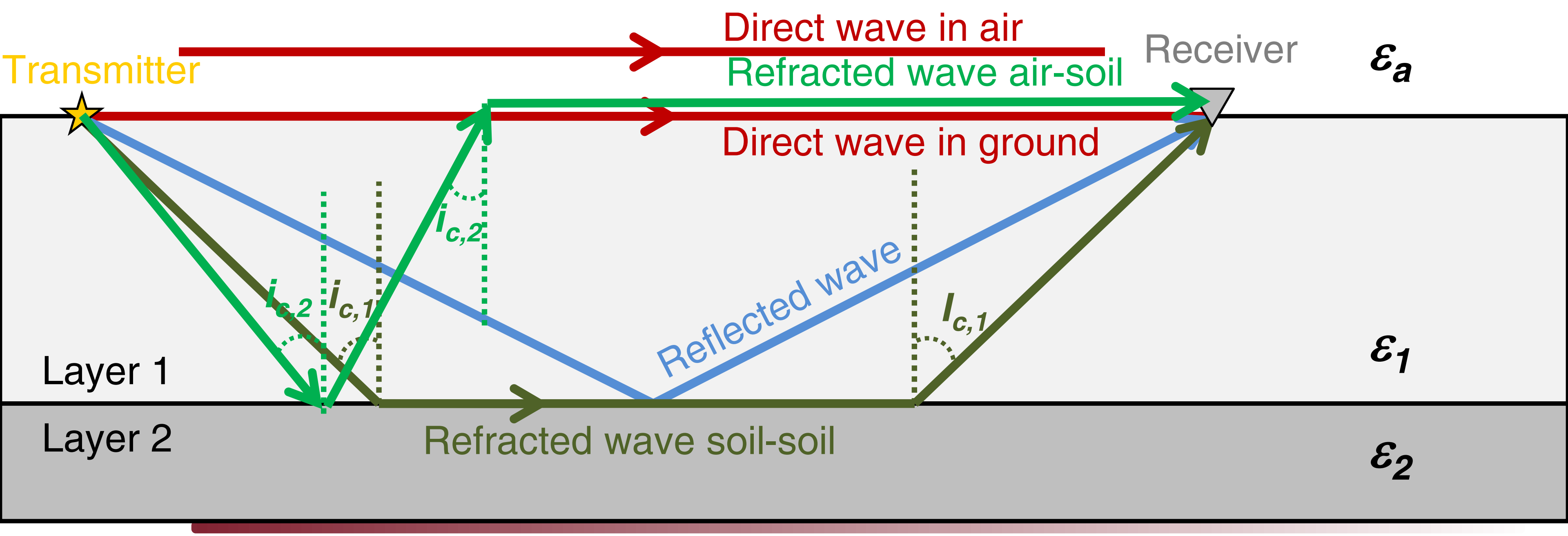
Which contribution does arrive first?

EM signal

Why here the wave travelling in air is important?

1. For seismic: $v_{air} \approx$ 330m/s < v_{ground} (not always, think about S-waves…)

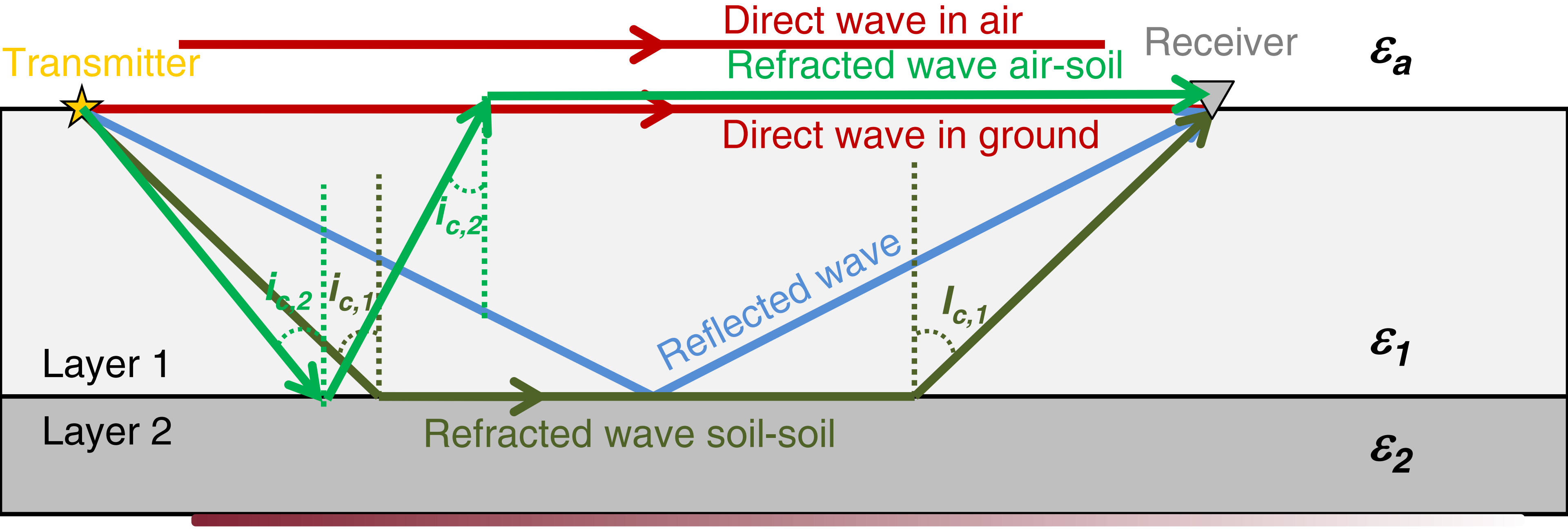
   For EM: $v_{air} \approx 3\text{x}10^8$ m/s > v_{ground} (It always arrives first!)
2. For seismic if you bury the geophones, you should blunt this effect.



EM signal

Q. Why in this case I have the wave critically refracted to the free surface?

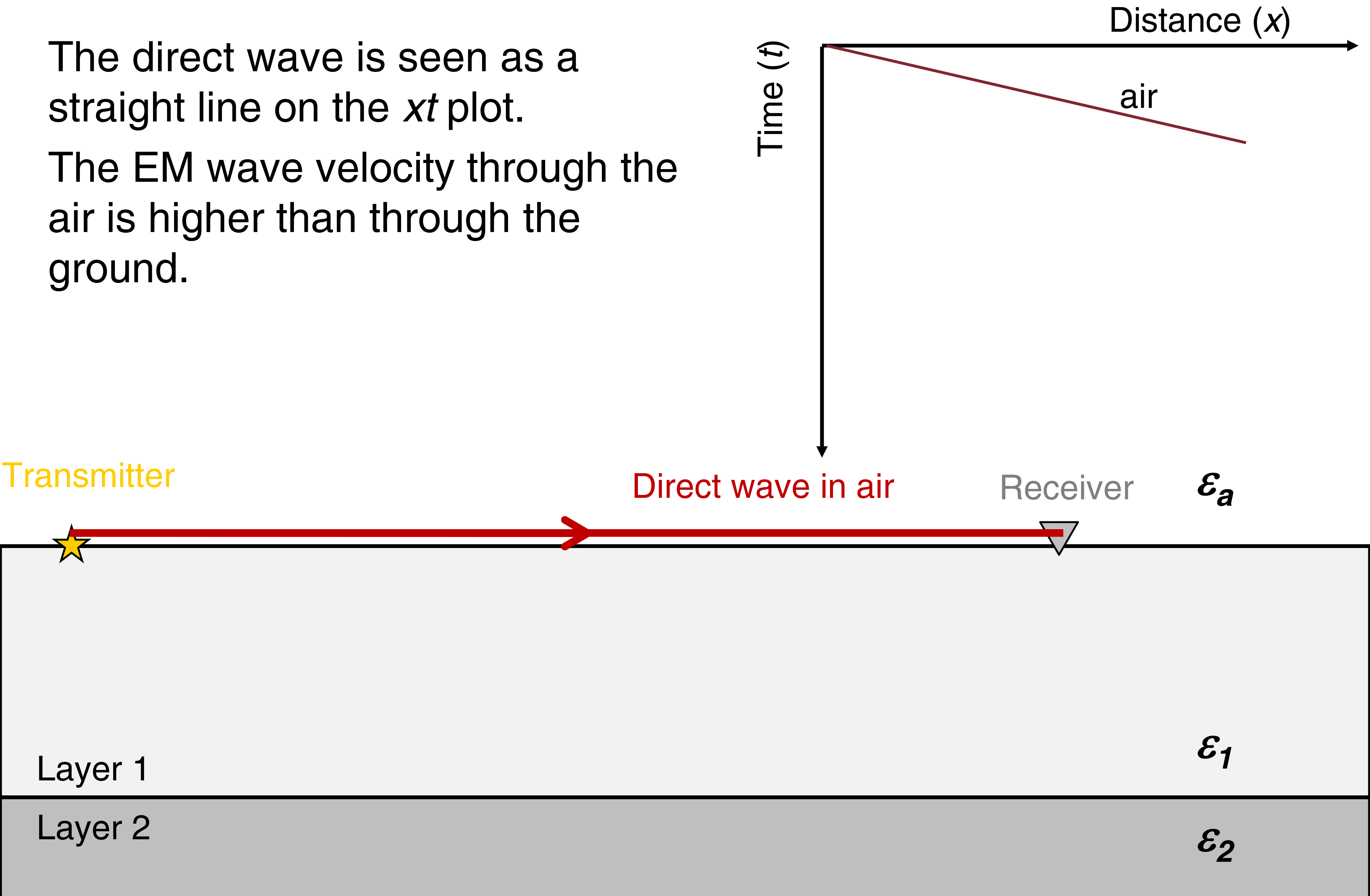
A. Because, the dielectric constant of the air is around 1 while for ground is higher. Therefore, the EM wave velocity through the air (approx. the speed of light) will always be higher than the EM wave velocity through the ground and there will be a critical refraction on the free surface, which does not occur in seismic.



Time-distance plot – Direct wave in air and in ground

The direct wave is seen as a straight line on the *xt* plot.

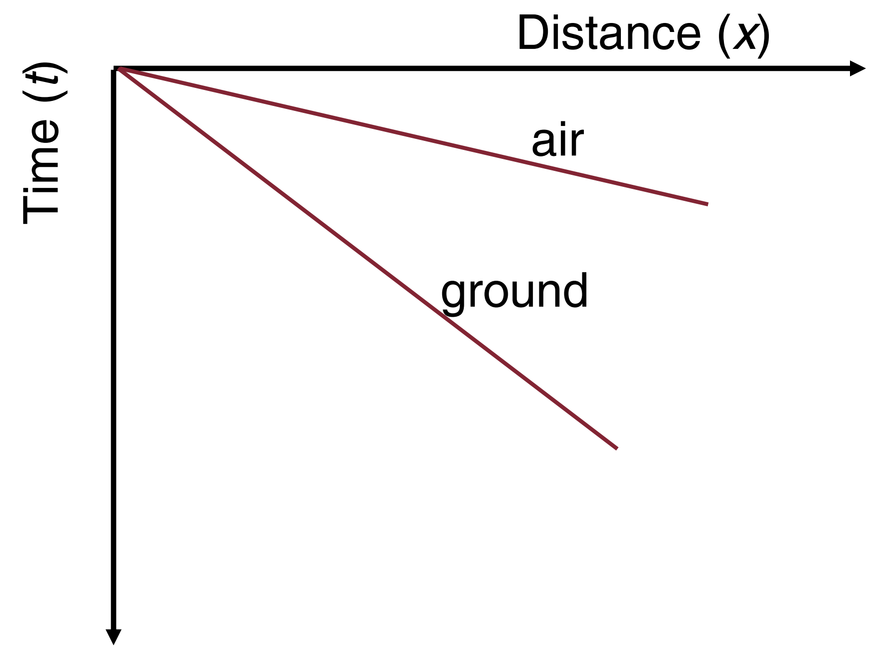
The EM wave velocity through the air is higher than through the ground.

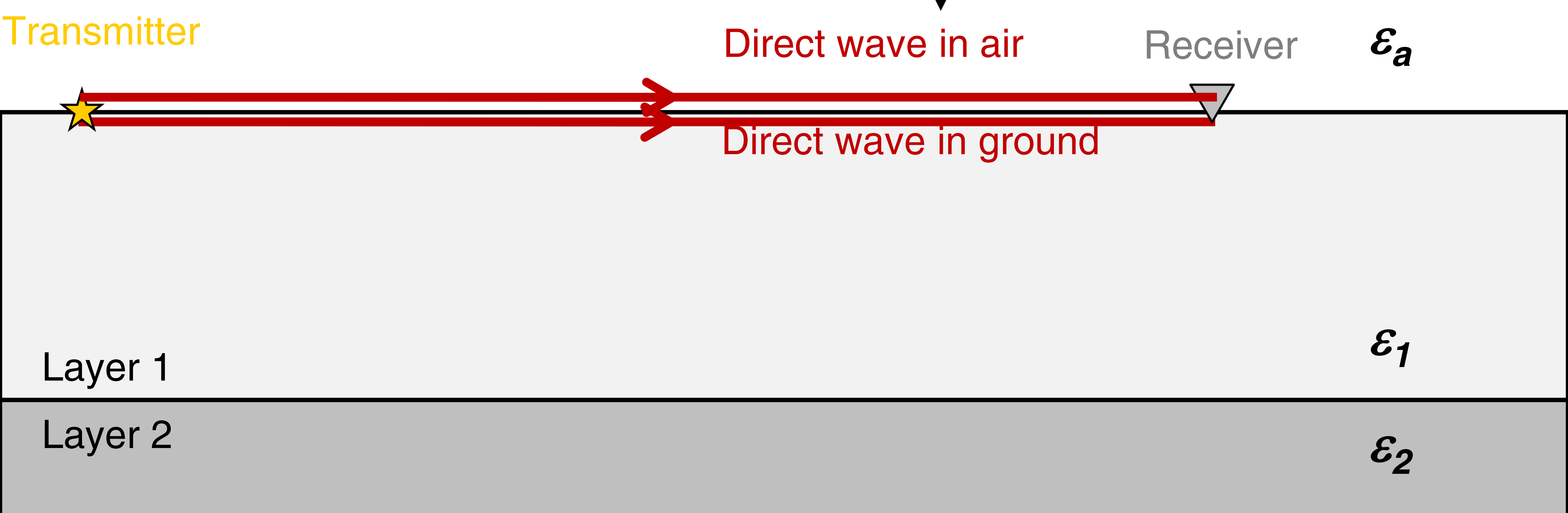


Time-distance plot – Direct wave in air and in ground

The direct wave is seen as a straight line on the *xt* plot.

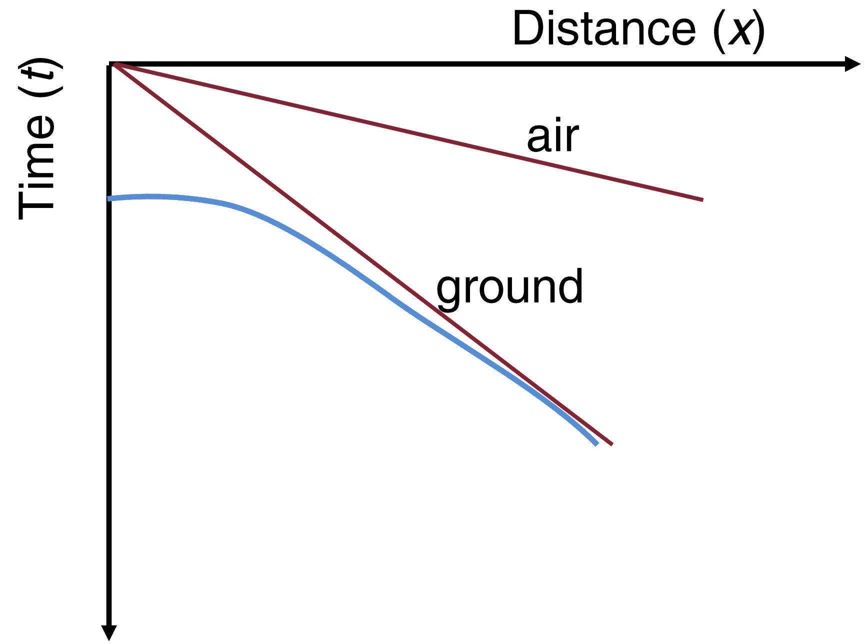
The EM wave velocity through the air is higher than through the ground.

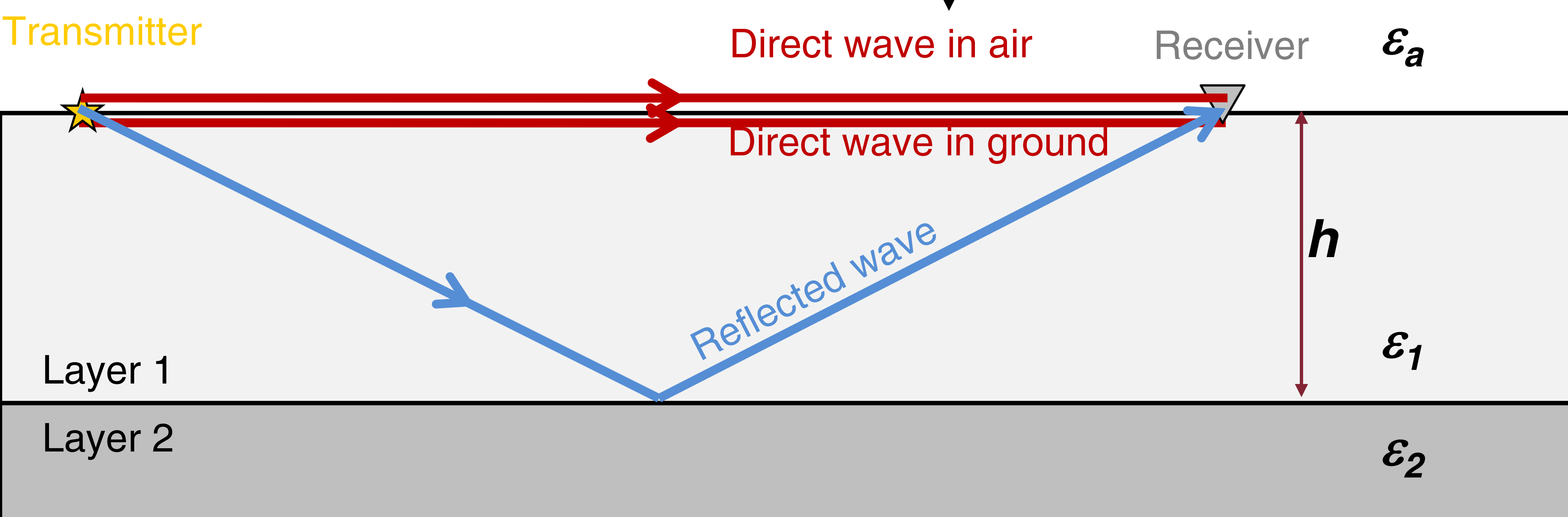




Time-distance plot – Reflected wave

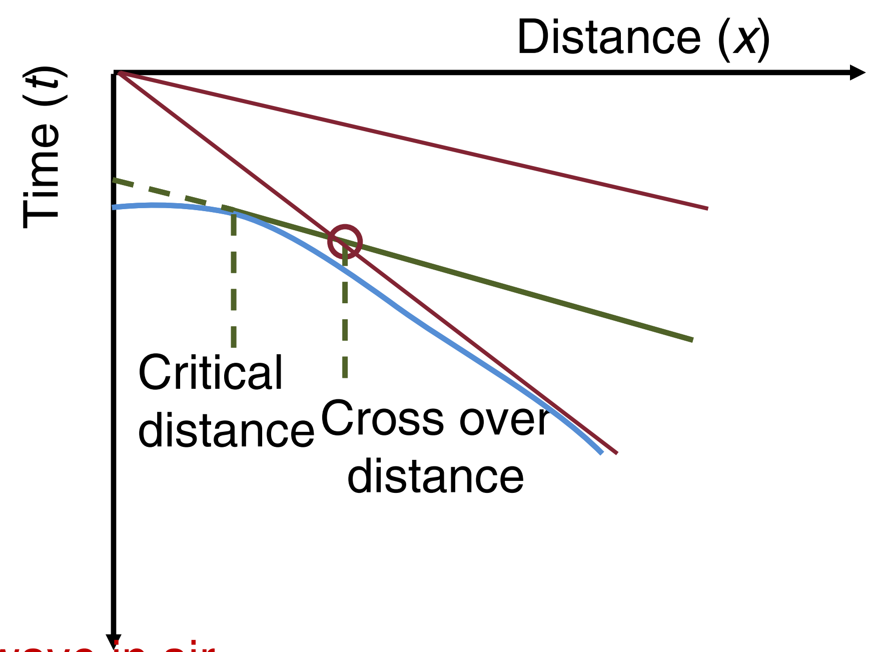
Reflection is never a first arrival and appears as a hyperbola on the *x*-*t* plot.
It is asymptotic with the direct wave in the ground.
The *t*-intercept (*x*=0) gives the thickness of the shallow layer.

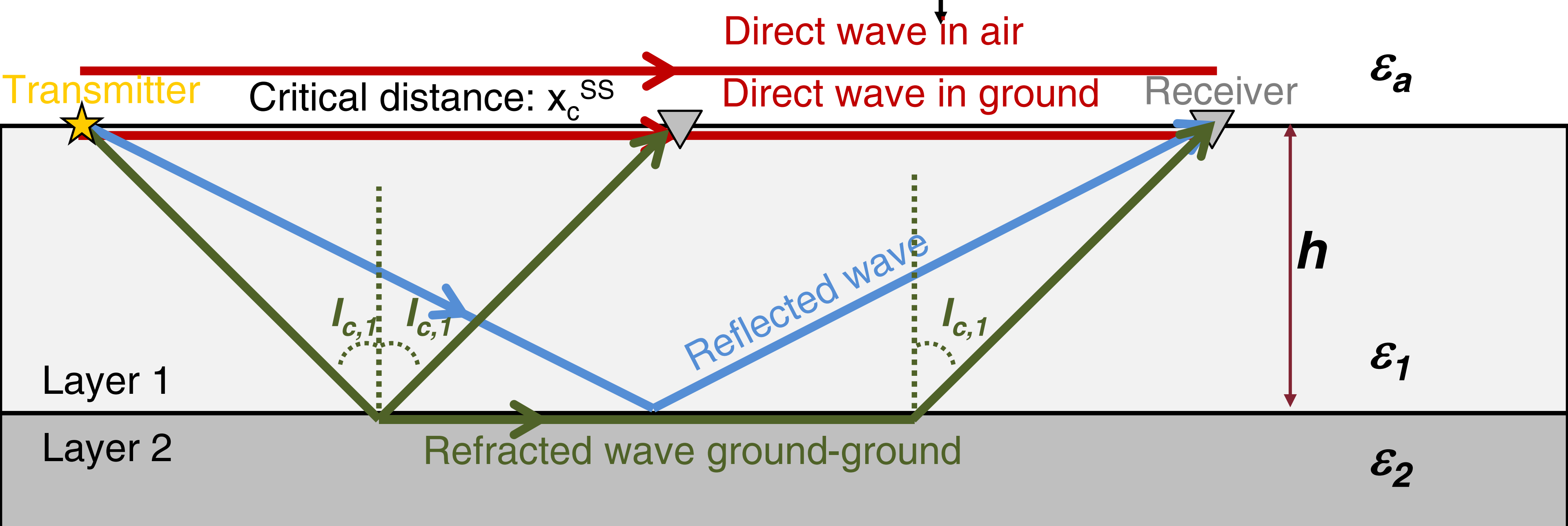




Time-distance plot – Refracted wave ground-ground

It is a linear path on *x-t* diagram.
It travels both in the upper and in the lower layer.
It arrives only after the **critical distance** x_c^{SS} **and it is the** first arrival only after the **cross over distance.**

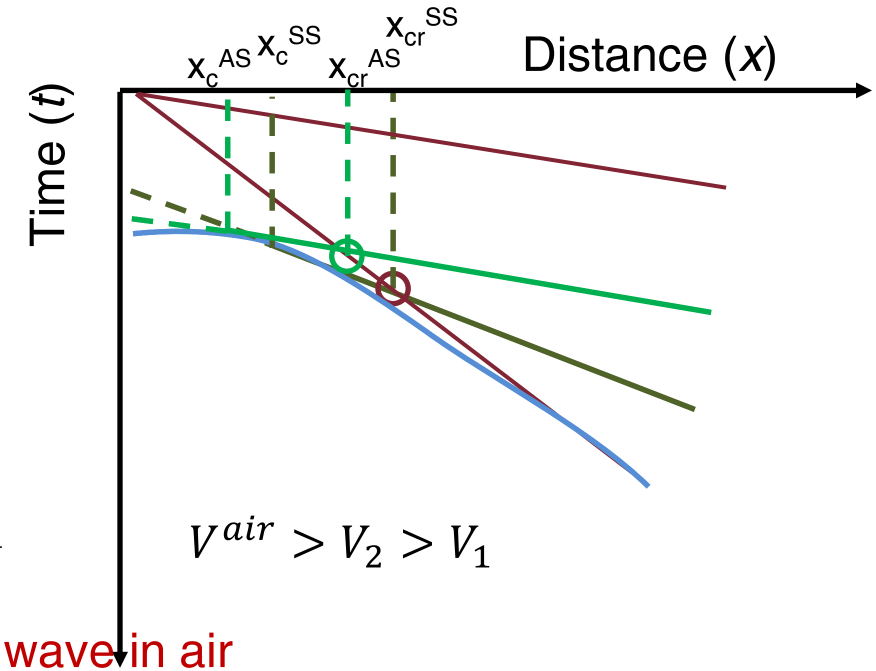




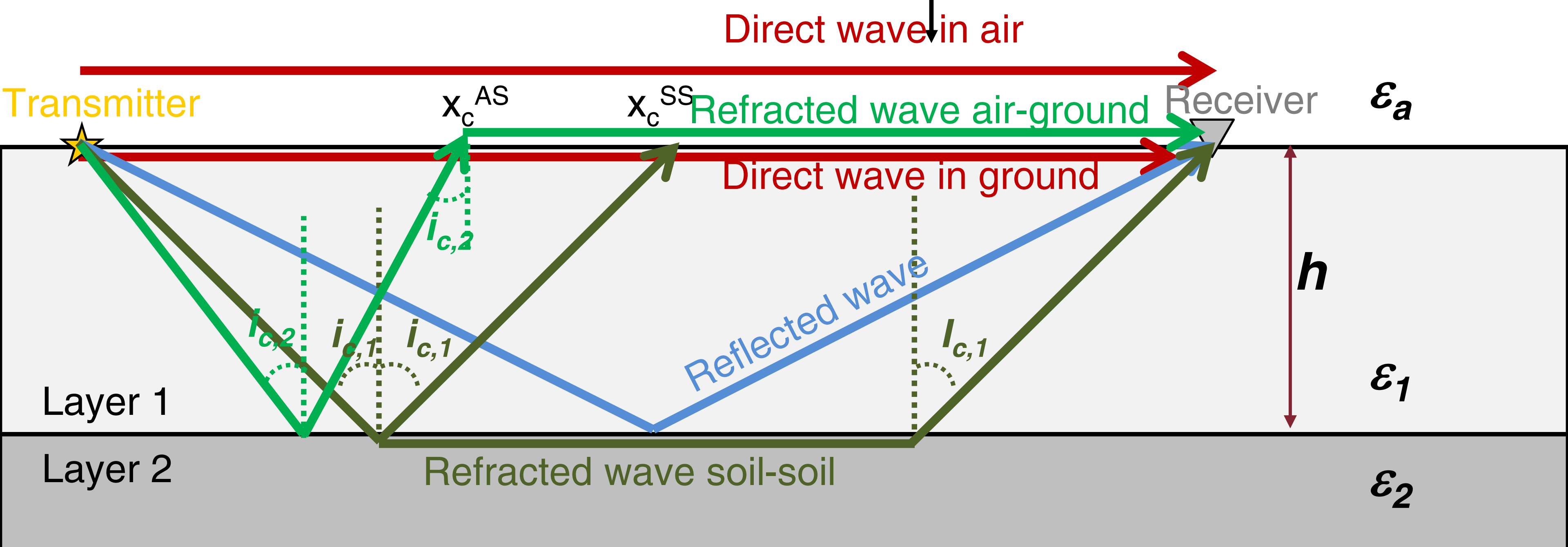
Time-distance plot – Refracted wave air-ground

It is a linear path on the time-distance plot. It travels partly in the upper layer and partly in air.
It arrives only after the critical distance AS and it is the first arrival only after the cross-over distance AS, both smaller than those related to the soil-soil refraction (SS).

$$\sin i_{c,1} = \frac{V_1}{V_2} \qquad \sin i_{c,2} = \frac{V_1}{V^{air}} < \sin i_{c,1} \rightarrow i_{c,2} < i_{c,1}$$



$$V^{air} > V_2 > V_1$$

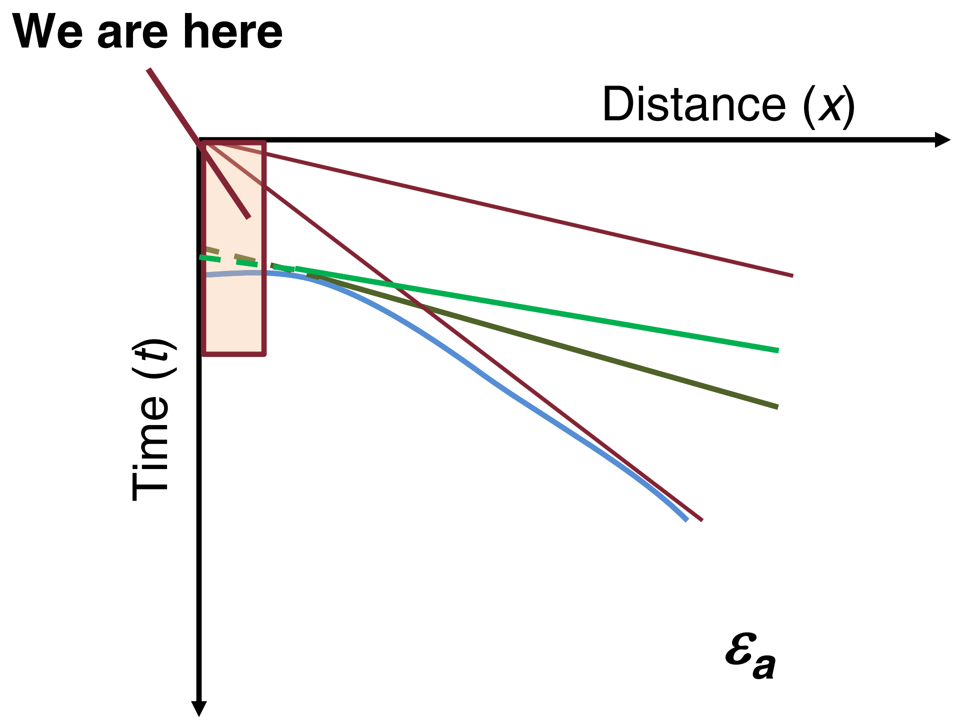


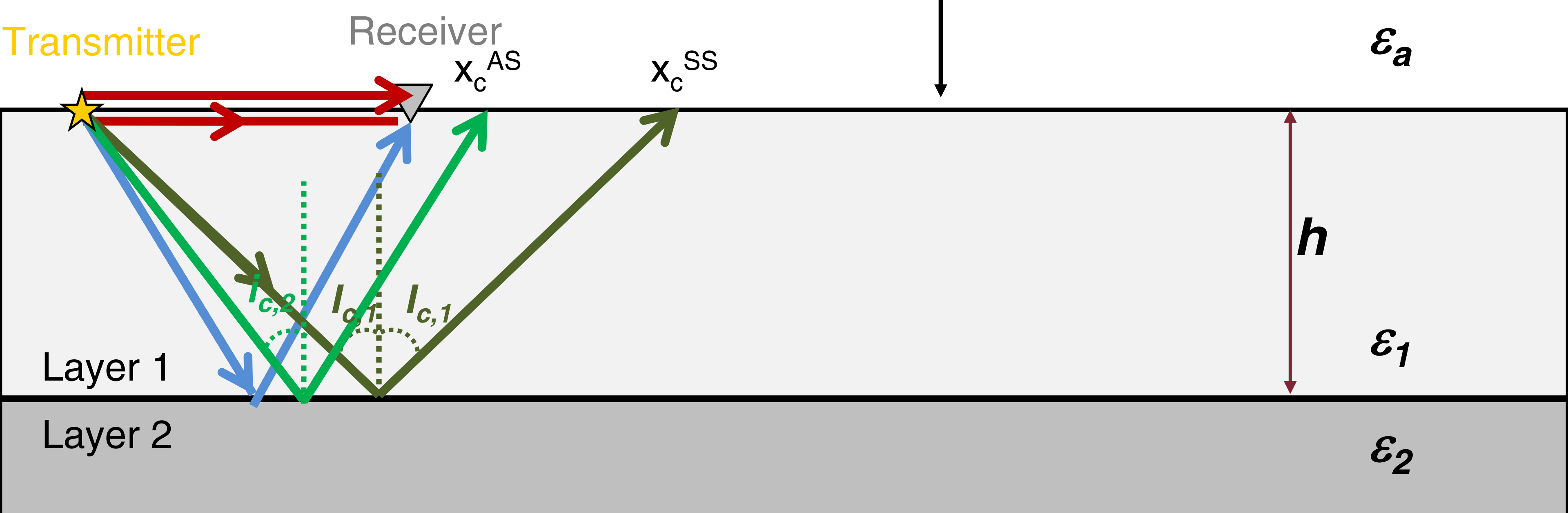
GPR application

For GPR survey we often deal only with direct and reflected wave, beacuse:

1. Source and receiver are not much spaced apart for technical reasons and we are below x_c

2. Often we have not critical refraction (EM wave velocity does not have a codified trend with depth as for seismic)



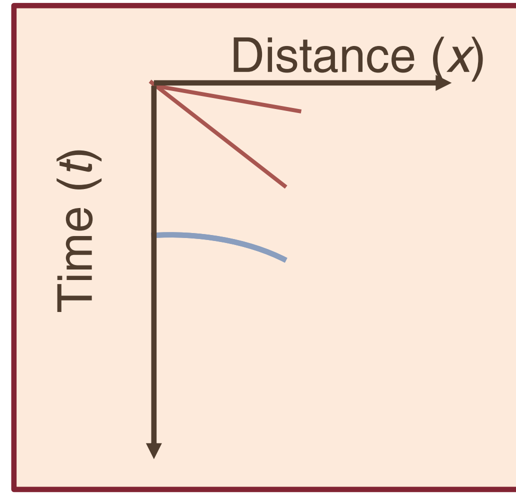


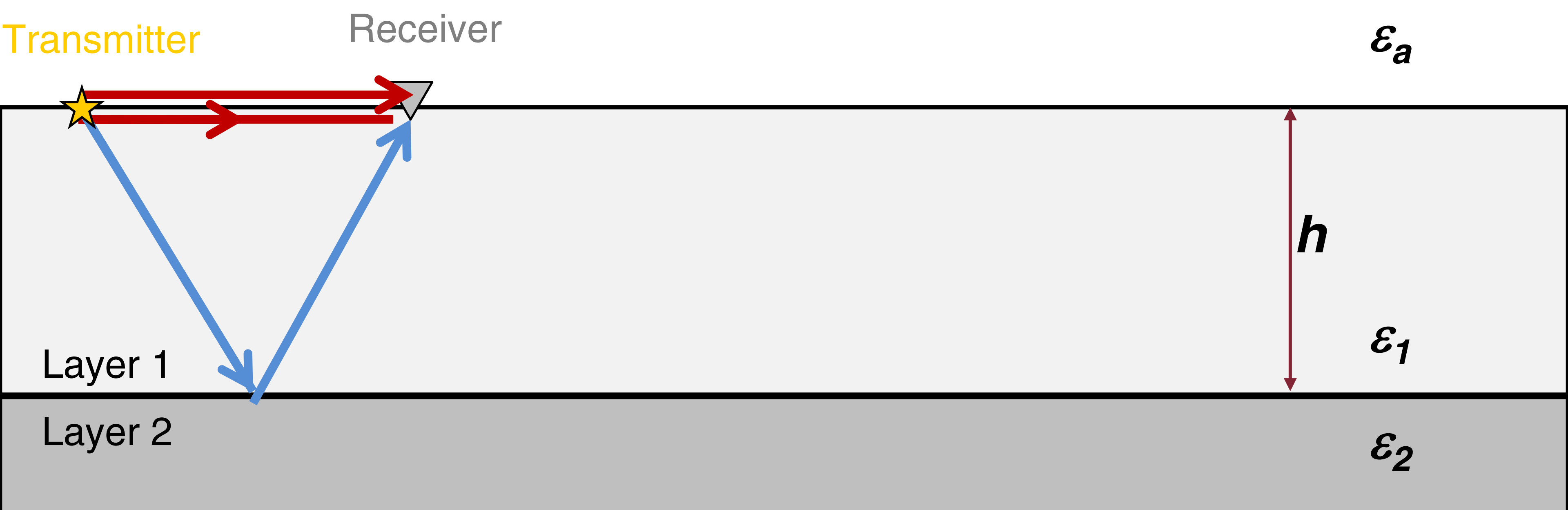
GPR application

For GPR survey we often deal only with direct and reflected wave, beacuse:

1. Source and receiver are not much spaced apart for technical reasons and we are below x_c

2. Often we have not critical refraction (EM wave velocity does not have a codified trend with depth as for seismic)

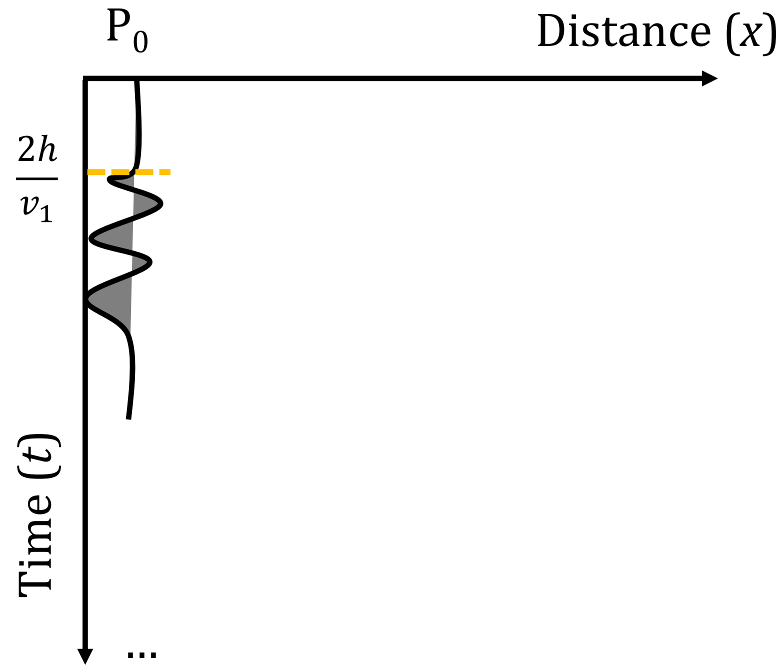


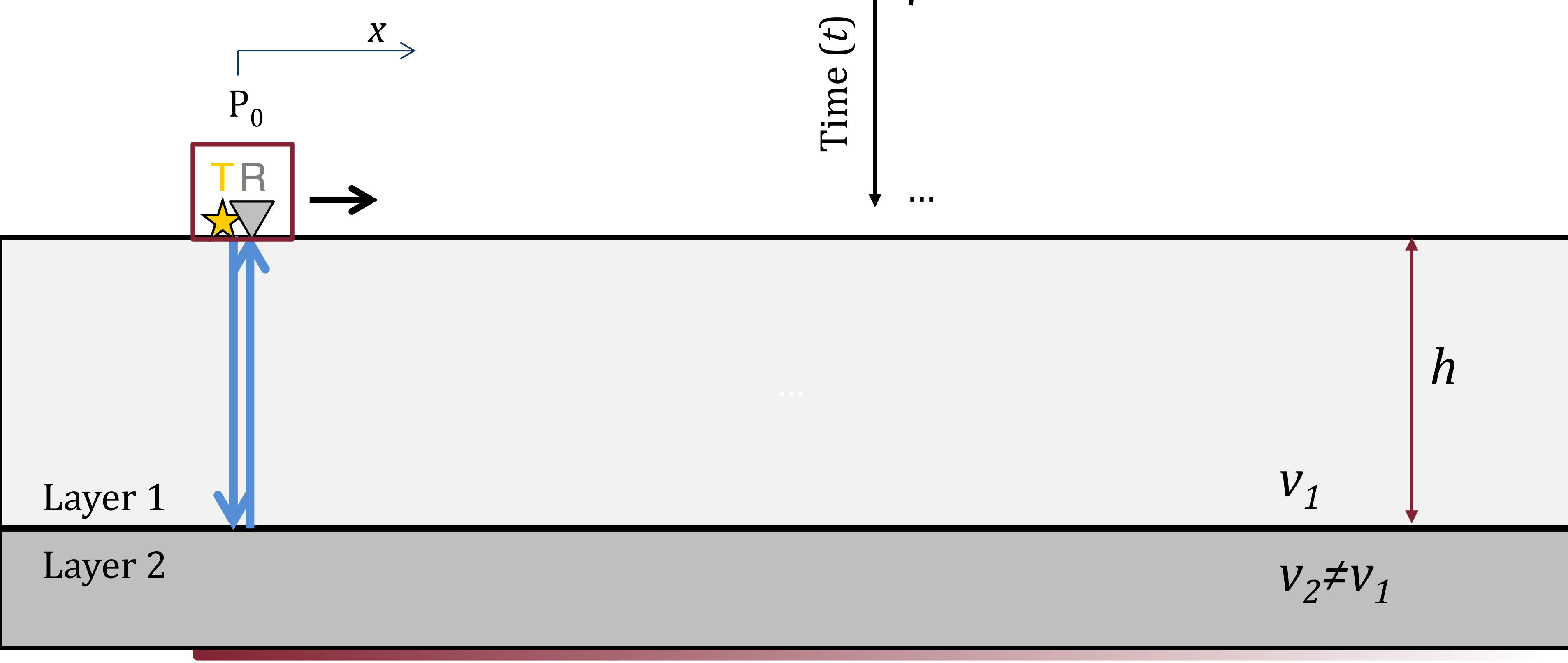


GPR – Zero-offset configuration

Q. If the offset is close to zero?

A. I will have a trace for each position with a spatial sampling Δx

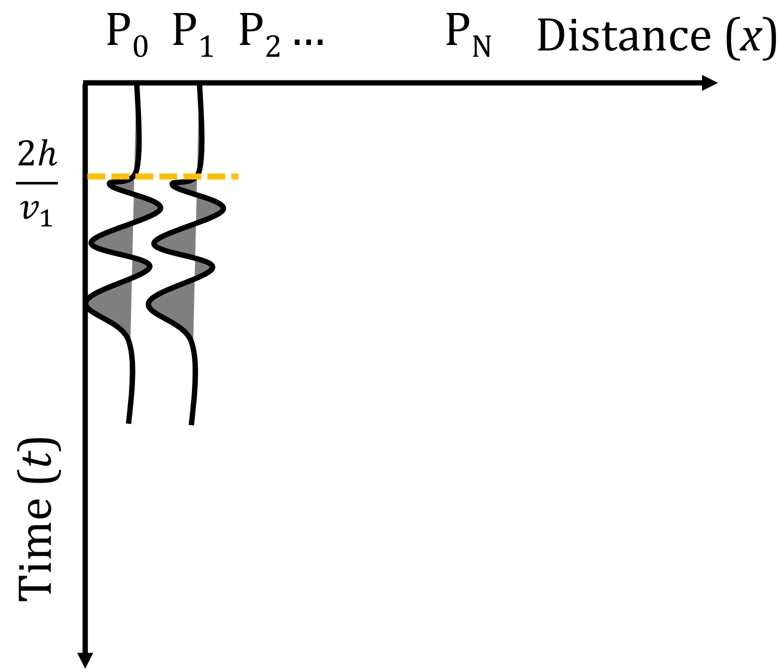


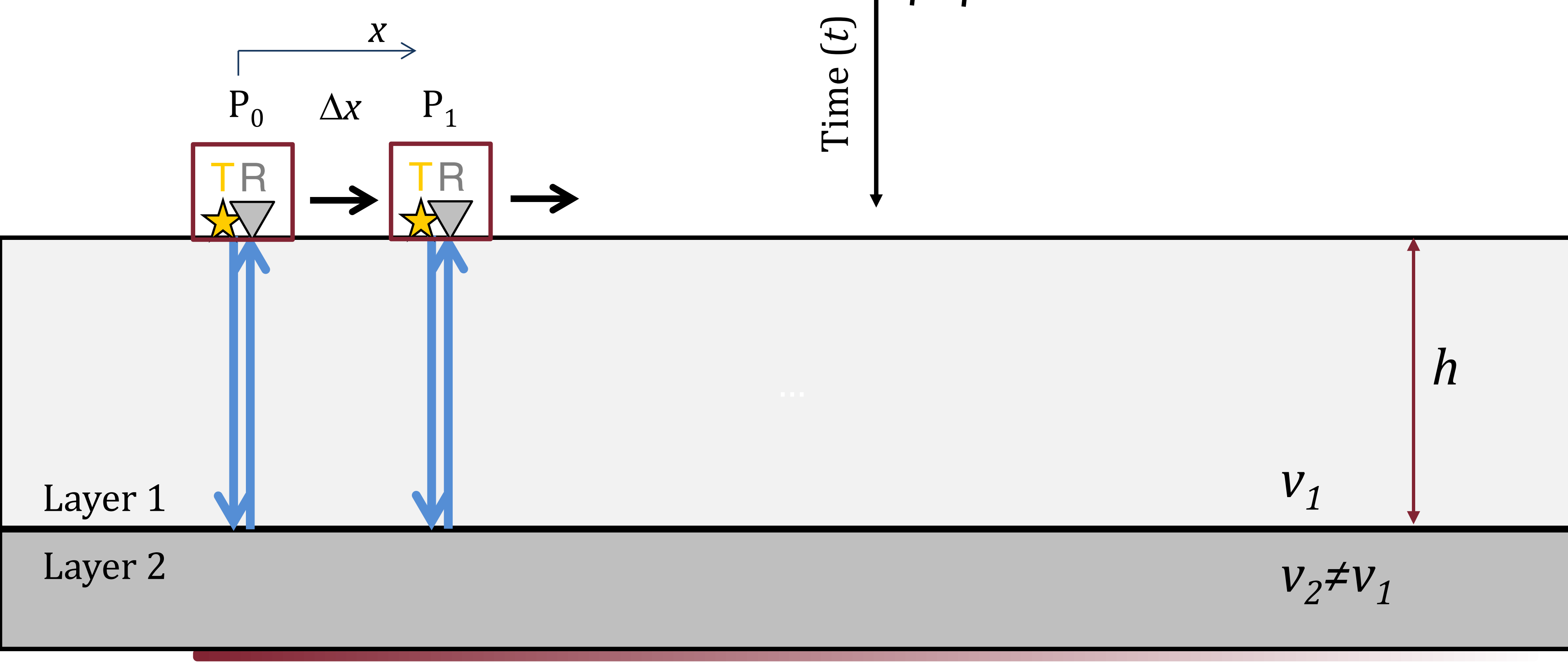


GPR – Zero-offset configuration

Q. If the offset is close to zero?

A. I will have a trace for each position with a spatial sampling Δx

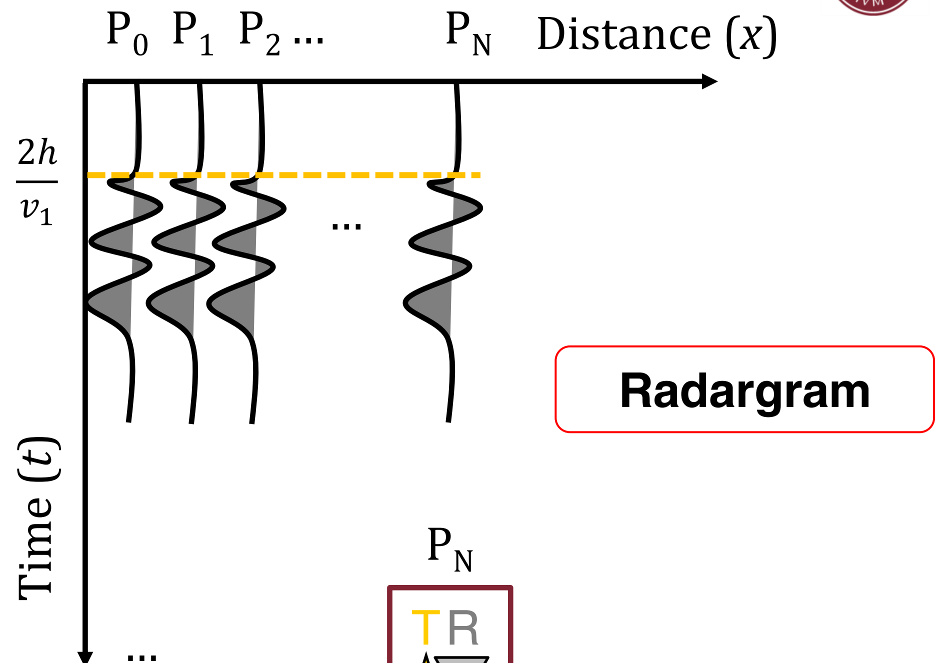


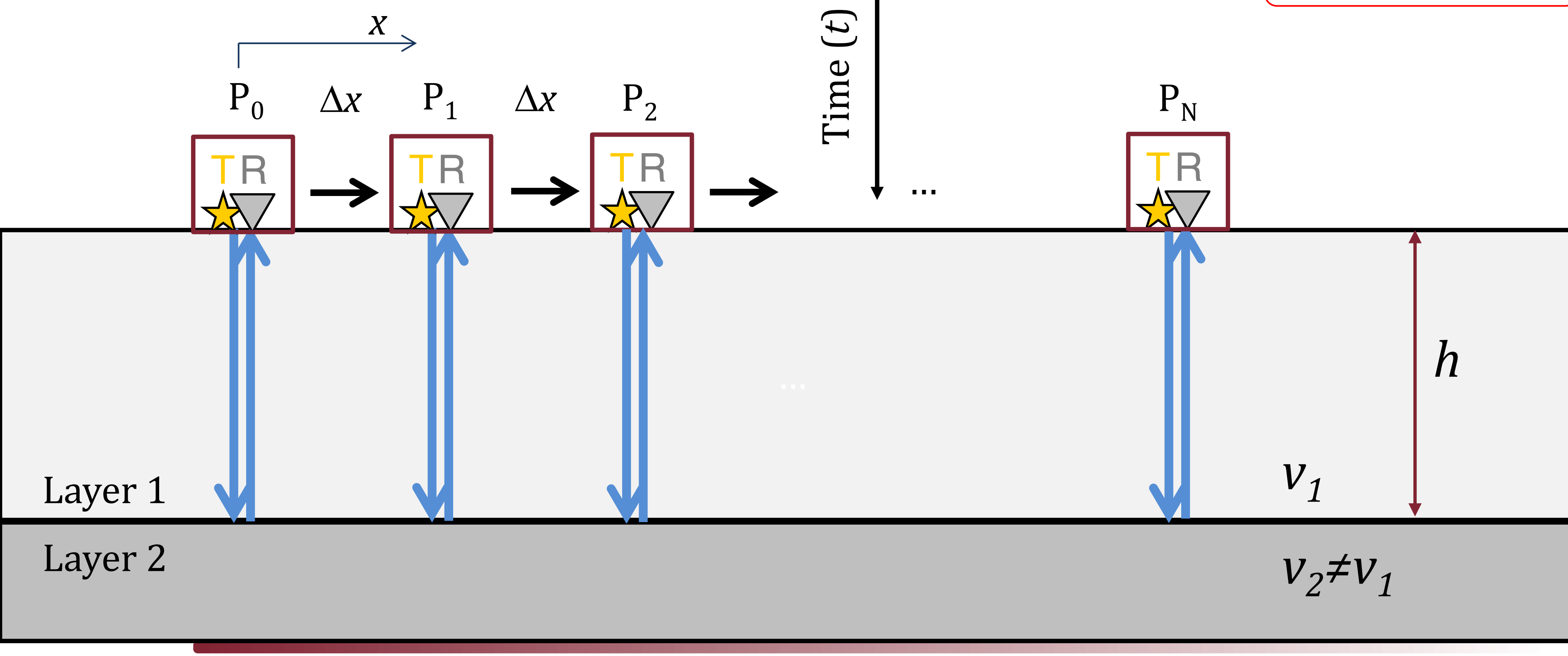


GPR – Zero-offset configuration

Q. If the offset is close to zero?

A. I will have a trace for each position with a spatial sampling Δx

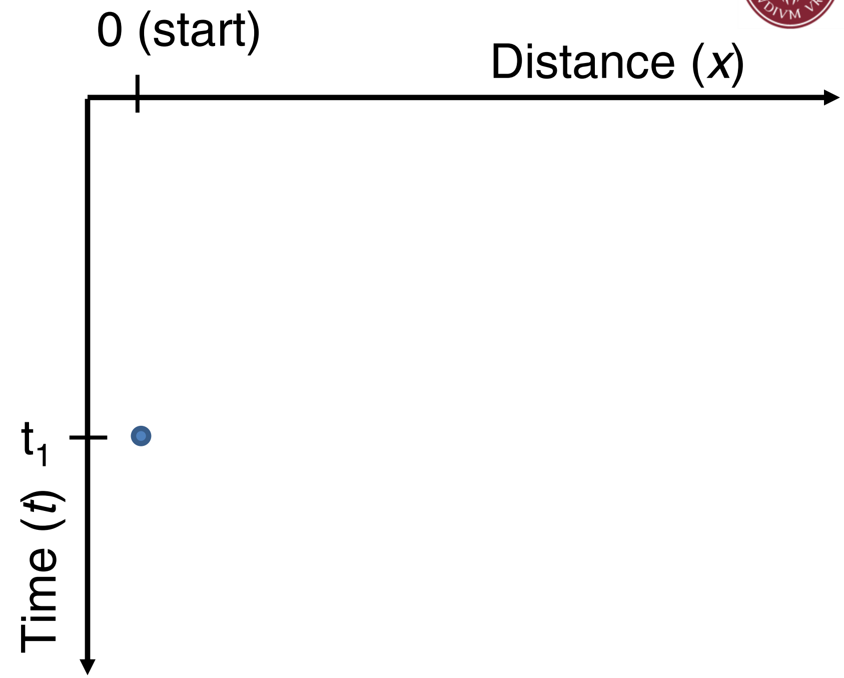


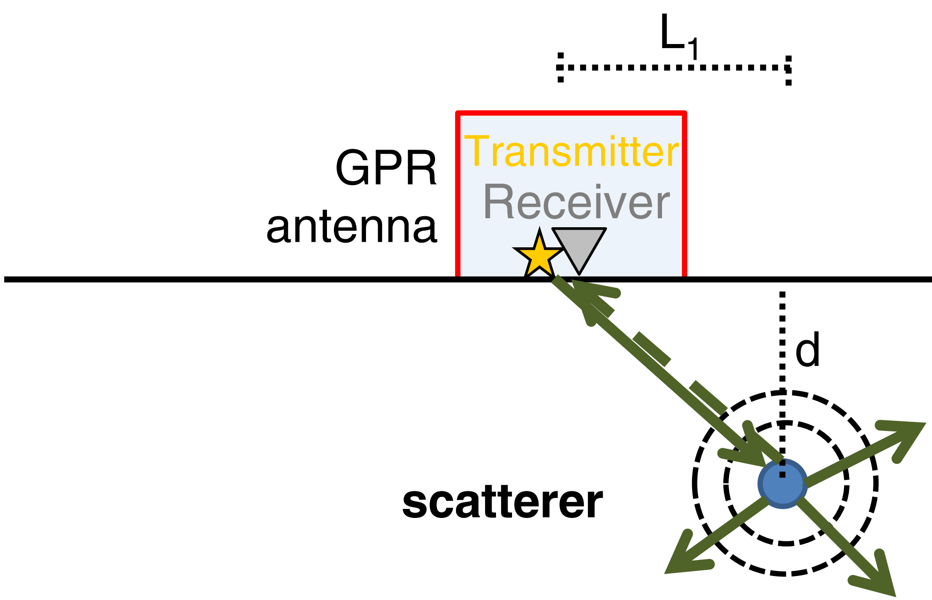


Scattering

Q. How does the scattering object is visible on a time-distance plot?
A. By the so-called **diffraction hyperbola!**

Hp. Source and receiver almost in the same position (GPR antenna).





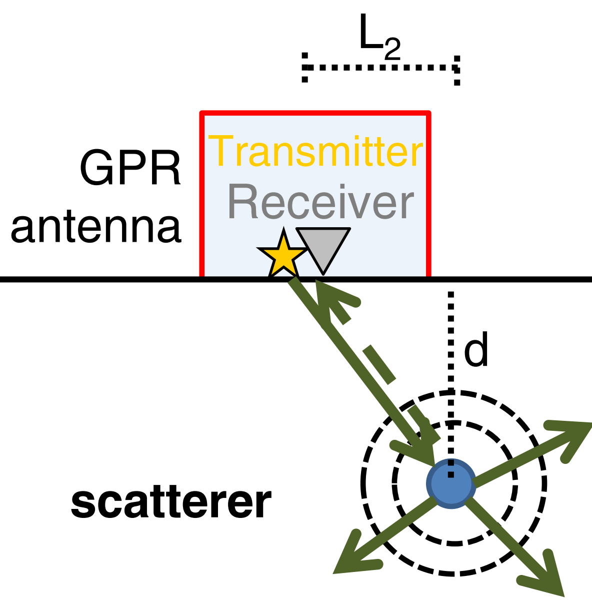
Travel time (from and to)

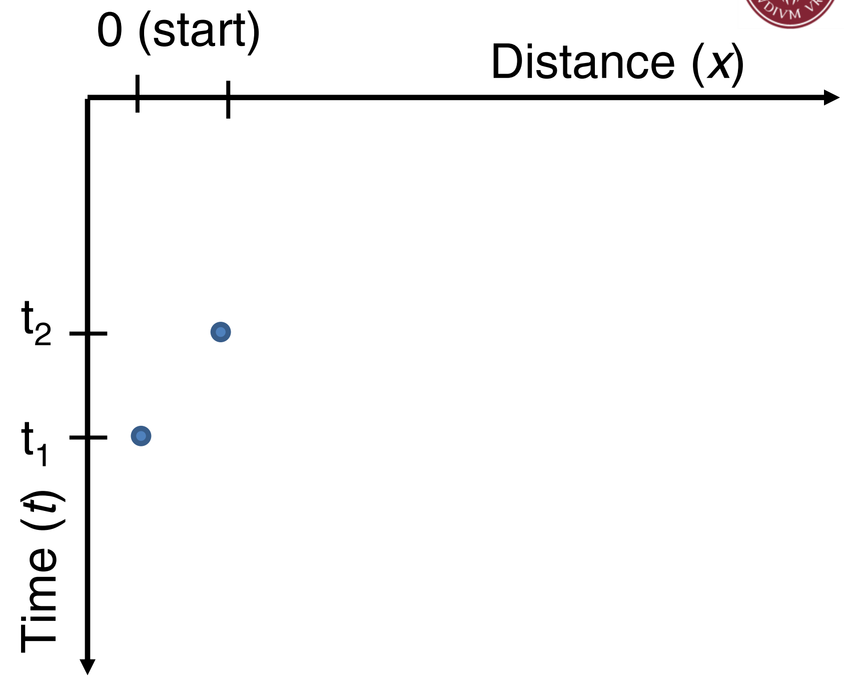
$$t_1 = \frac{2\sqrt{({L_1}^2 + d^2)}}{v}$$

Scattering

Q. How does the scattering object is visible on a time-distance plot?
A. By the so-called **diffraction hyperbola!**

Hp. Source and receiver almost in the same position (GPR antenna).





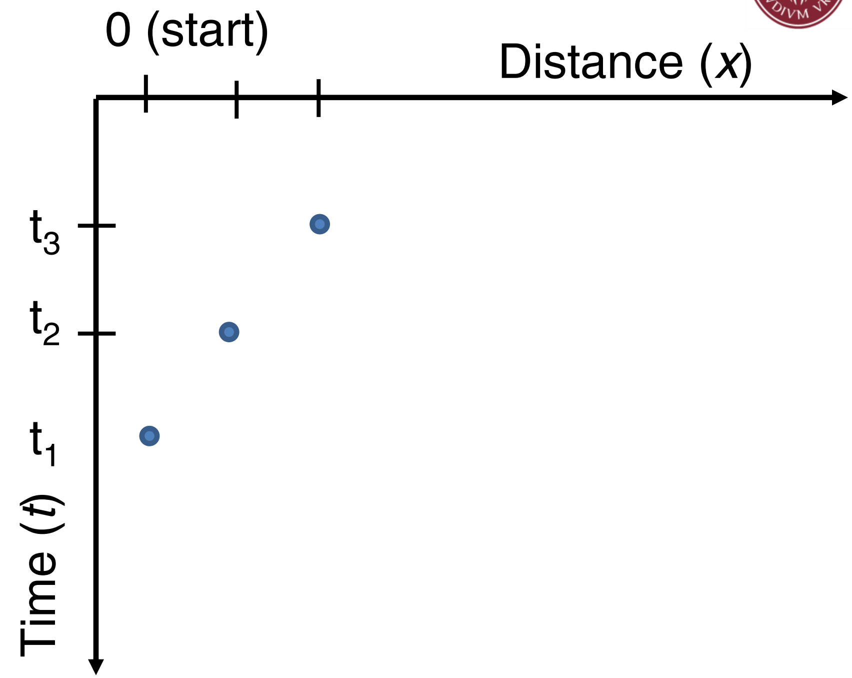
Travel time (from and to)

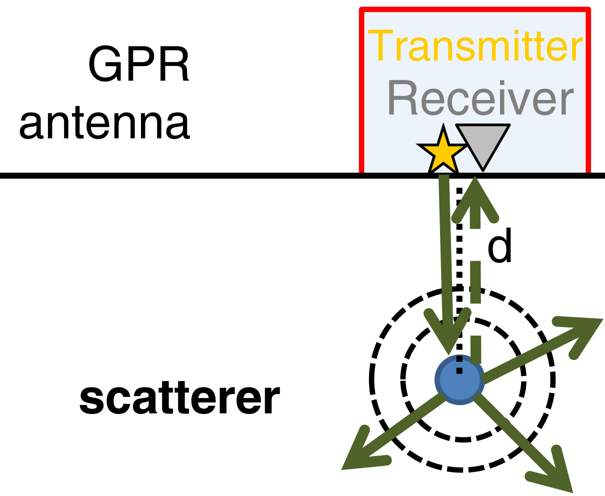
$$t_2 = \frac{2\sqrt{(L_2{}^2 + d^2)}}{v} < t_1 \quad L_2 < L_1$$

Scattering

Q. How does the scattering object is visible on a time-distance plot?
A. By the so-called **diffraction hyperbola!**

Hp. Source and receiver almost in the same position (GPR antenna).





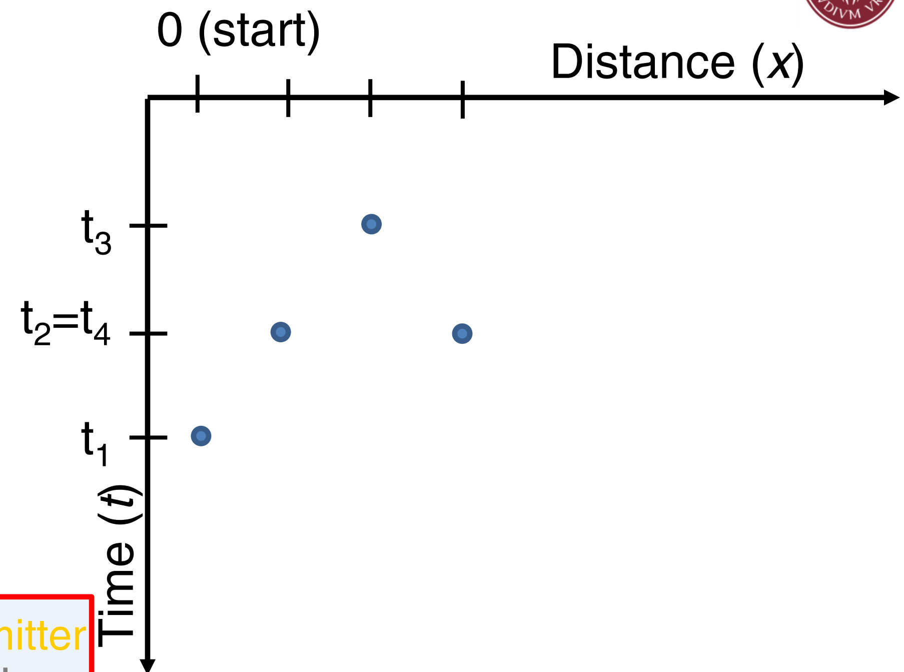
Travel time (from and to)

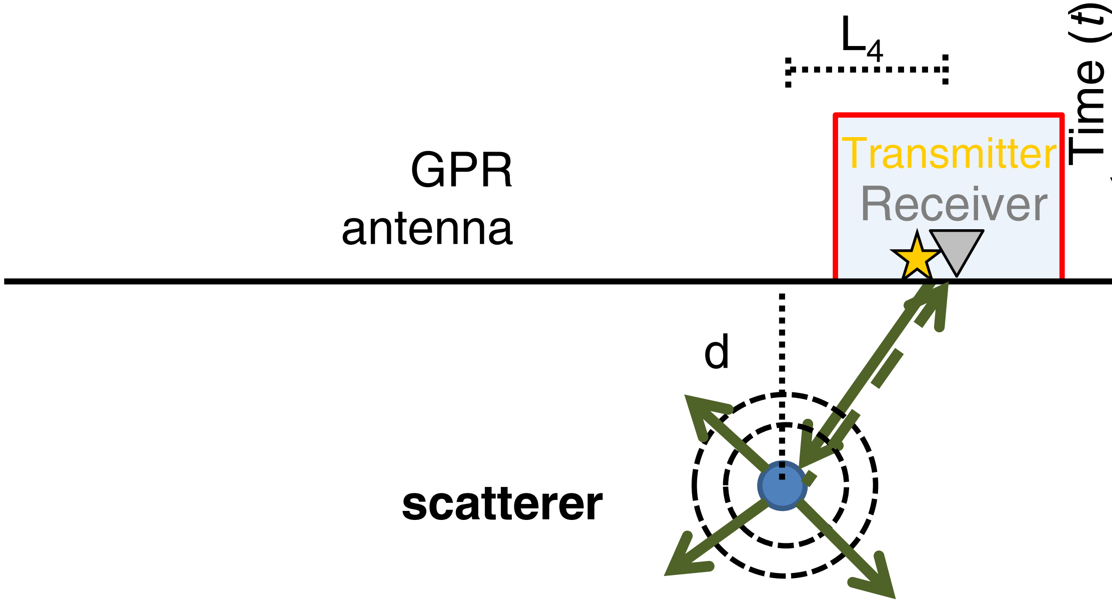
$$t_3 = \frac{2\sqrt{(0^2 + d^2)}}{v} = \frac{2d}{v} < t_2$$

Scattering

Q. How does the scattering object is visible on a time-distance plot?
A. By the so-called **diffraction hyperbola!**

Hp. Source and receiver almost in the same position (GPR antenna).





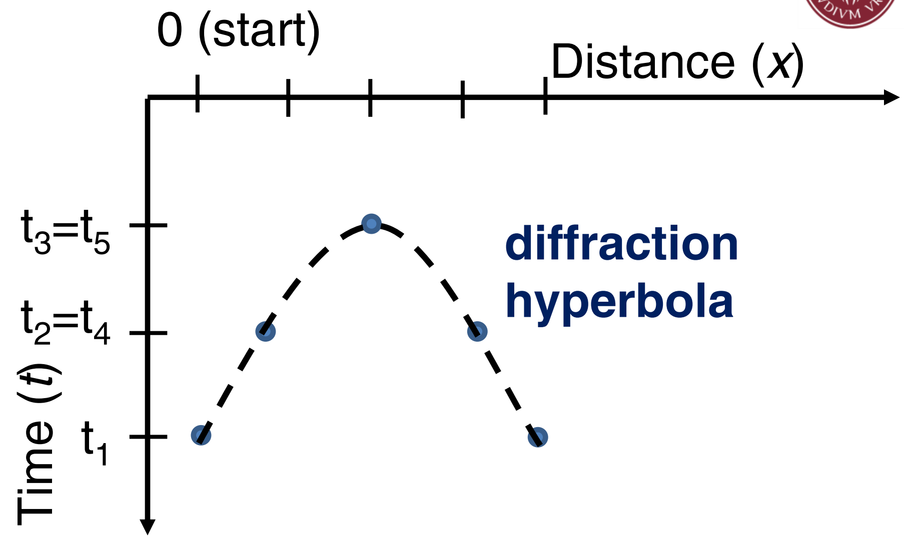
Travel time (from and to)

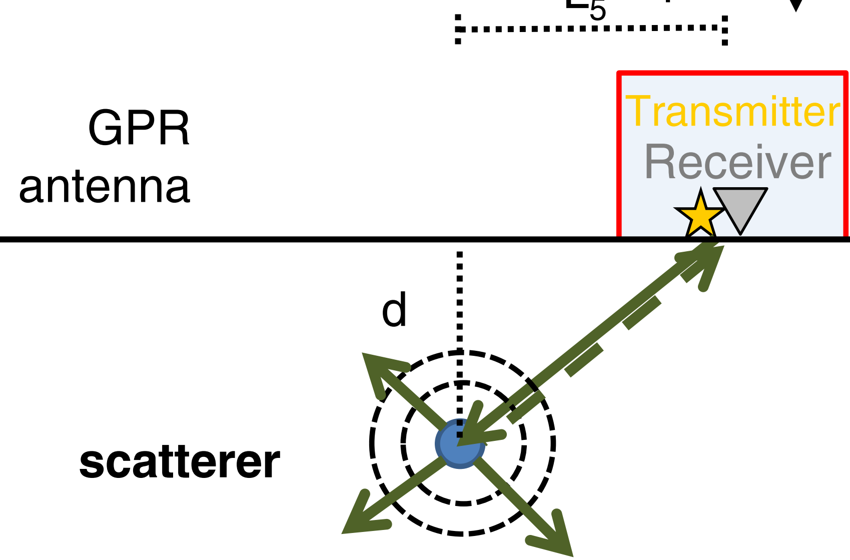
$$t_4 = \frac{2\sqrt{(L_4{}^2 + d^2)}}{v} = t_2\ L_4 = L_2$$

Scattering

Q. How does the scattering object is visible on a time-distance plot?
A. By the so-called **diffraction hyperbola!**

Hp. Source and receiver almost in the same position (GPR antenna).





Travel time (from and to)

$$t_5 = \frac{2\sqrt{(L_5{}^2 + d^2)}}{v} = t_3\ L_5 = L_3$$

GPR – Zero-offset configuration

$$t_{SCATT} = \frac{2\sqrt{(L-x)^2+d^2}}{v}$$