SAPIENZA
UNIVERSITÀ DI ROMA

Environmental geophysics

Giorgio De Donno

1. Seismic methods

Seismic waves at interfaces: refraction method

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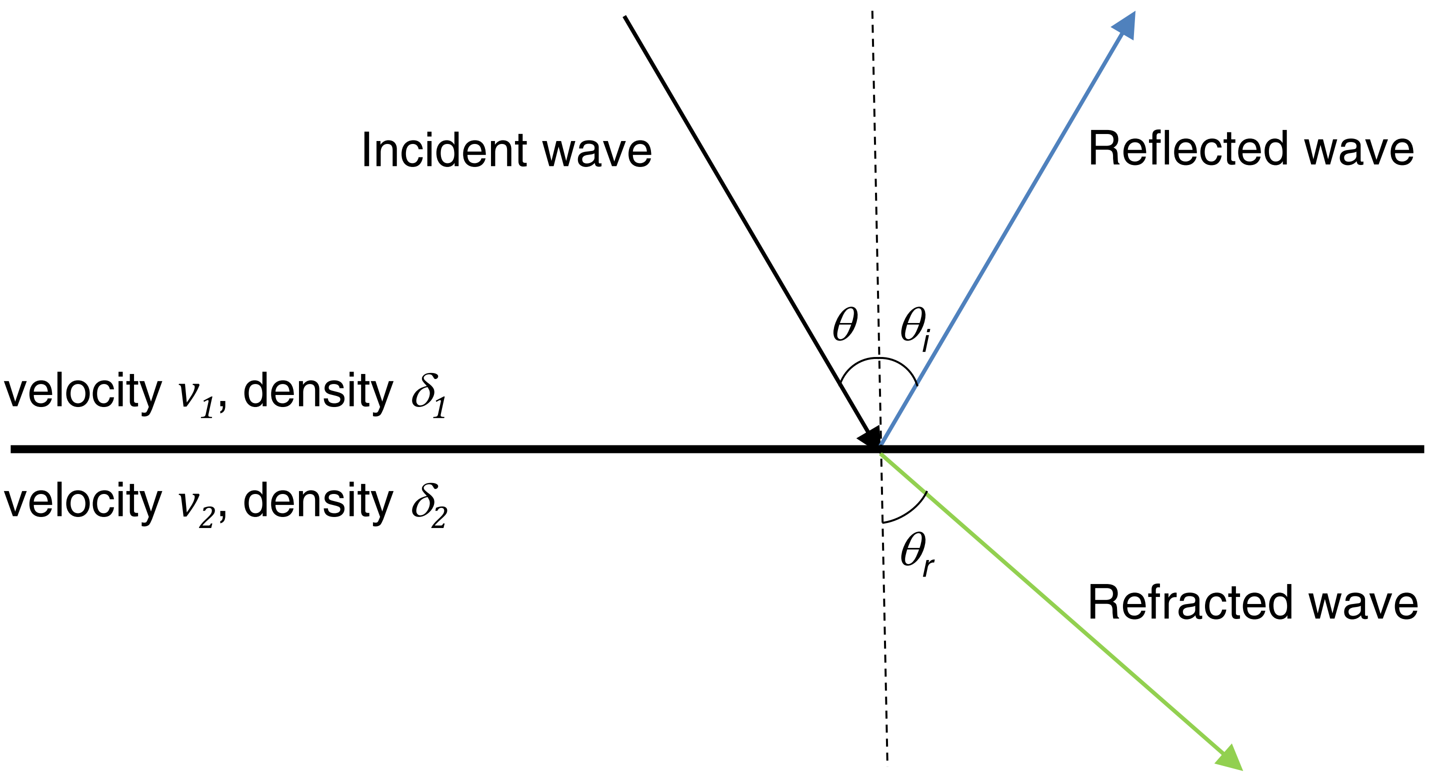
Seismic waves at interfaces

Seismic waves propagate according to geometric optics:

Snell's law

$$\frac{\sin\theta}{\sin\theta_r} = \frac{v_1}{v_2}$$

$$\theta = \theta_i$$

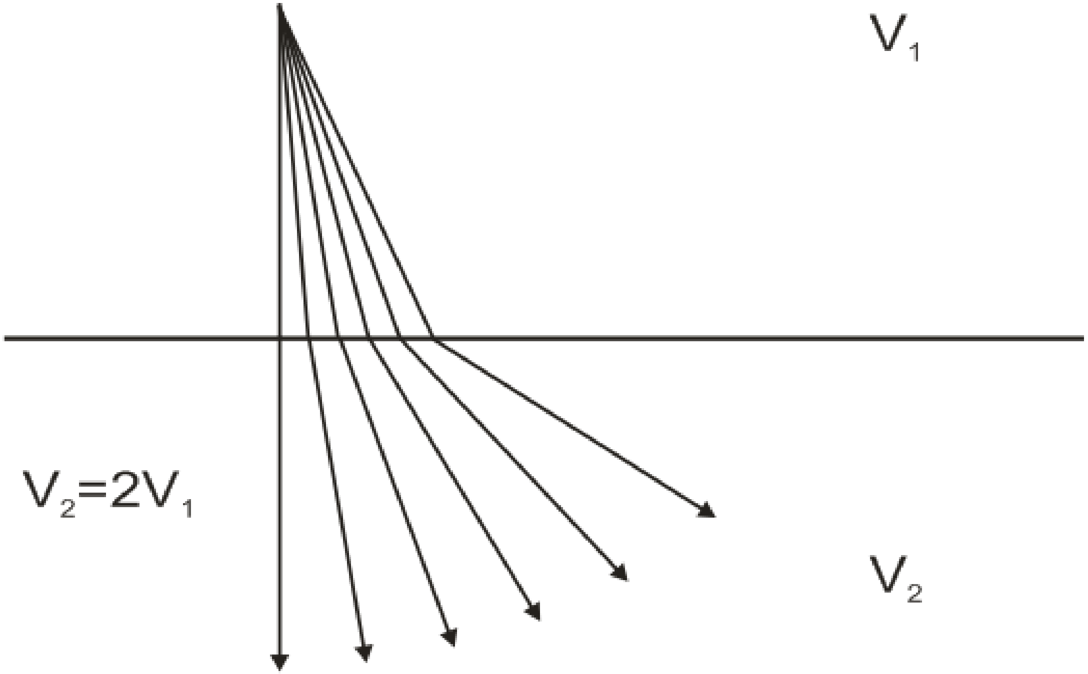


Seismic waves at interfaces

If $v_2 > v_1$, then as θ increases θ_r increases faster

Snell's law

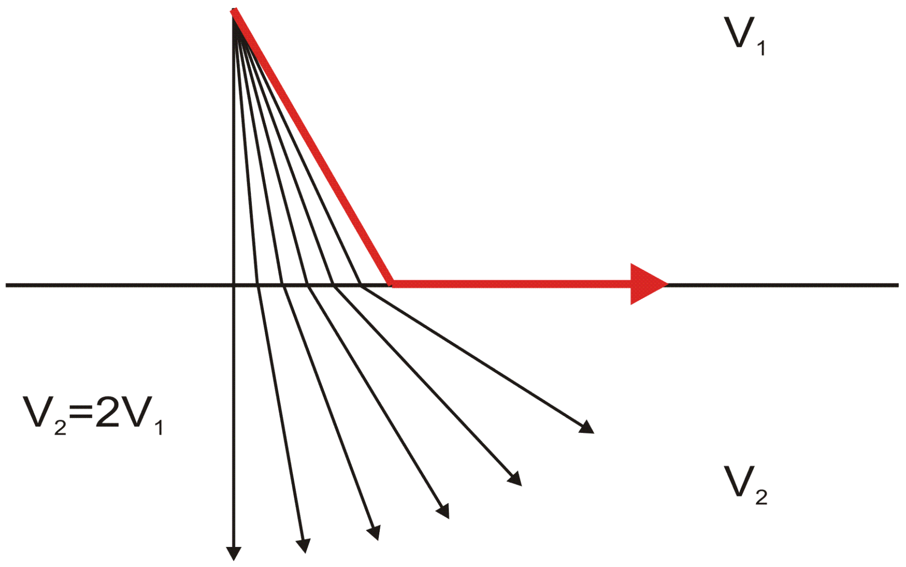
$$\frac{\sin\theta}{\sin\theta_r} = \frac{v_1}{v_2}$$



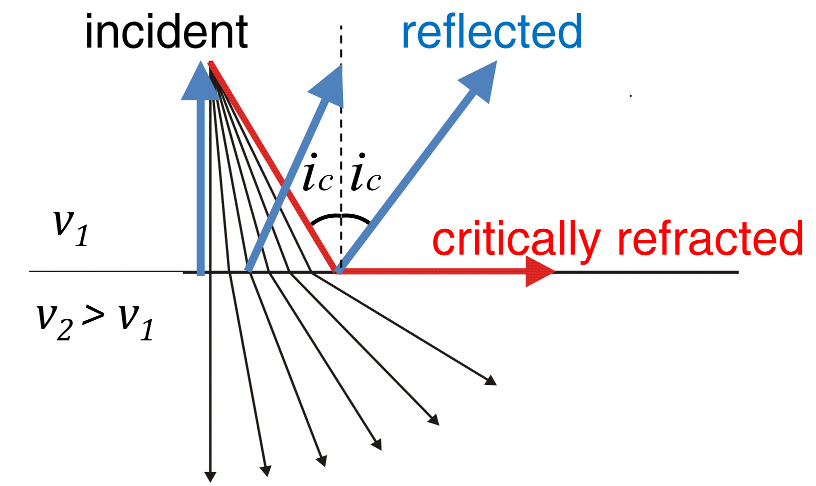
Seismic waves at interfaces

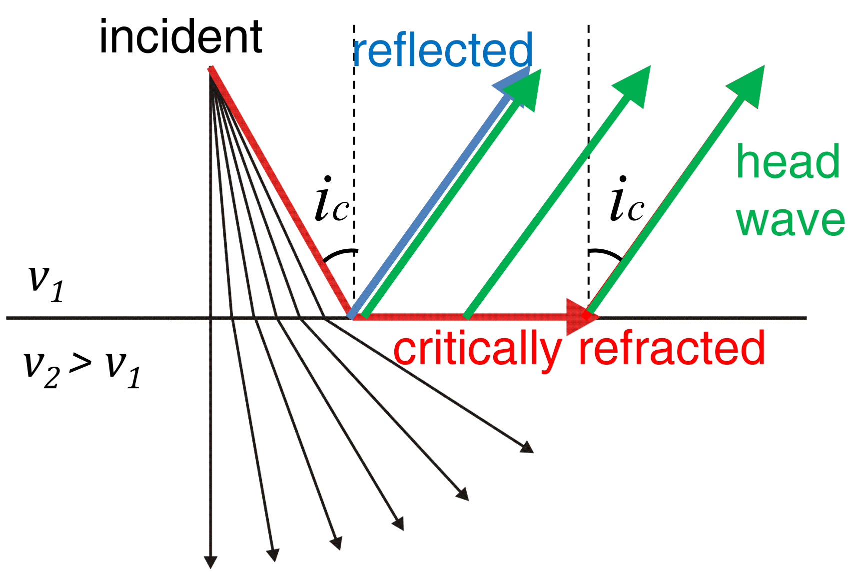
There will be a critical angle of incidence i_c, for which the refraction angle θ_r = 90°

$$\frac{\sin i_c}{\sin \frac{\pi}{2}} = \frac{v_1}{v_2} \Rightarrow \sin i_c = \frac{v_1}{v_2} \Rightarrow i_c = \sin^{-1}\frac{v_1}{v_2}$$



Critical refraction





The critically refracted wave can be approximated (according to the Huygens principle) as a series of point sources at the interface between the two layers.

Q1. Will the refracted wave come back on surface?
Q2. If yes, at which angle?

A1: Yes, as a head wave
A2: At the critical angle

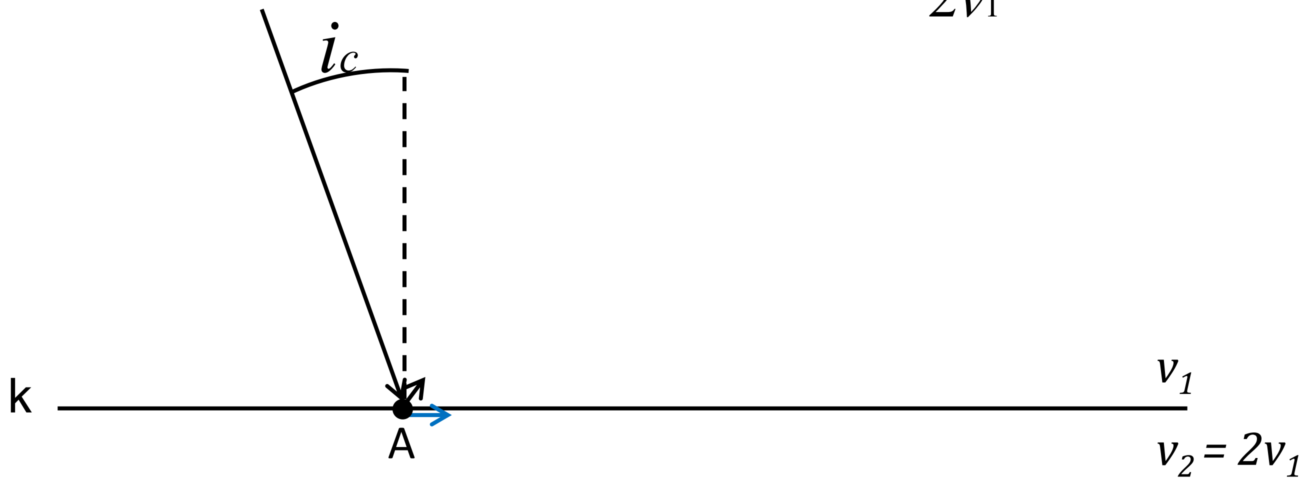
Critical refraction – Head wave

A1: Yes, as a head wave

Ex. 1 Two layers with underlying velocity double than shallow velocity

$$i_c = \sin^{-1}\frac{v_1}{2v_1} = 30°$$

t=0 time of incidence



Time 0: critical incidence in A and critical wave generation (blue arrow) travelling with velocity v_2. Point A is a new point source of spherical wavefronts (back-directed to the surface) according to Huygens' principle (black arrow) with velocity v_1

Critical refraction – Head wave

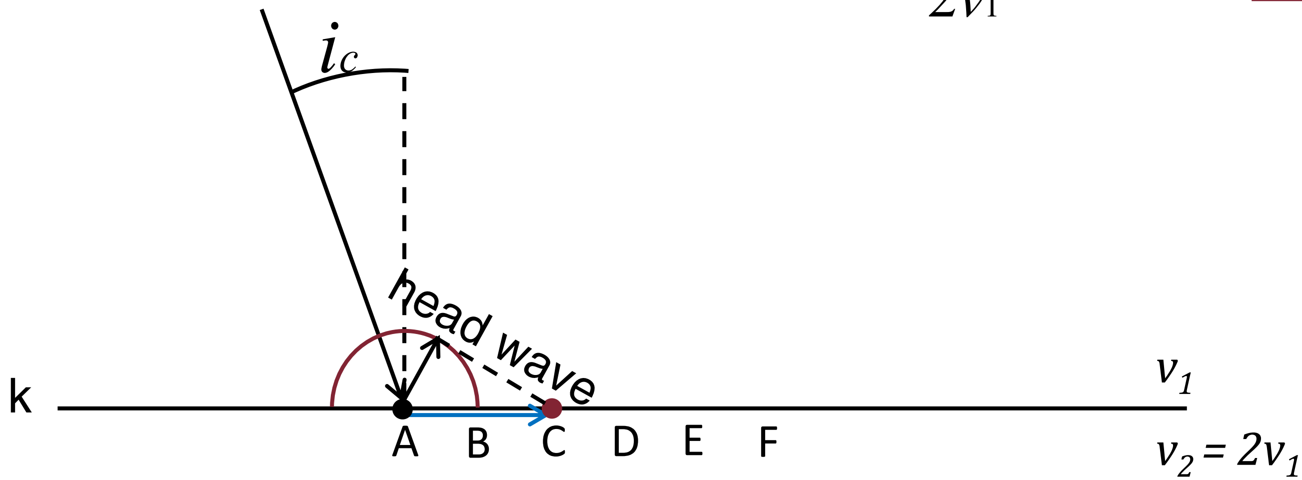
A1: Yes, as a head wave

Ex. 1 Two layers with underlying velocity double than shallow velocity

$$i_c = \sin^{-1}\frac{v_1}{2v_1} = 30^\circ$$

t=0 time of incidence

Δt



Time 0: critical incidence in A and critical wave generation. Point A is a new point source of spherical wavefronts (back-directed to the surface) according to Huygens' principle

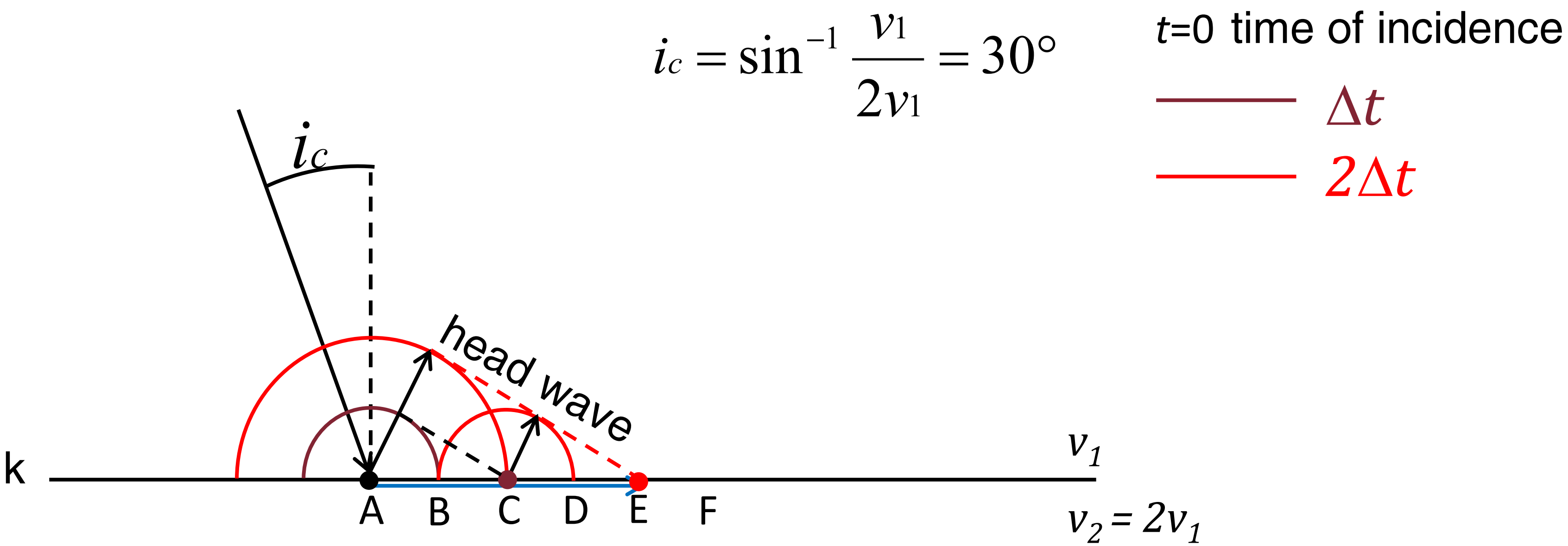
Time Δt: critically refracted wave arrives in C, here generating a new wave according to Huygens' principles. The back-directed wavefront (generated in A at t_0) is now in B.

Critical refraction – Head wave

A1: Yes, as a head wave

Ex. 1 Two layers with underlying velocity double than shallow velocity

$$i_c = \sin^{-1}\frac{v_1}{2v_1} = 30°$$



Time 0: critical incidence in A and critical wave generation. Point A is a new point source of spherical wavefronts (back-directed to the surface) according to Huygens' principle

Time Δt: critically refracted wave arrives in C, here generating a new wave according to Huygens' principles. The back-directed wavefront (generated in A at t_0) is now in B.

Time 2Δt: critically refracted wave arrives in E, while the wavefront generated in A at t_0 is now in C and that generated in C is now in D. From the envelope of wavefronts generated in A and C, a new plane wavefront (**head wave** – *dashed line*) arises, returning back to the surface.

Critical refraction – Head wave

A1: Yes, as a head wave

Ex. 1 Two layers with underlying velocity double than shallow velocity

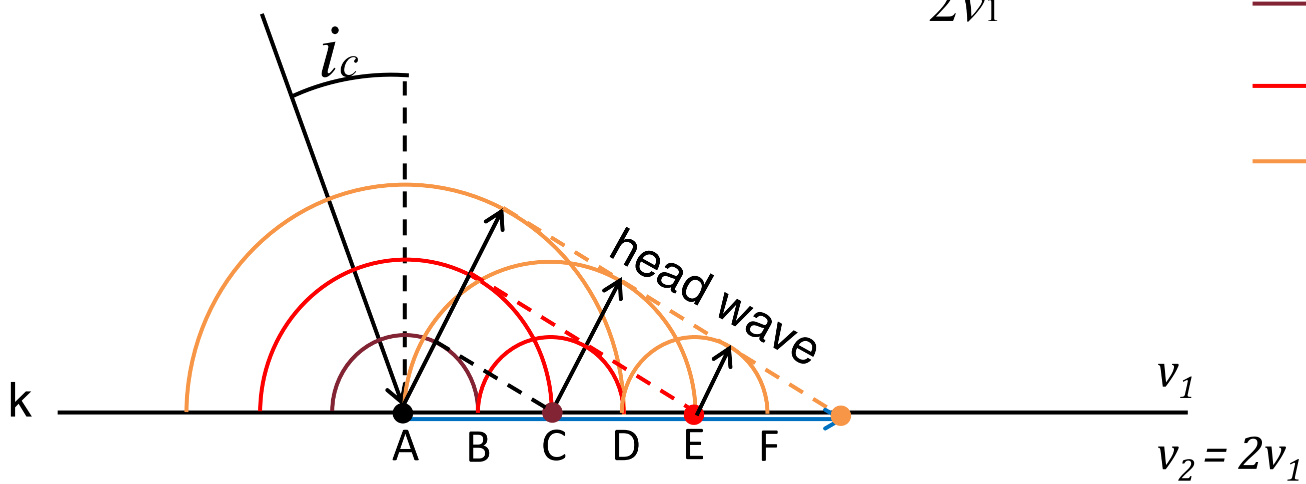
$$i_c = \sin^{-1}\frac{v_1}{2v_1} = 30°$$

t=0 time of incidence

Δt

$2\Delta t$

$3\Delta t$



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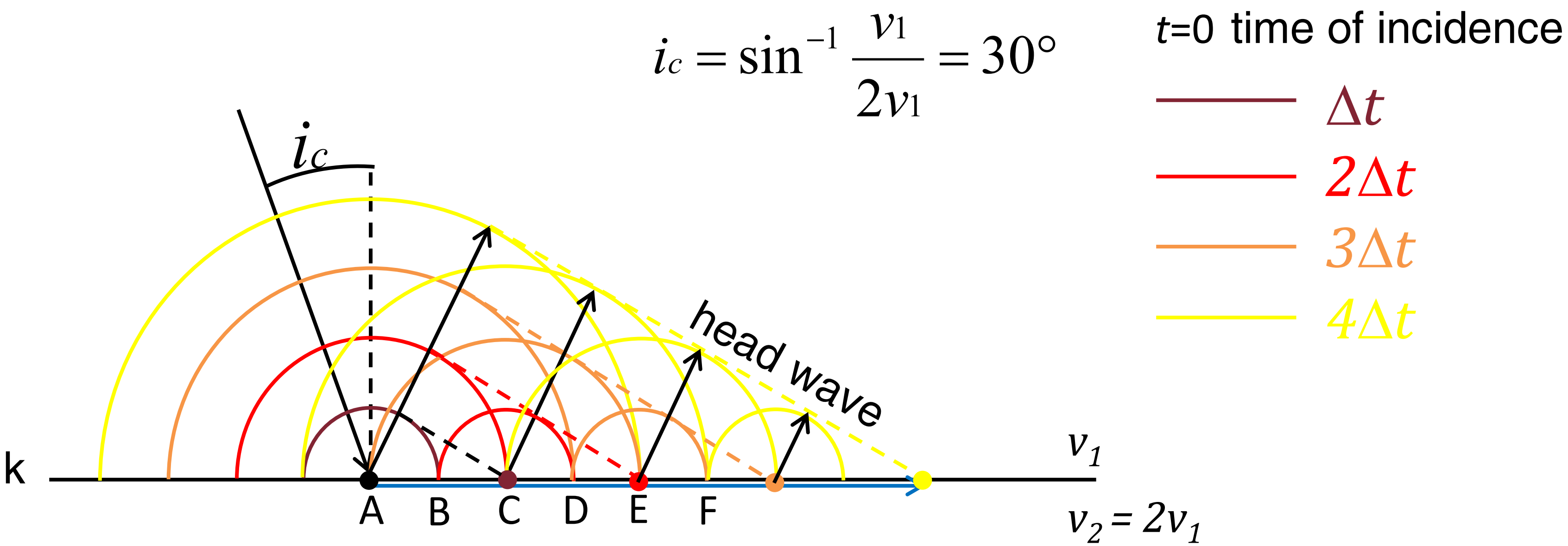
And so on…

Critical refraction – Head wave

A1: Yes, as a head wave

Ex. 1 Two layers with underlying velocity double than shallow velocity

$$i_c = \sin^{-1}\frac{v_1}{2v_1} = 30°$$



Time 0: critical incidence in A and critical wave generation. Point A is a new point source of spherical wavefronts (back-directed to the surface) according to Huygens' principle

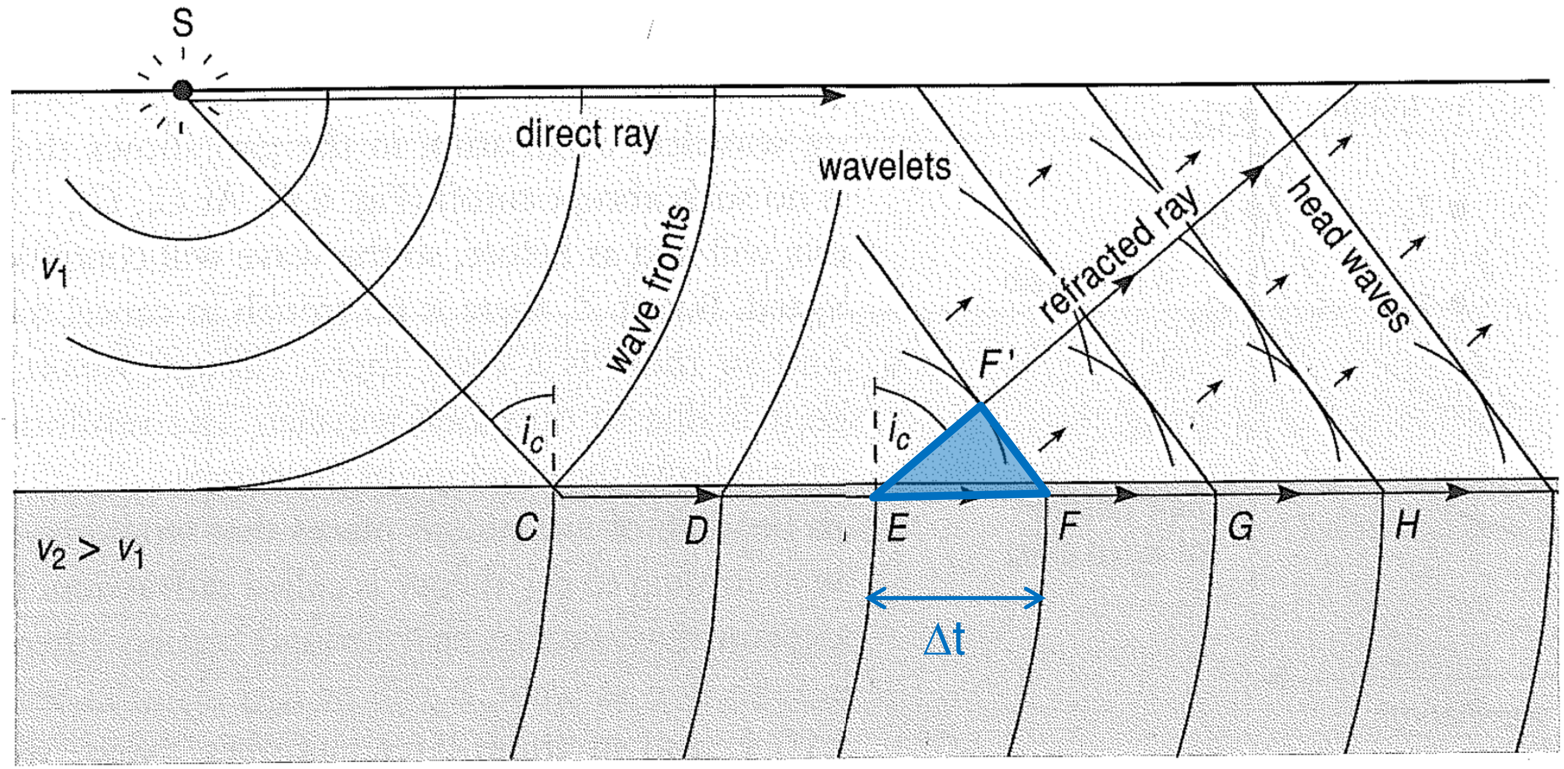
Time Δt: critically refracted wave arrives in C, here generating a new wave according to Huygens' principles. The back-directed wavefront (generated in A at t_0) is now in B.

Time 2Δt: critically refracted wave arrives in E, while the wavefront generated in A at t_0 is now in C and that generated in C is now in D. From the envelope of wavefronts generated in A and C, a new plane wavefront (**head wave** – *dashed line*) arises, returning back to the surface.

And so on…

Critical refraction – Head wave

A2: At the critical angle

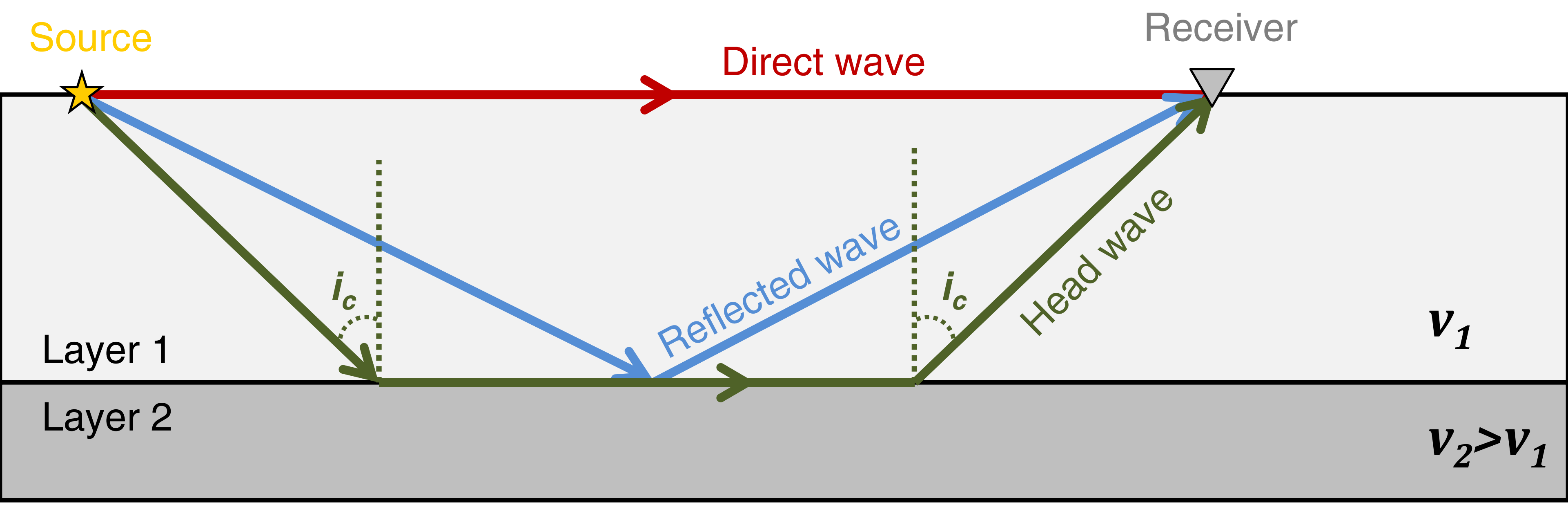


$$\overline{EF'} = \overline{EF}\sin i_{head} \Rightarrow \sin i_{head} = \frac{\overline{EF'}}{\overline{EF}} = \frac{v_1 \Delta t}{v_2 \Delta t} = \frac{v_1}{v_2} = \sin i_c$$

Surface seismic survey

When seismic signal is received on surface at a certain distance from the shot point, I have basically 3 main contributions on the recorded signal:

- ✓ Direct wave
- ✓ Reflected wave
- ✓ Head wave (critically refracted)

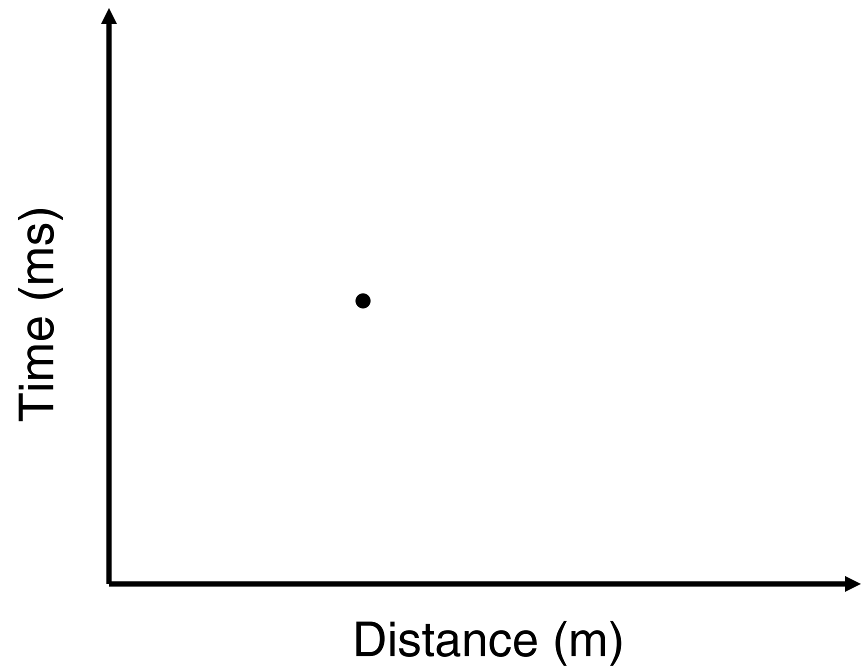


Q. Which contribution does arrive first?

Time-distance plot

To this aim we need to plot the first breaks (picked times) vs. the shot-receiver distance (**offset**).

Understanding the **time-distance plot** is pivotal for seismic refraction.



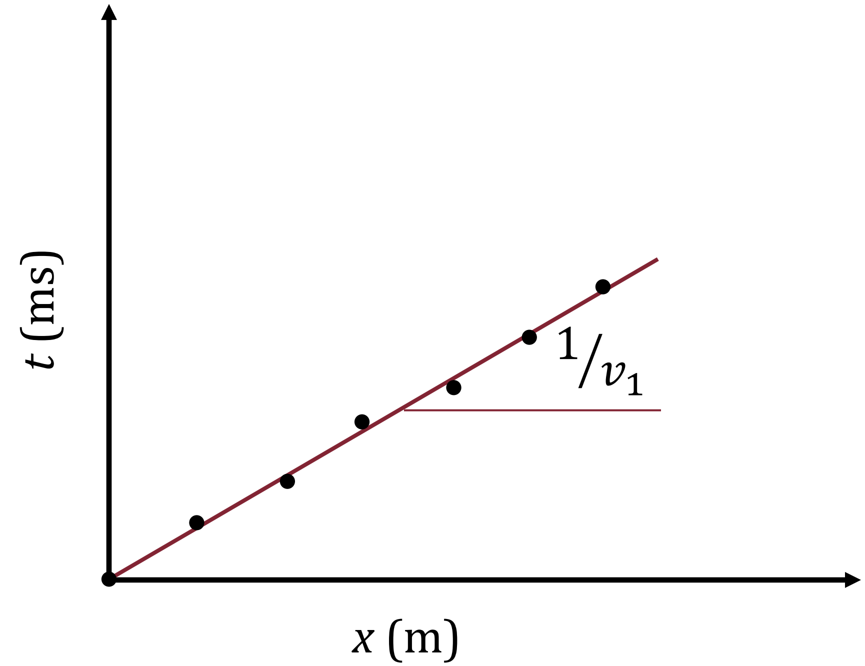
If we have only a single receiver, we have only a single point on the *x-t* plot and we cannot assess if it is related to direct or head wave (**reflection is never a first arrival!).**

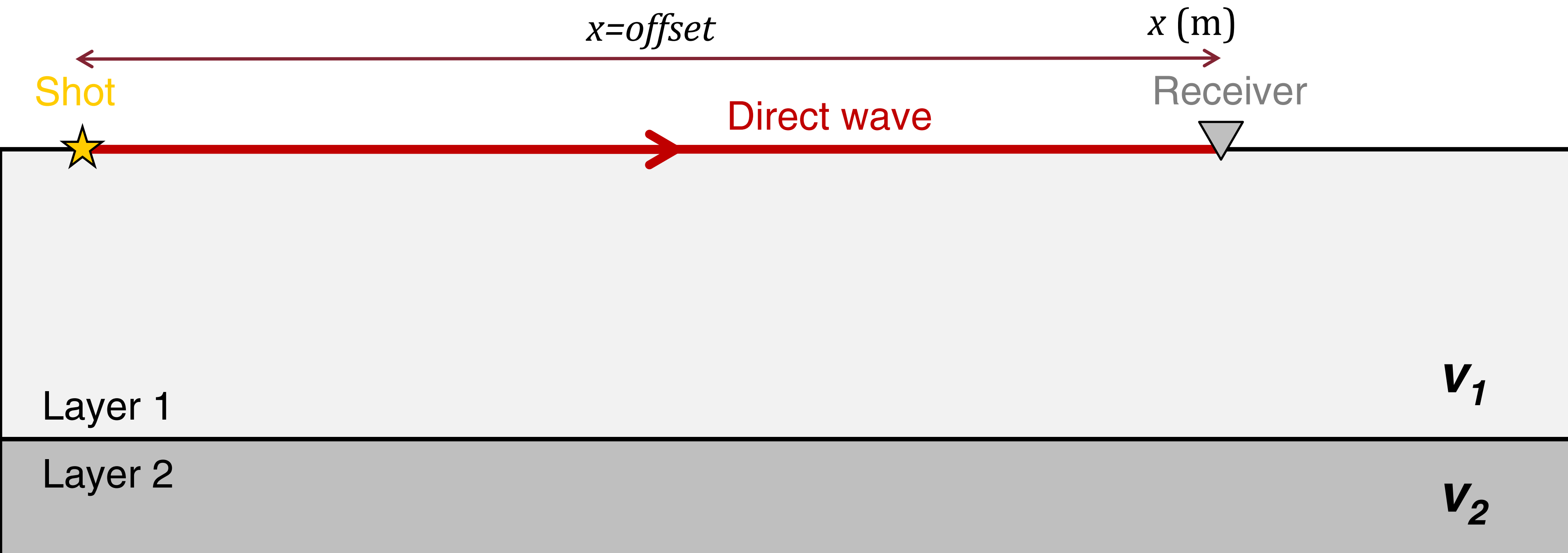
Therefore, we need more (equally spaced) receivers in order to have more points: consequently we can interpolate these points with i.e. straight lines…

Time-distance plot – Direct wave

The direct wave is seen as a straight line on the *x-t* plot, with angular coefficient equal to the inverse of the velocity of the shallow layer.

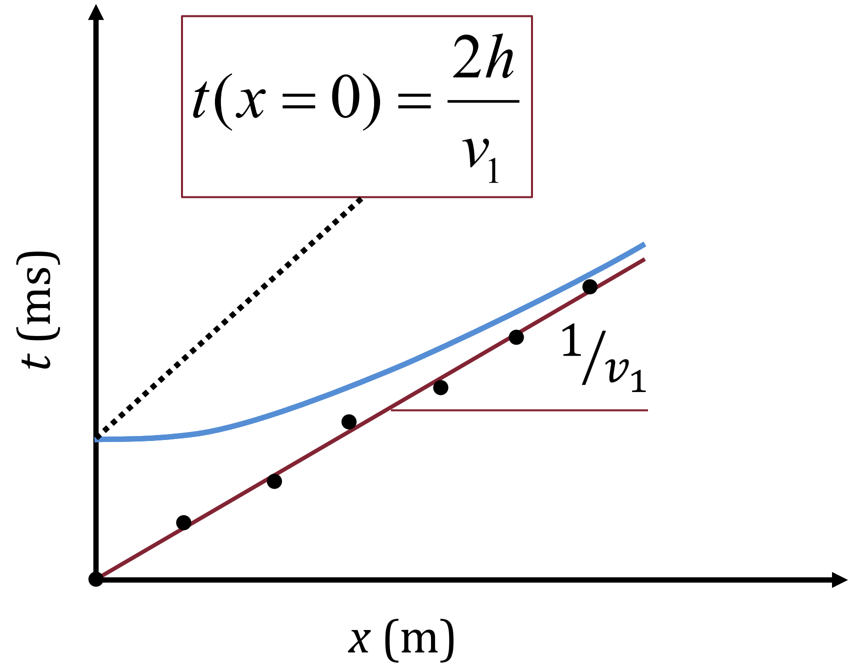
$$t_{direct} = \frac{x}{v_1}$$

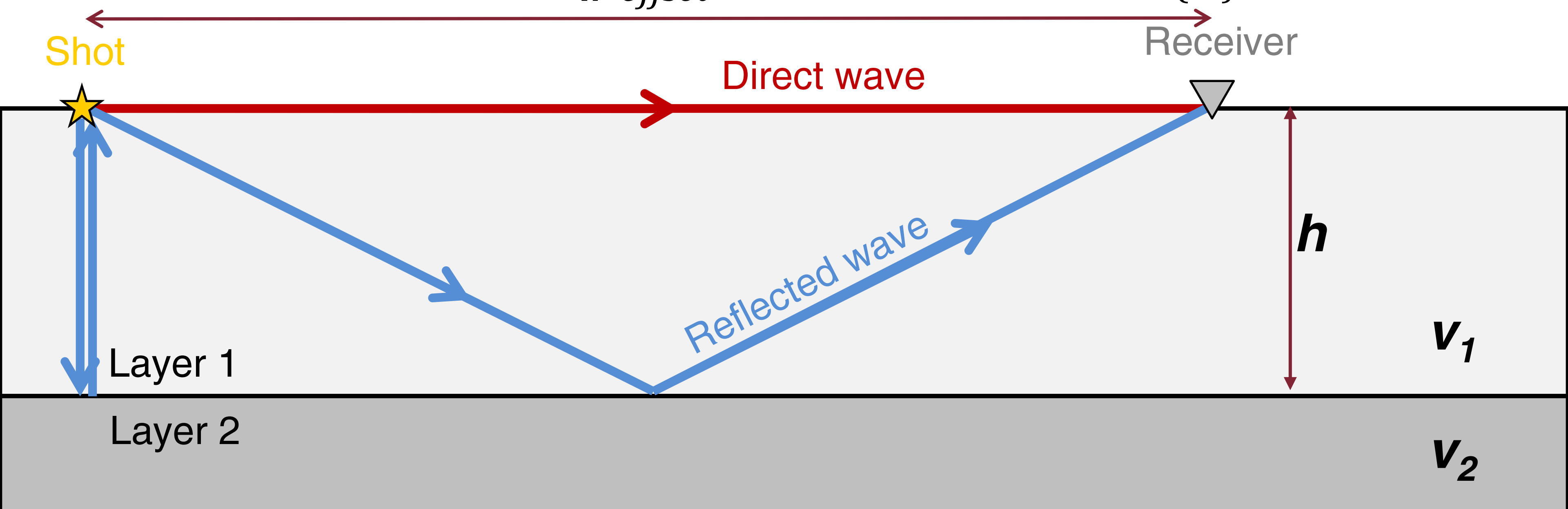




Time-distance plot – Reflected wave

Reflection is never a first arrival. However, it would appear as a hyperbola on the *x*-*t* plot.
It is asymptotic with direct wave for large offsets, while through the intercept time (*normal incidence*) one can assess the thickness of the first layer given its velocity.

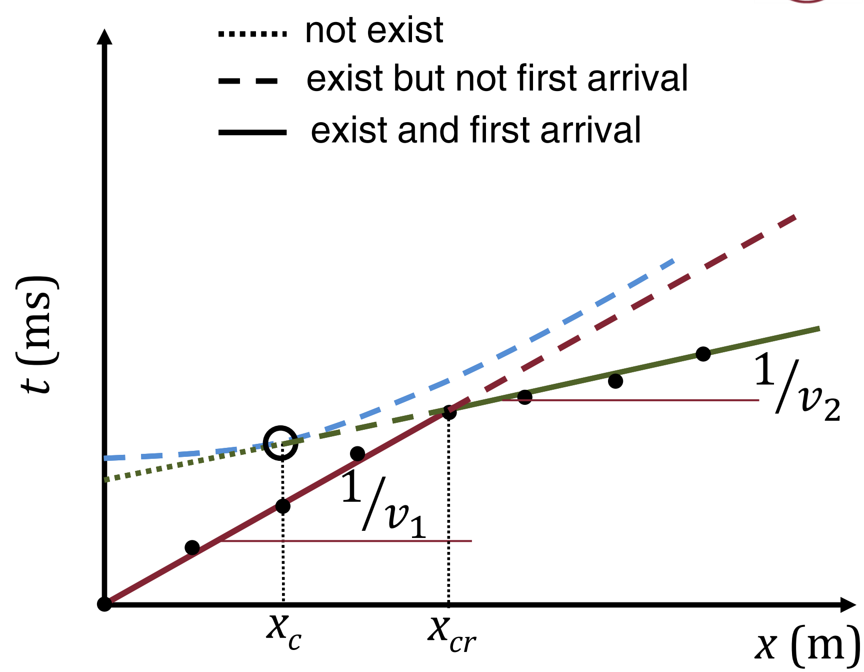


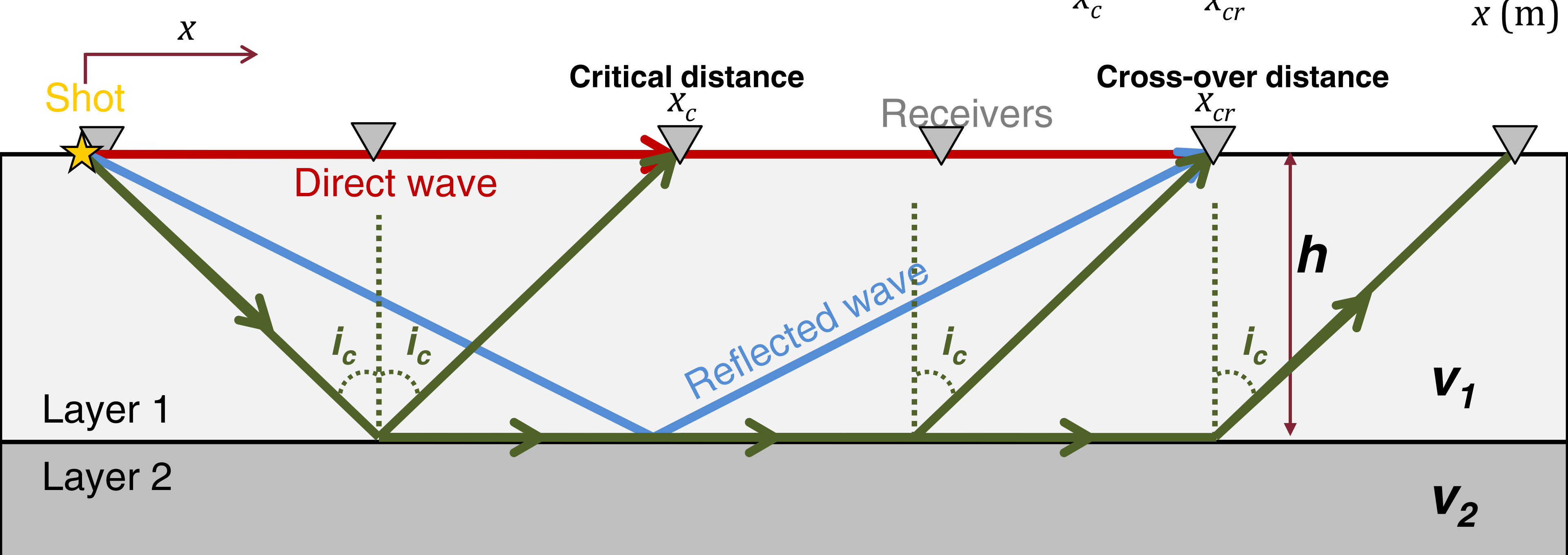


Time-distance plot – Refracted wave

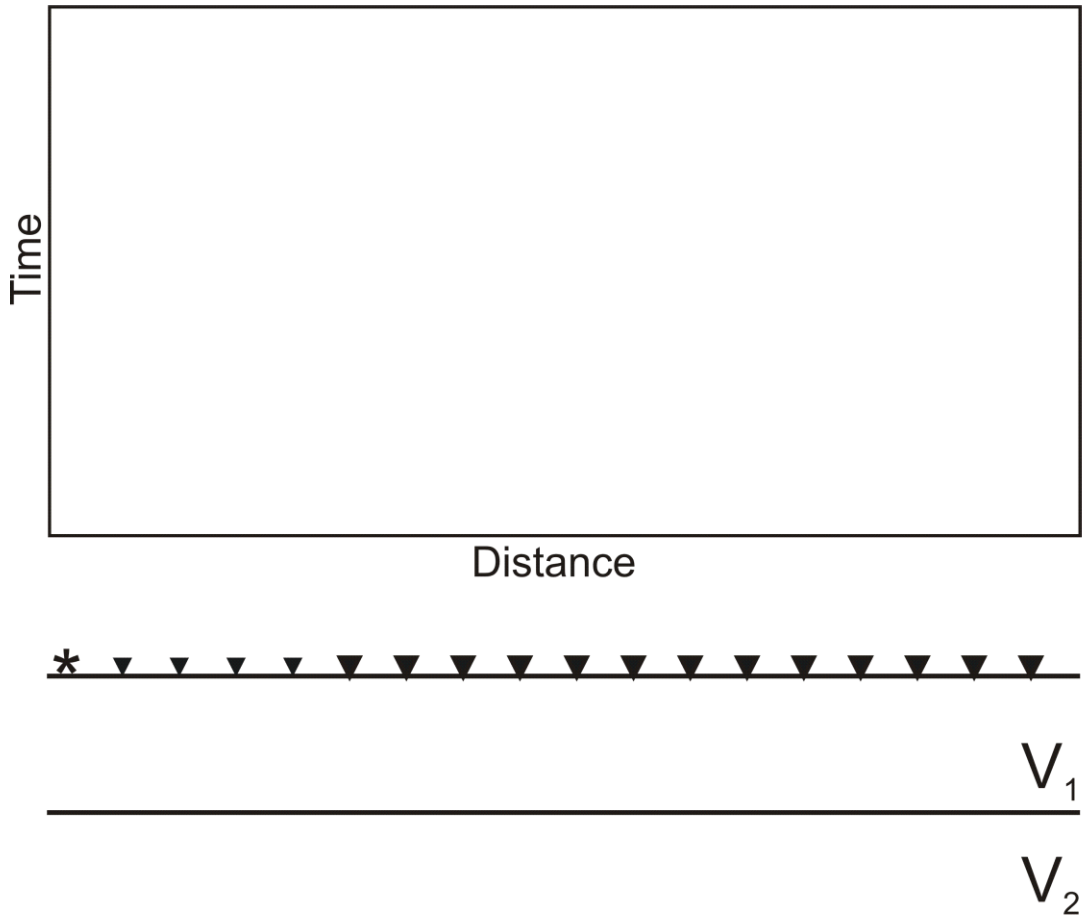
Critically refracted (*head*) wave is a straight line on *x*-*t* plot.

Only **exists** on surface after the **critical distance** x_c and it is the **first arrival only after the cross-over distance** $x_{cr.}$

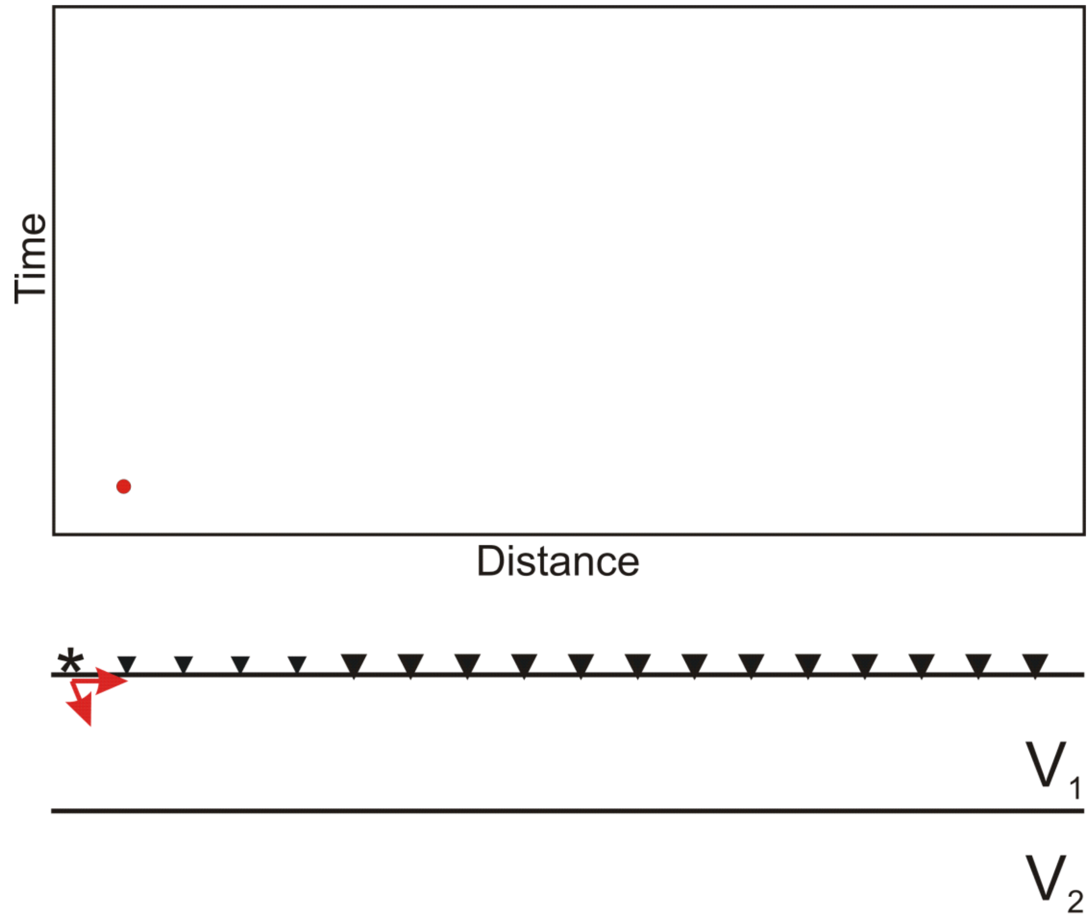




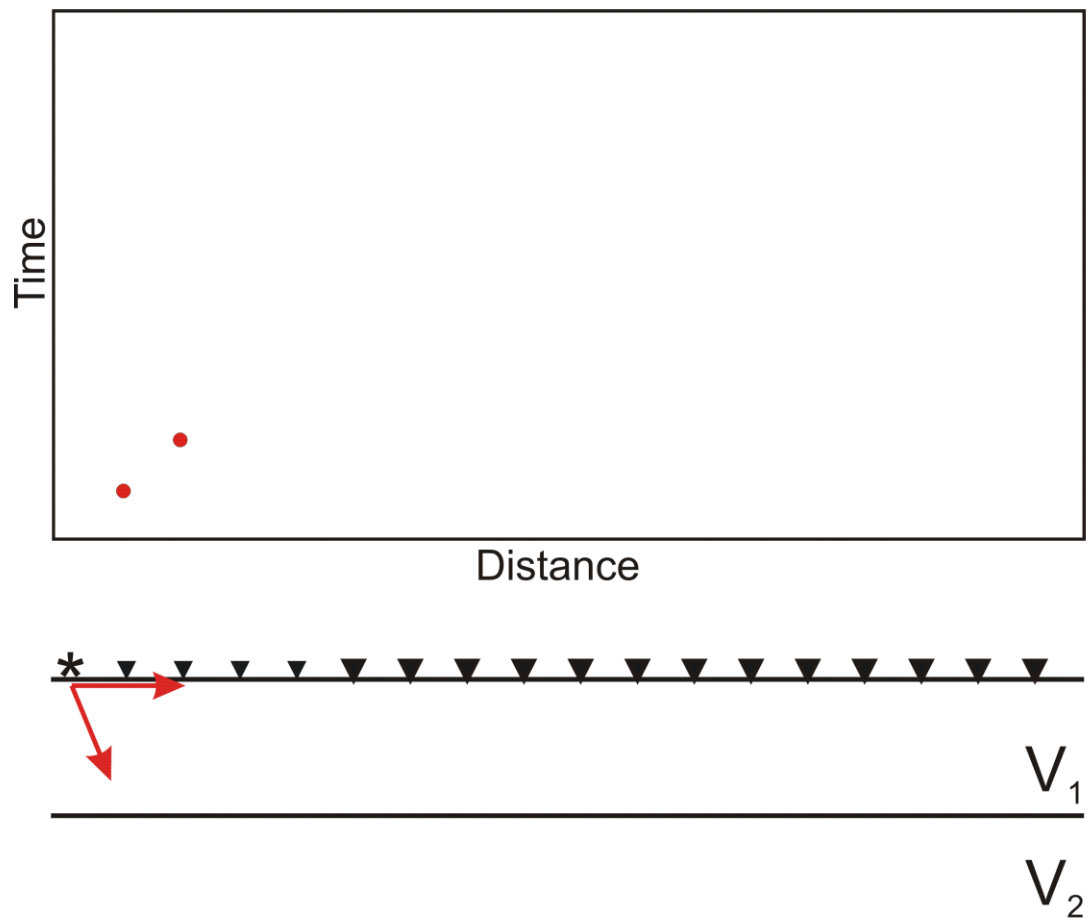
Refracted vs. Direct wave



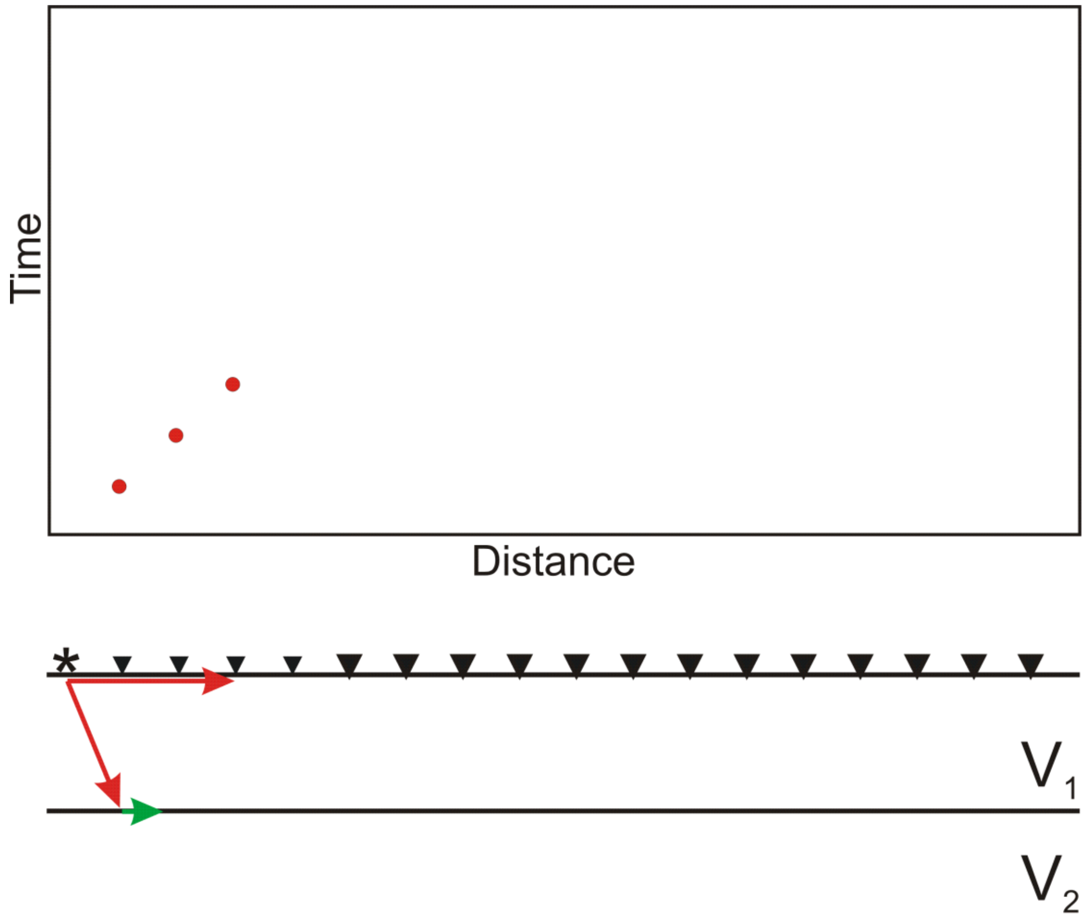
Refracted vs. Direct wave



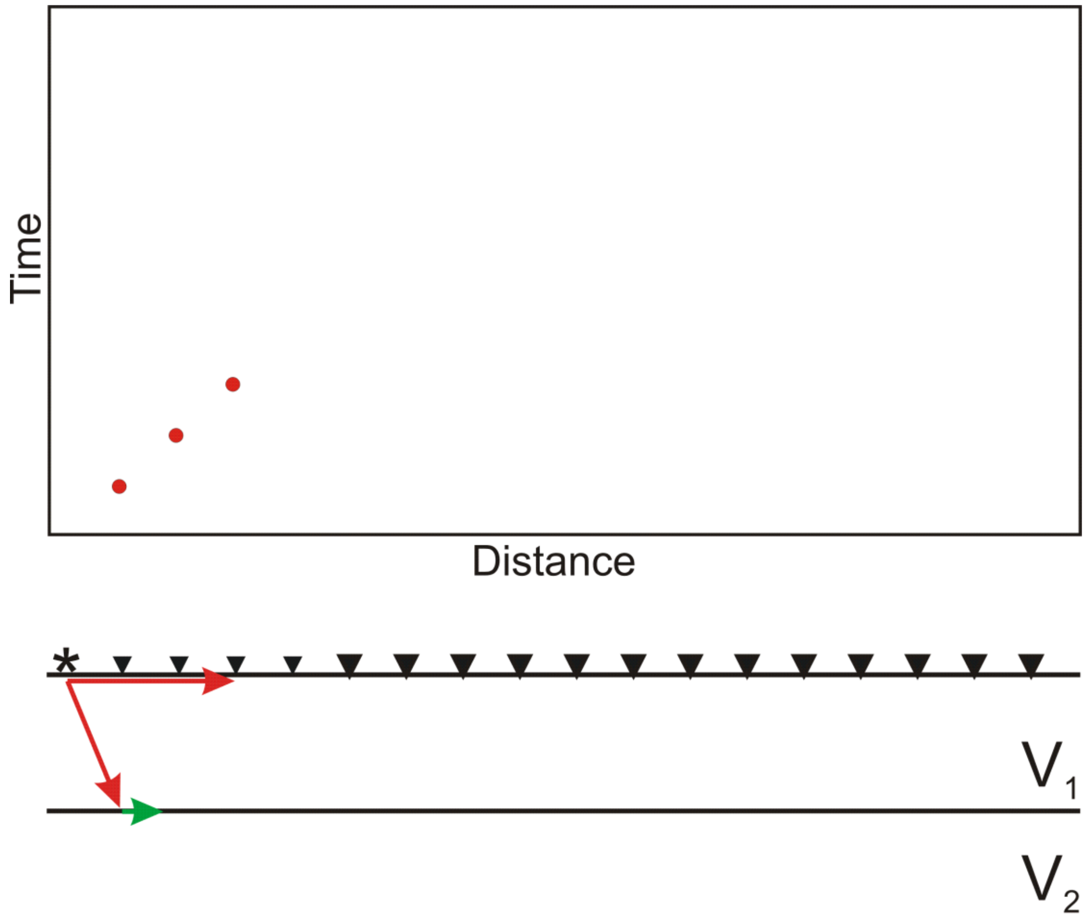
Refracted vs. Direct wave



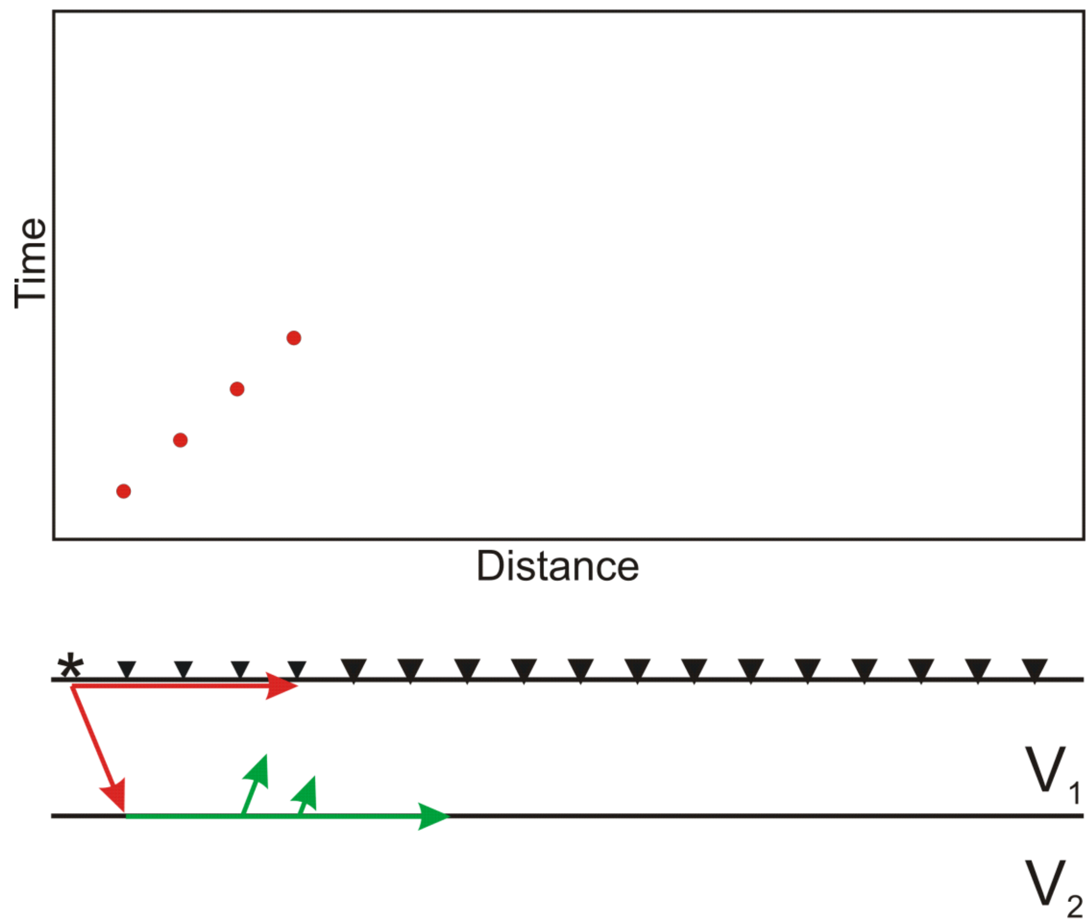
Refracted vs. Direct wave



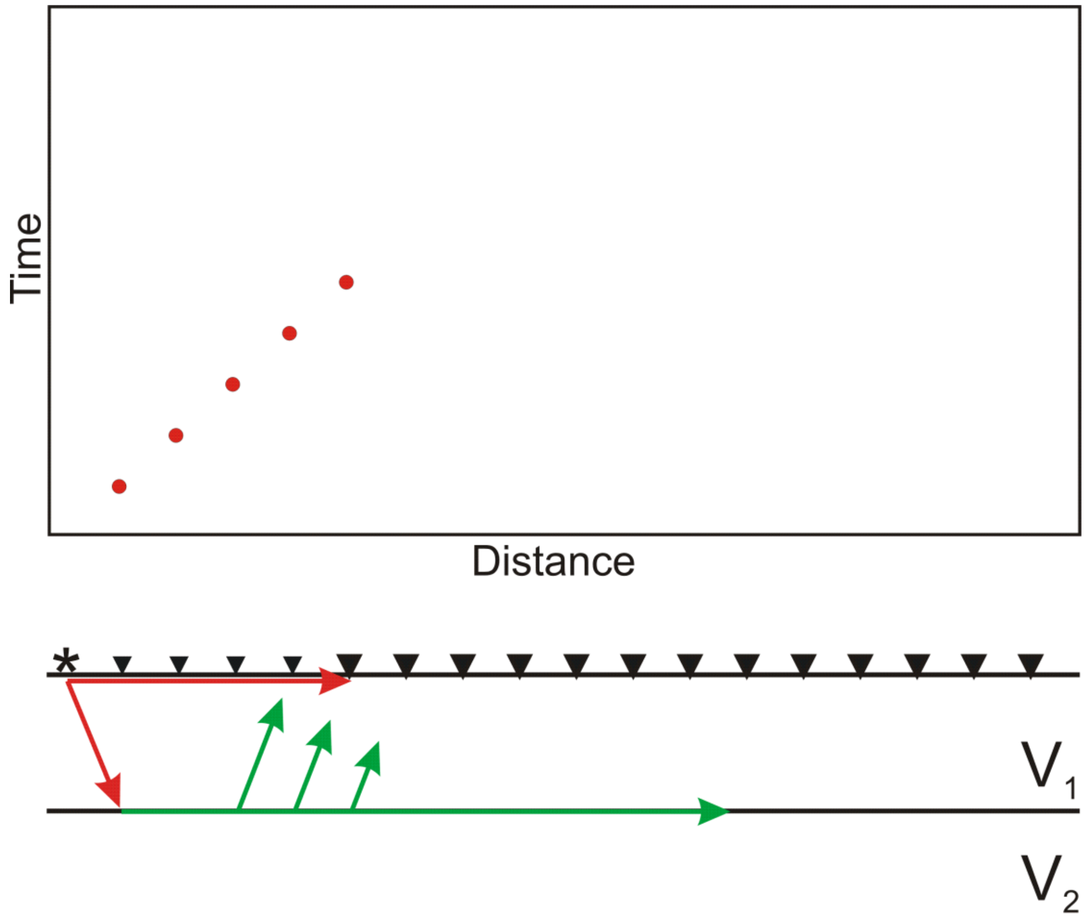
Refracted vs. Direct wave



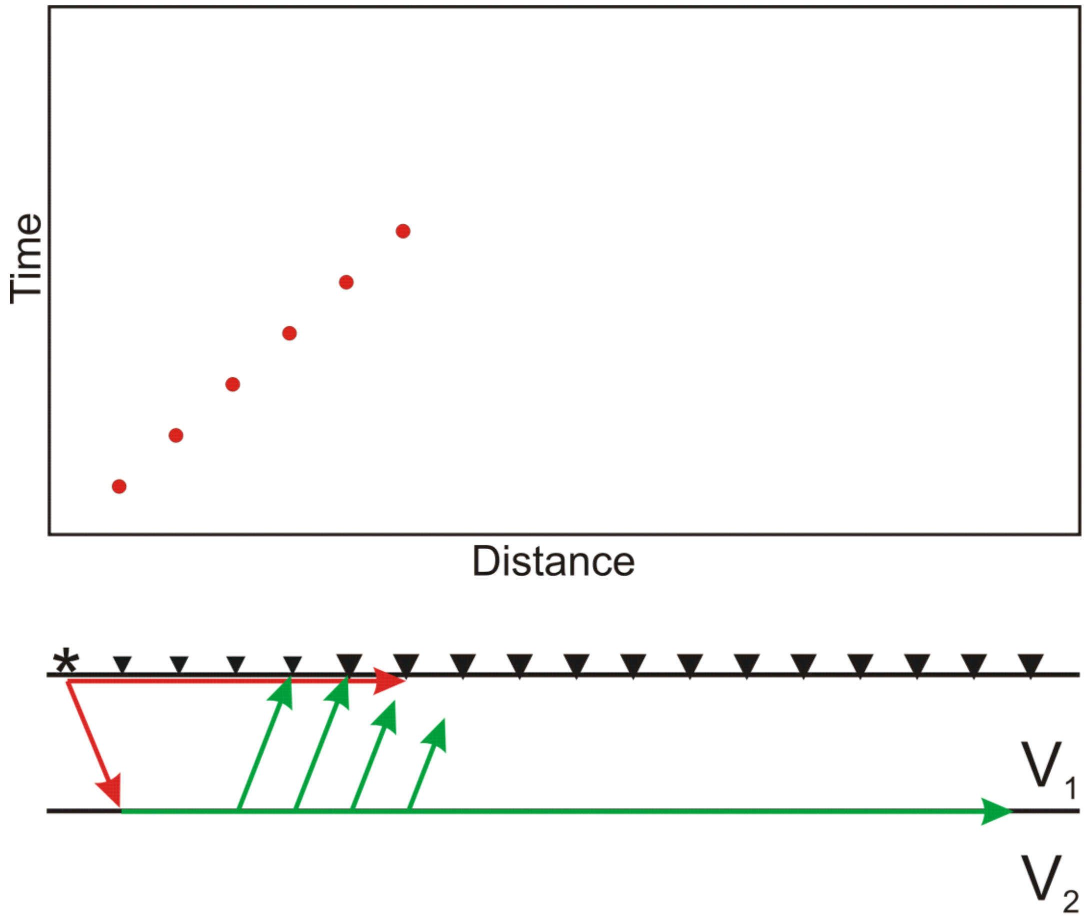
Refracted vs. Direct wave



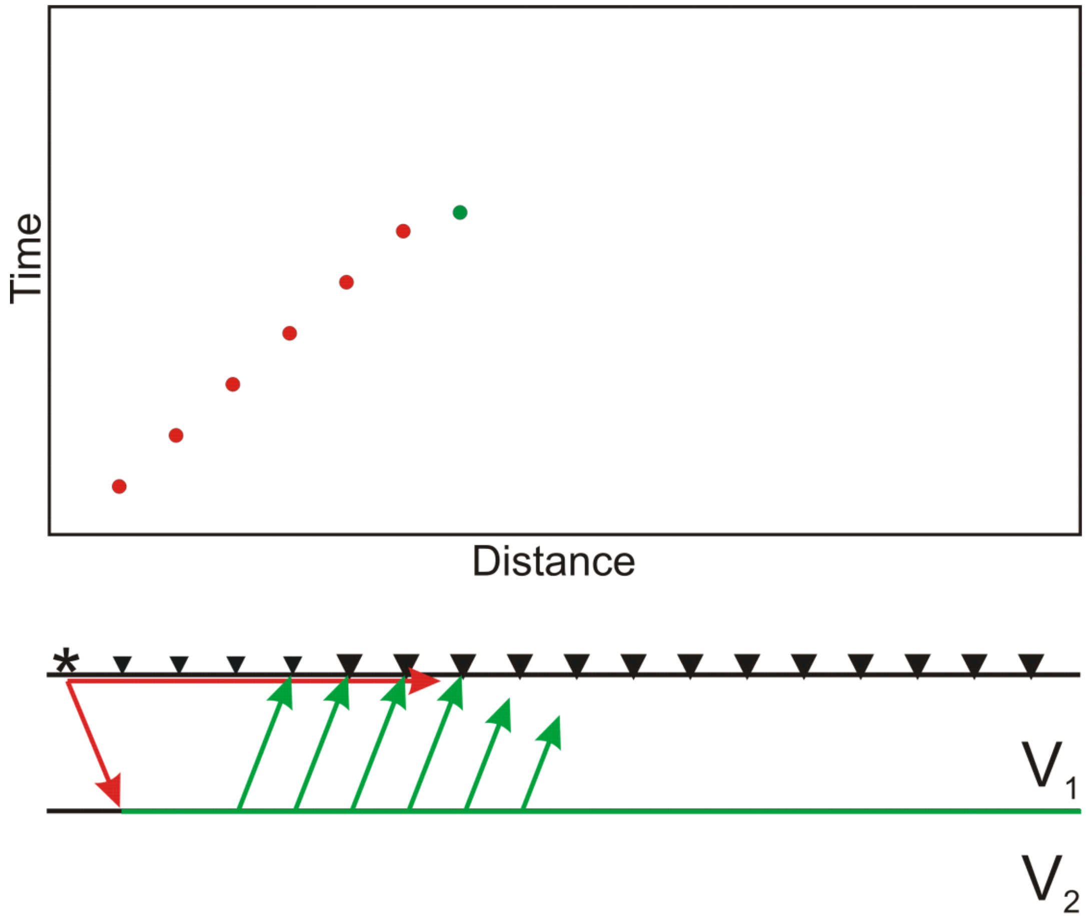
Refracted vs. Direct wave



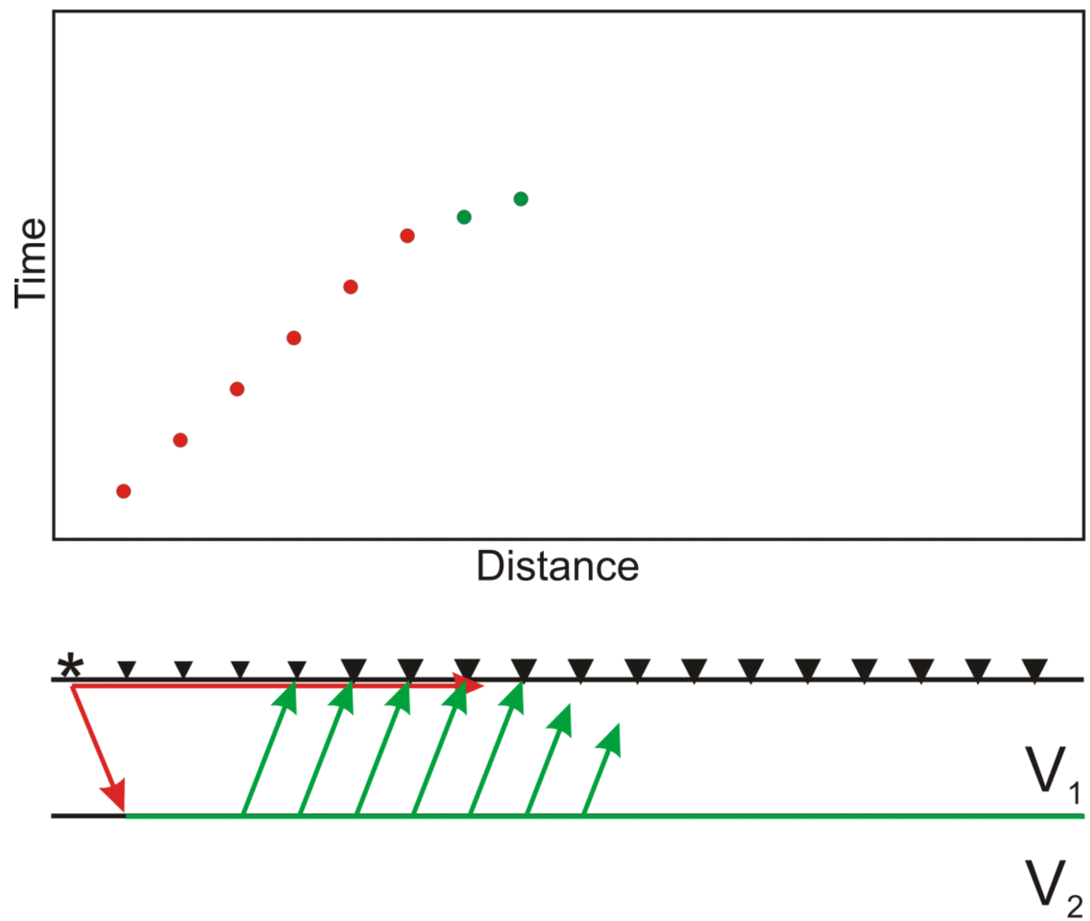
Refracted vs. Direct wave



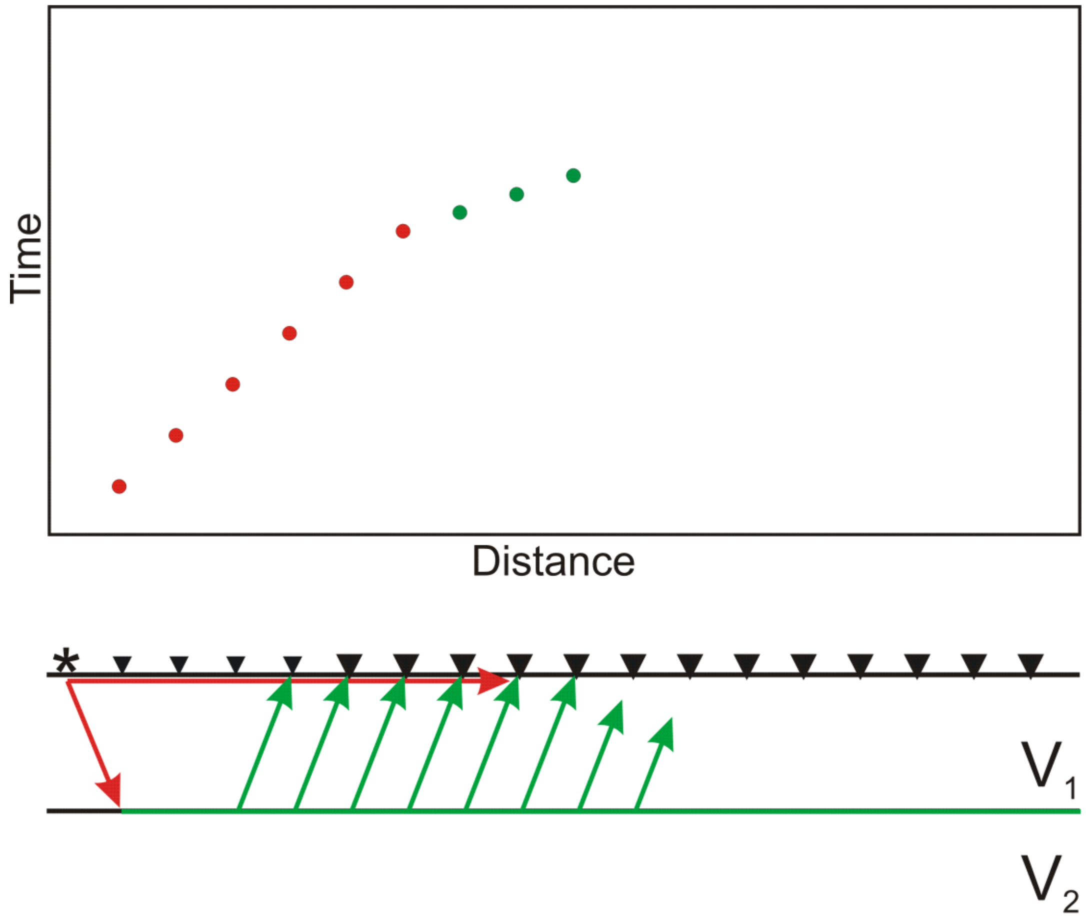
Refracted vs. Direct wave



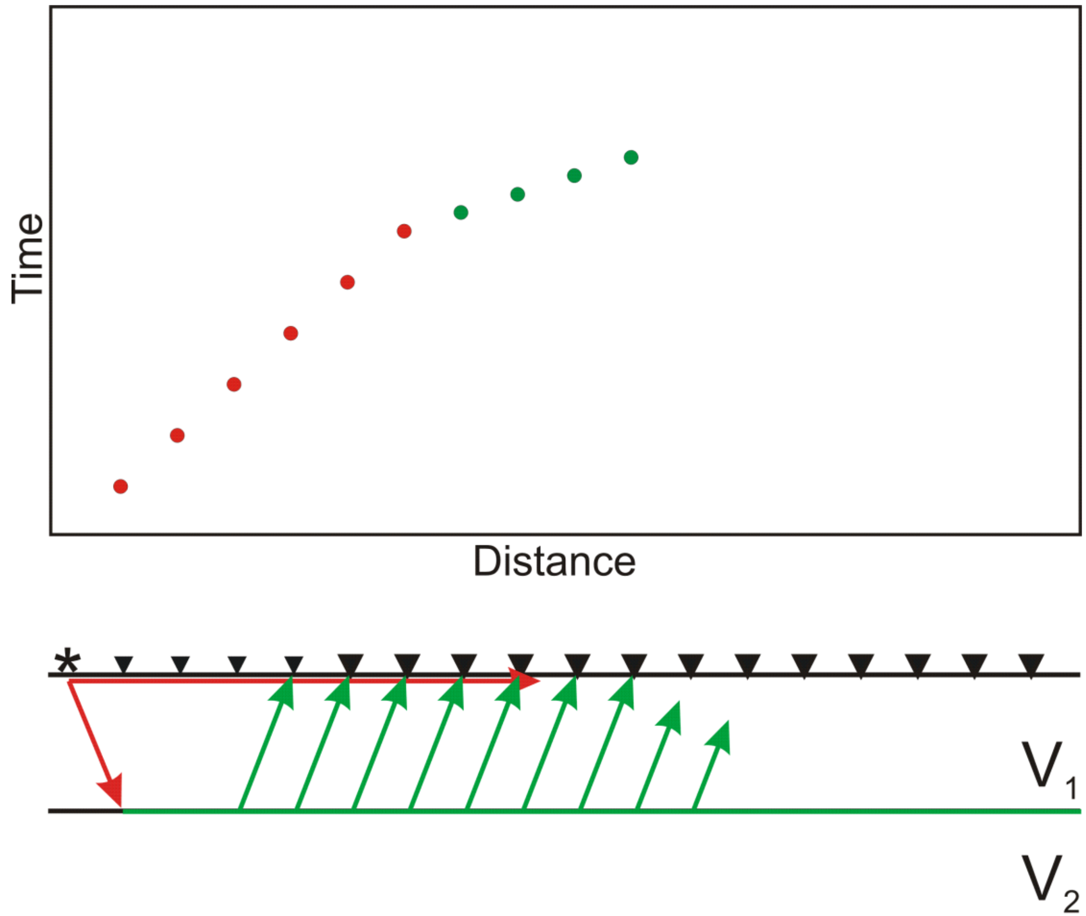
Refracted vs. Direct wave



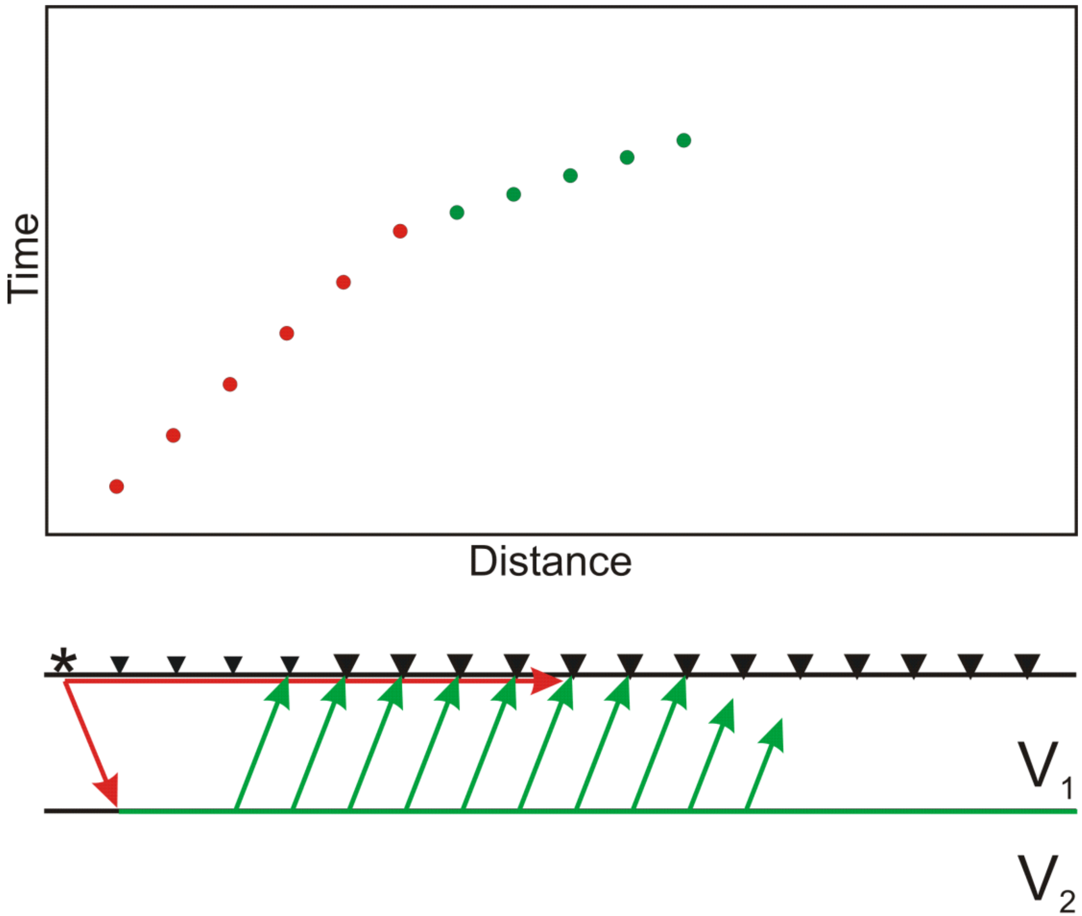
Refracted vs. Direct wave



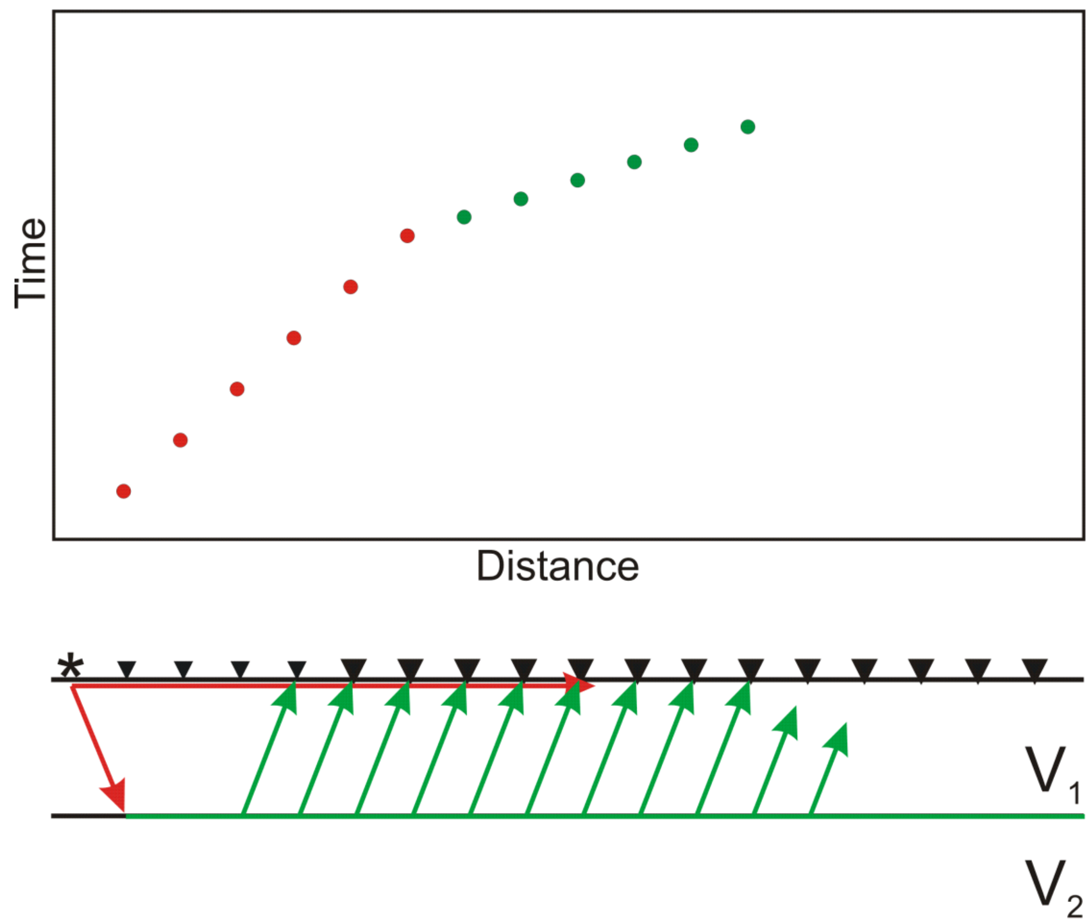
Refracted vs. Direct wave



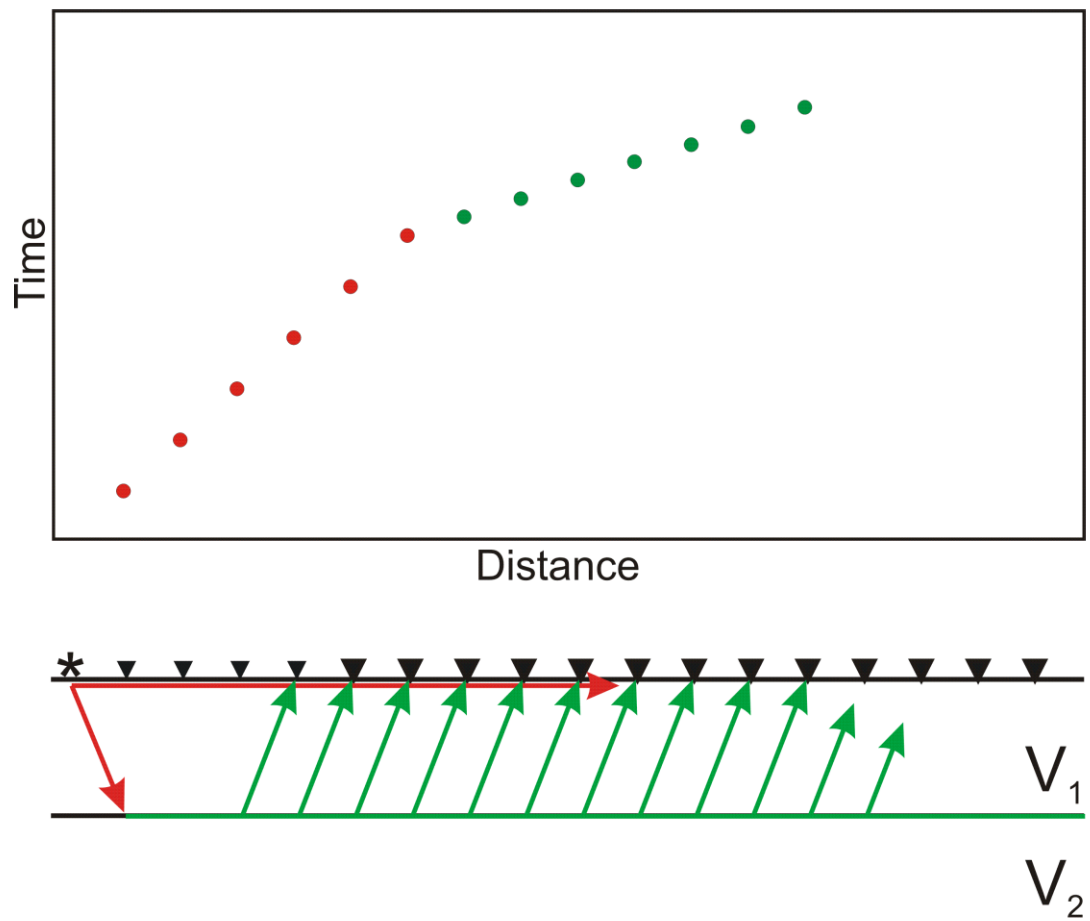
Refracted vs. Direct wave



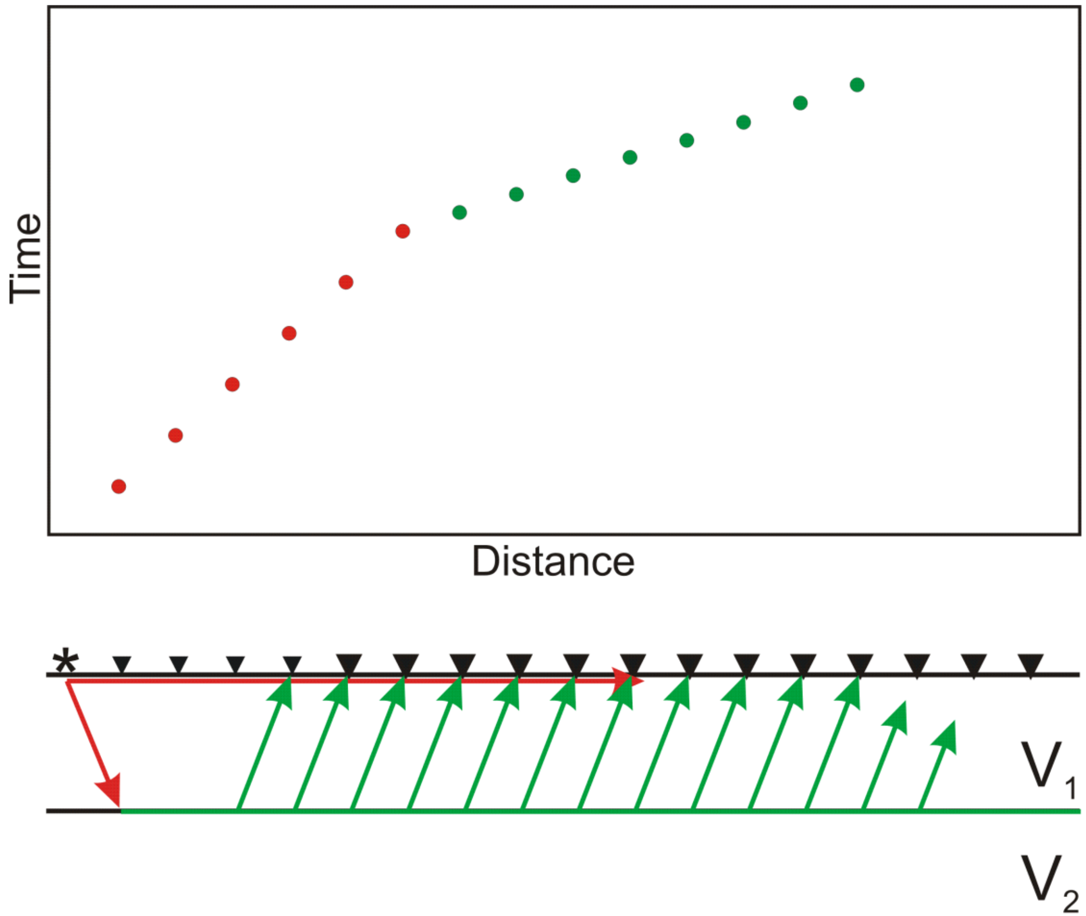
Refracted vs. Direct wave



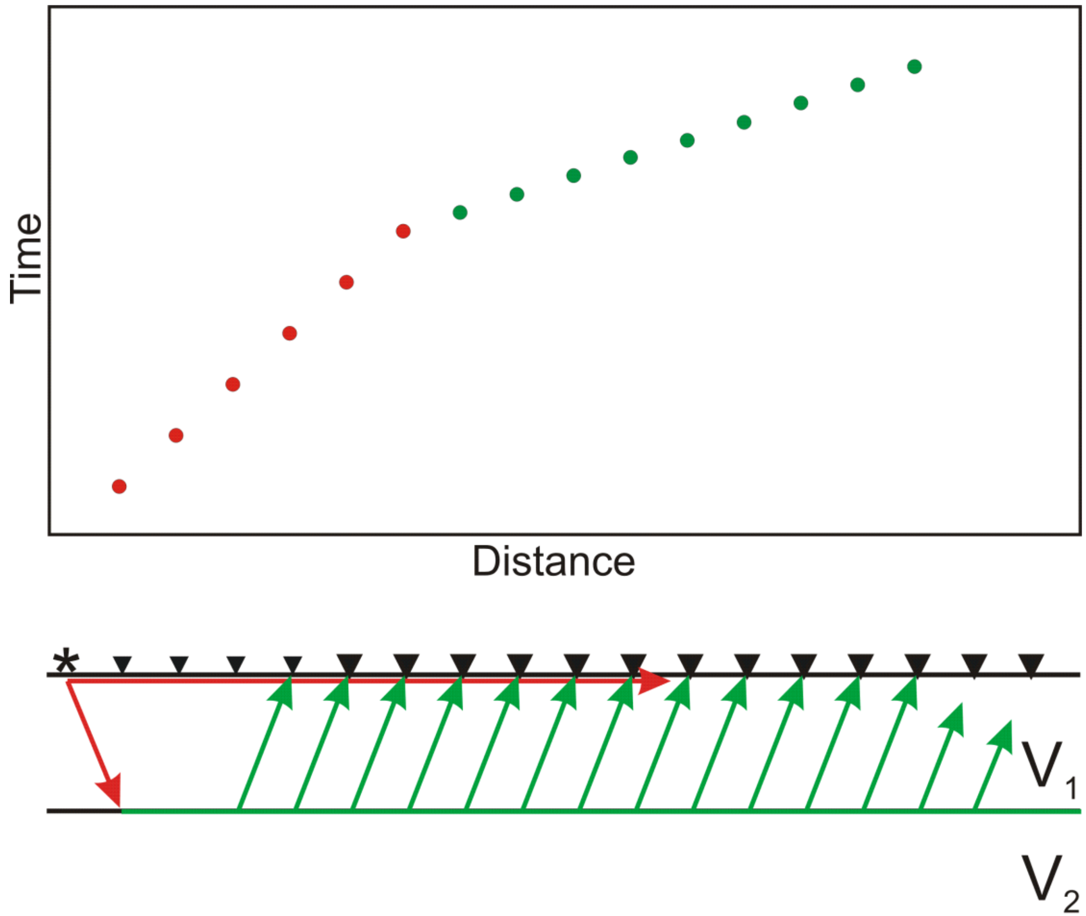
Refracted vs. Direct wave



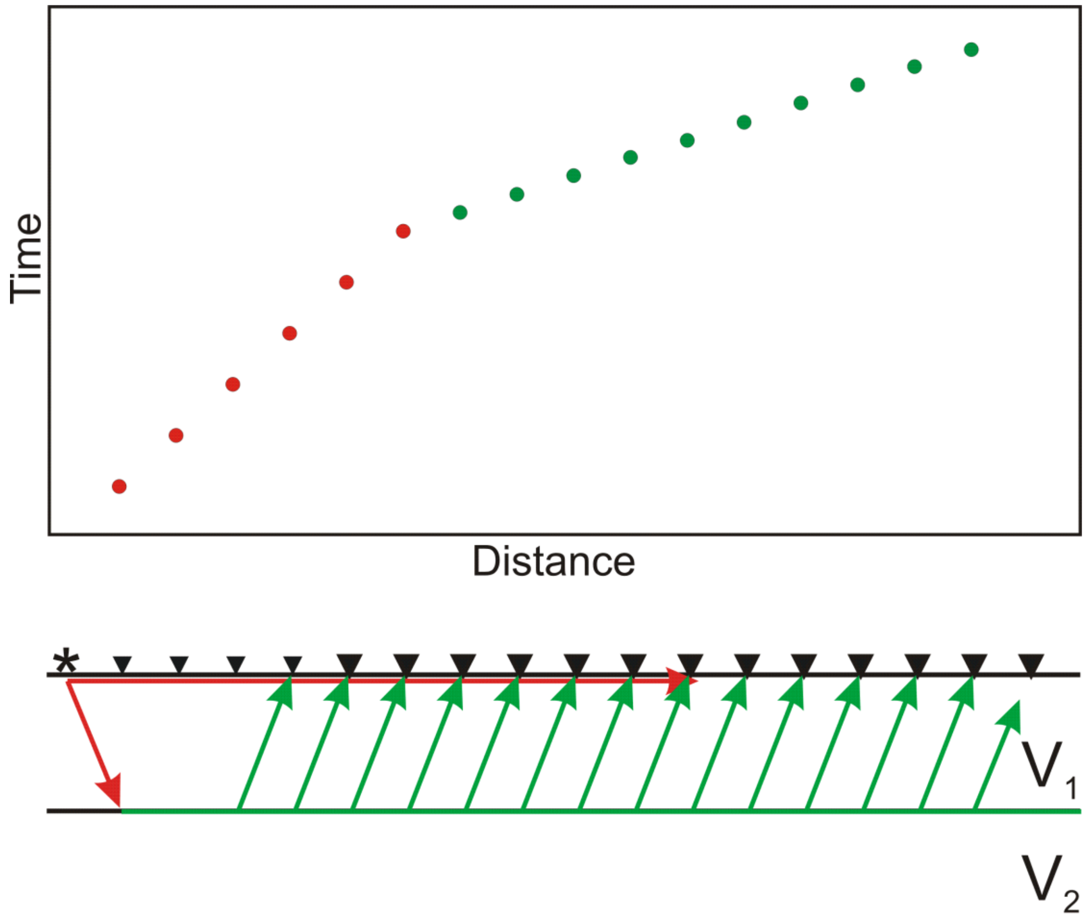
Refracted vs. Direct wave



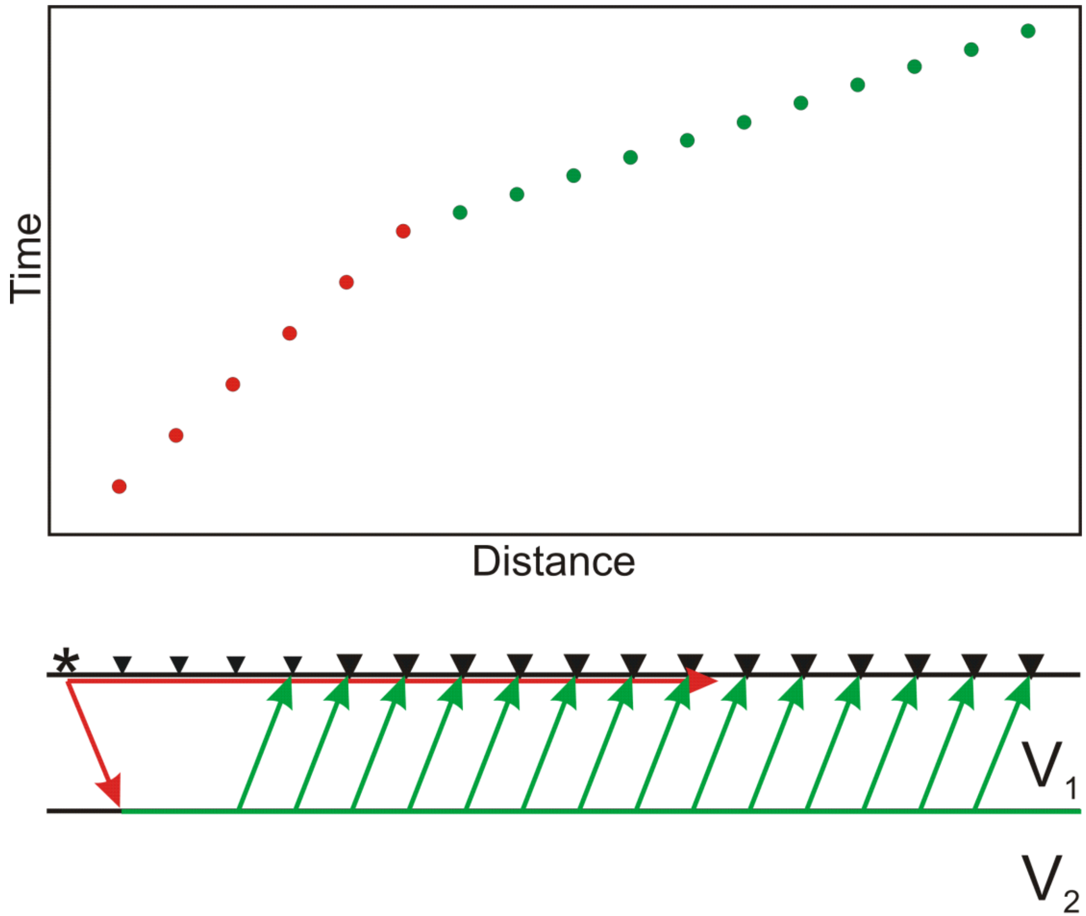
Refracted vs. Direct wave



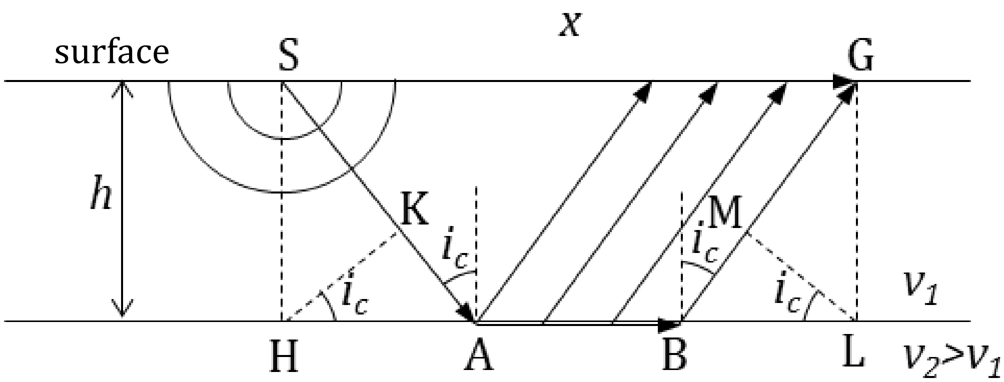
Refracted vs. Direct wave

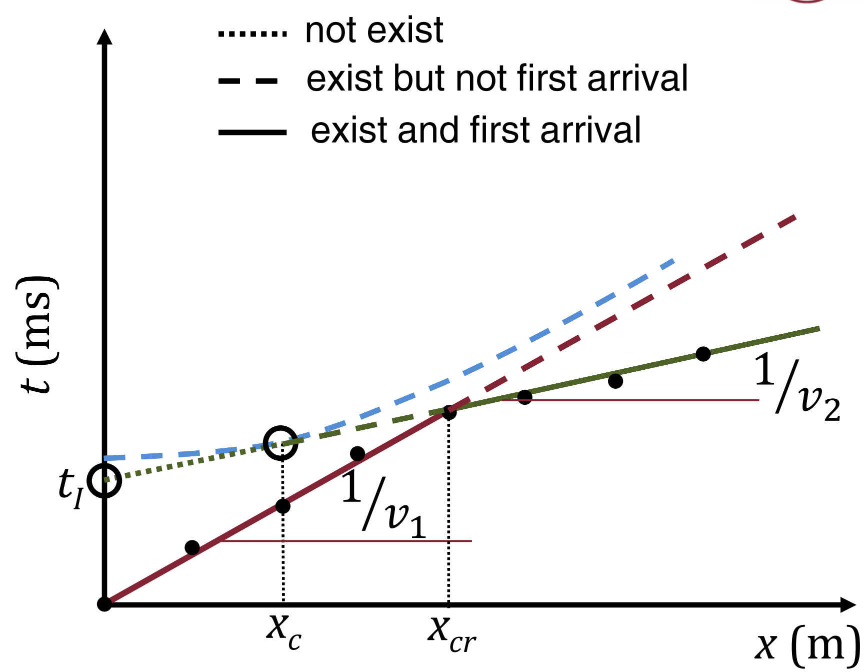


Refracted vs. Direct wave



Refracted wave





$$t^{REFR} = \frac{\overline{SA}}{v_1} + \frac{\overline{AB}}{v_2} + \frac{\overline{BG}}{v_1} = 2\frac{\overline{SA}}{v_1} + \frac{\overline{AB}}{v_2}$$

$$\frac{\overline{SA}}{v_1} = \frac{\overline{SK}}{v_1} + \frac{\overline{KA}}{v_1} \qquad \frac{\overline{SA}}{v_1} = \frac{\overline{SH}\cos i_c}{v_1} + \frac{\overline{HA}}{v_2}$$

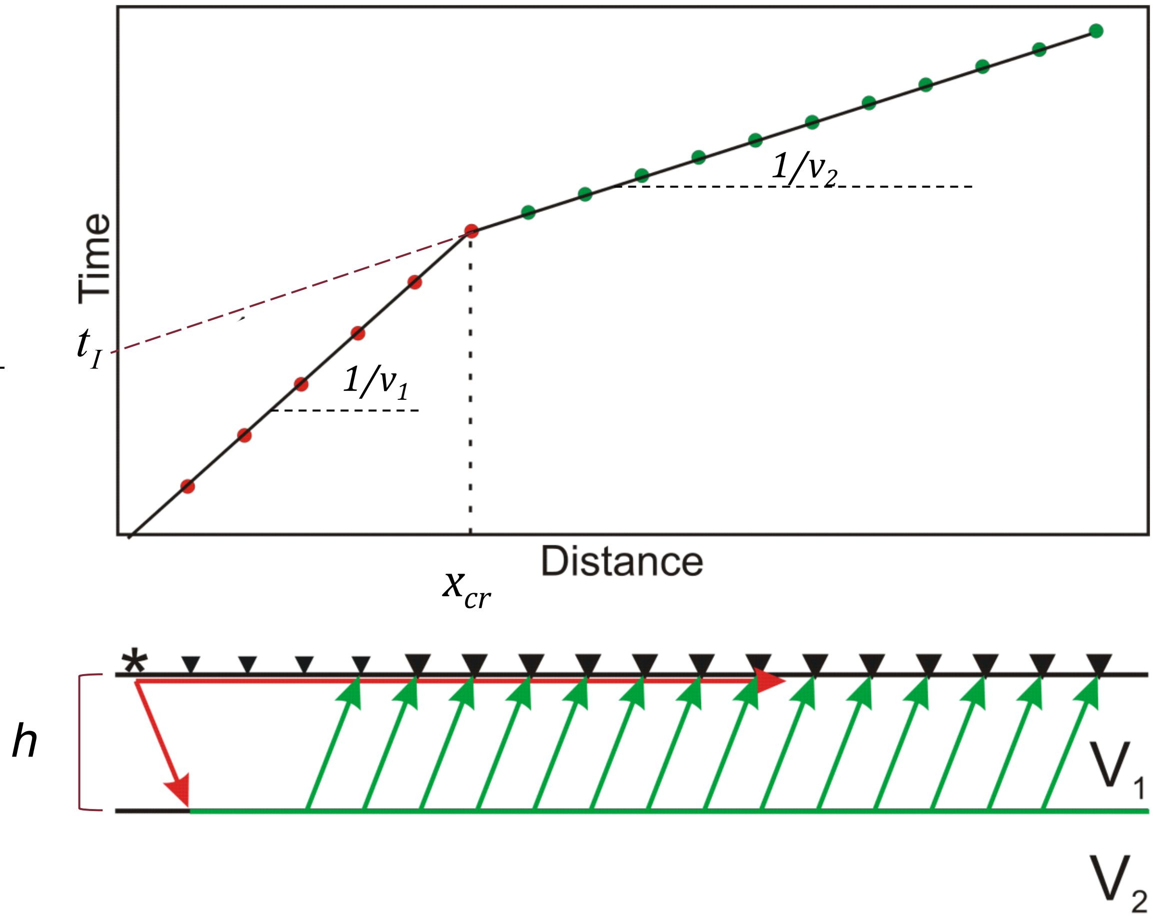
$$t^{REFR} = 2\frac{\overline{SH}\cos i_c}{v_1} + 2\frac{\overline{HA}}{v_2} + \frac{\overline{AB}}{v_2} \longrightarrow t^{REFR} = \underbrace{\frac{2h\cos i_c}{v_1}}_{t_I} + \frac{x}{v_2}$$

Graphical processing

Given t_i by graphical picking and v_1 and v_2 as the inverse of the angular coefficients we can retrieve *h*

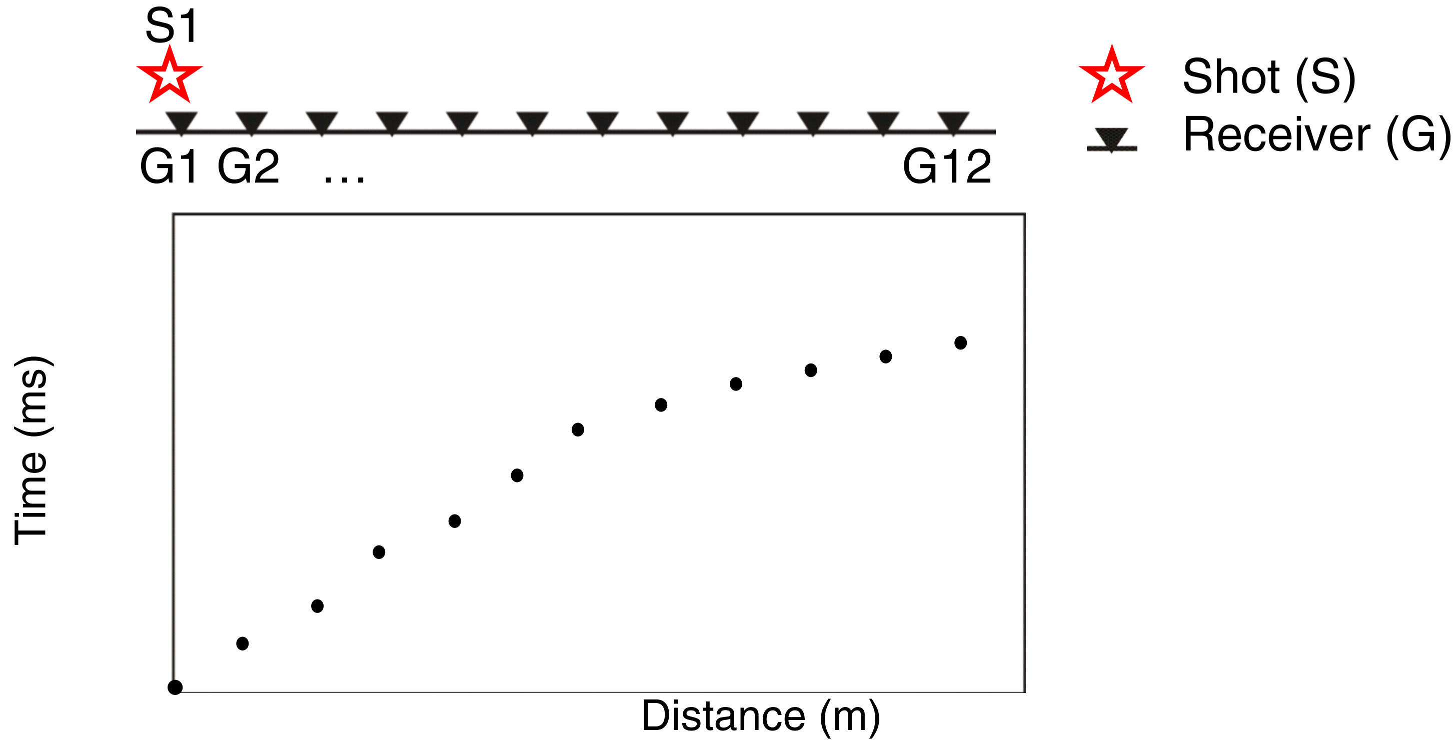
$$h = \frac{v_1 t_I}{2\cos i_c}$$

$$t_I = \frac{2h\cos i_c}{V_1}$$



Seismic refraction

Ex. 2. 12 geophones (G1-G12) spaced 1 m apart with seismic source on G1



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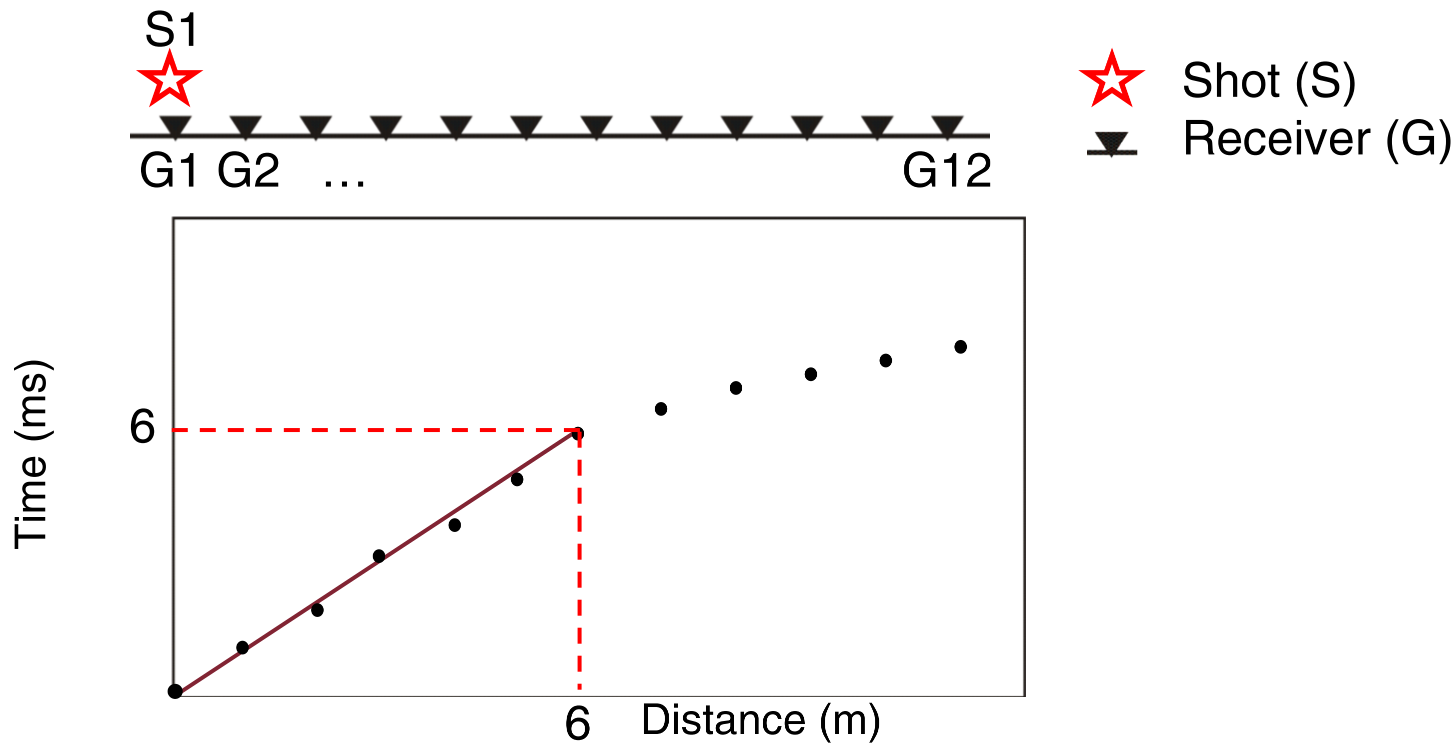
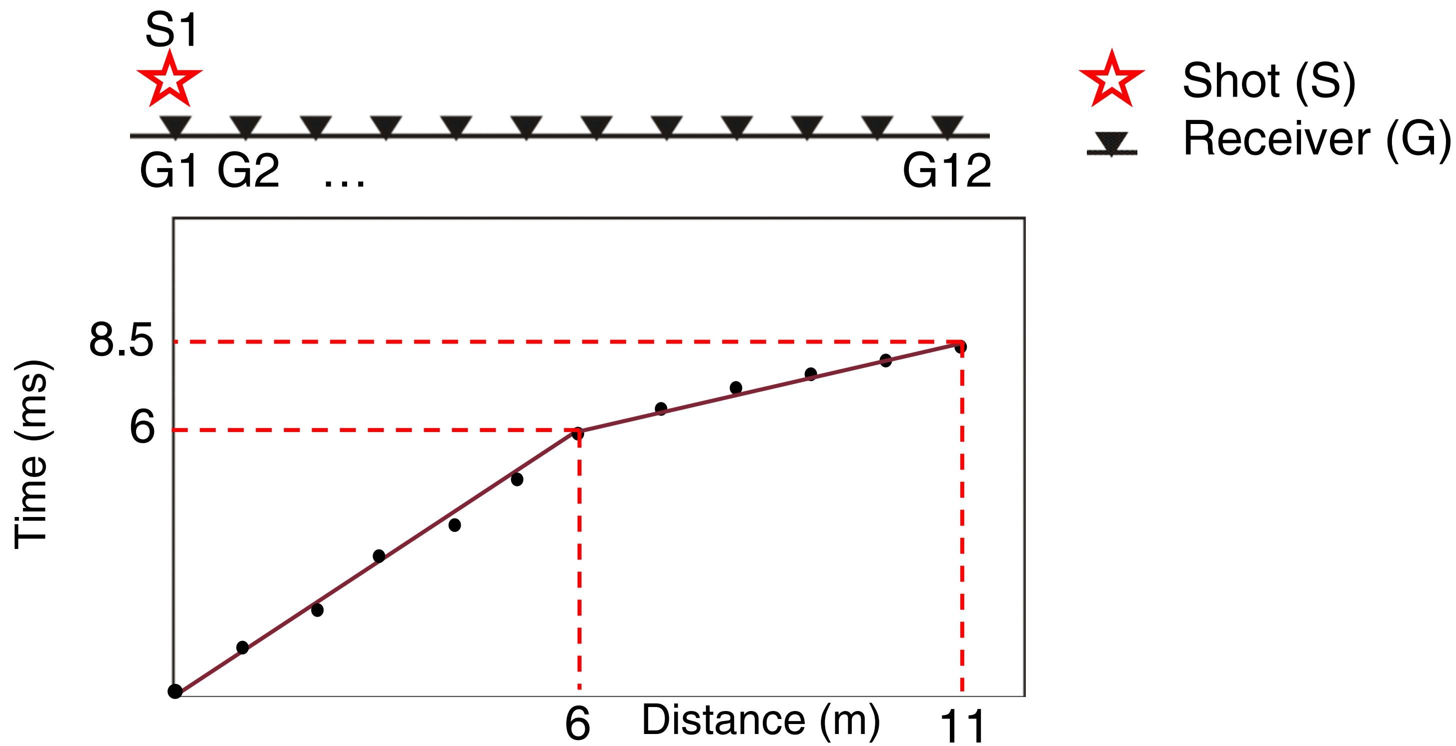


$$v_1 = \frac{1}{\tan^{-1}\left(\frac{6 \cdot 10^{-3}}{6}\right)} = \frac{1}{\tan^{-1} 10^{-3}} = 1000 \text{ m/s}$$

Seismic refraction

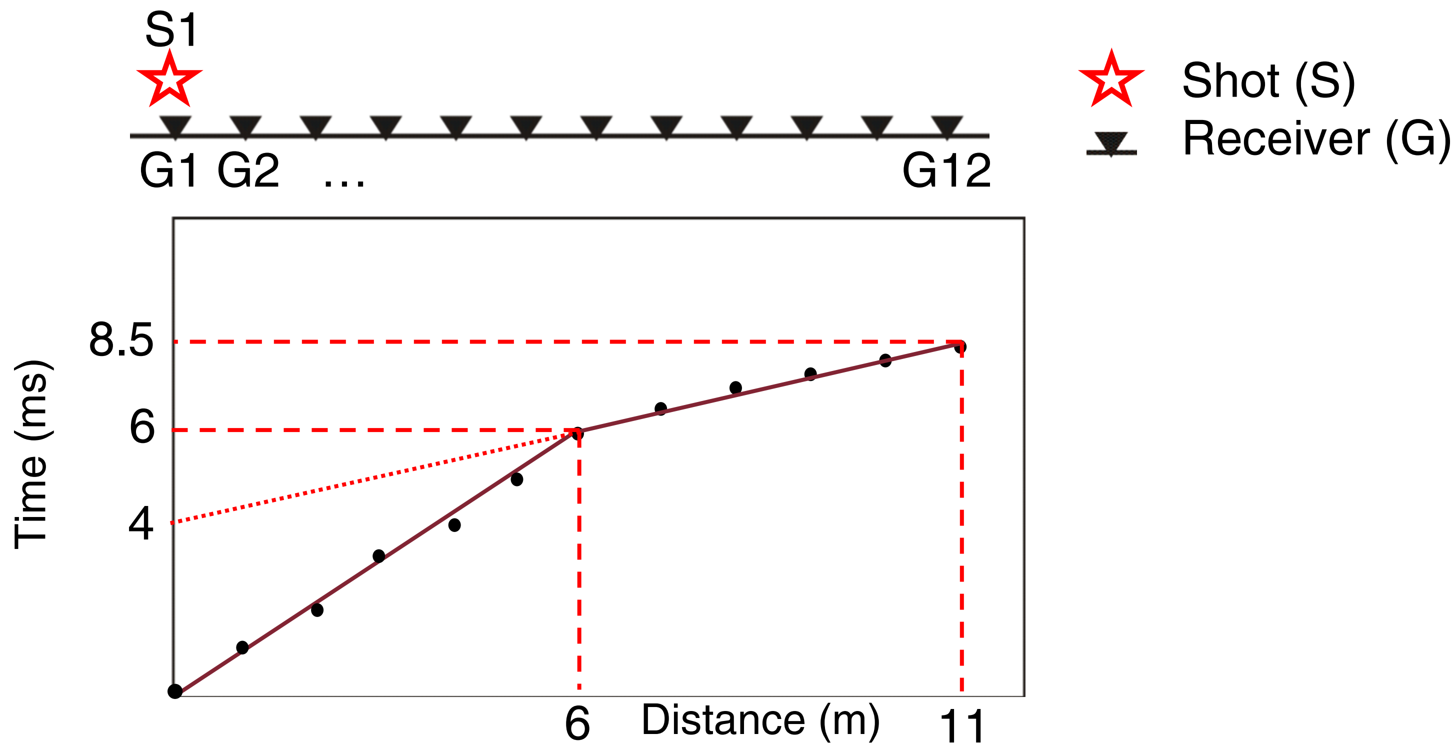
Ex. 2. 12 geophones (G1-G12) spaced 1 m apart with seismic source on G1



$$v_2 = \frac{1}{\tan^{-1}\left(\frac{(8.5-6)\cdot 10^{-3}}{11-6}\right)} = \frac{1}{\tan^{-1}\left(\frac{2.5\cdot 10^{-3}}{5}\right)} = \frac{1}{\tan^{-1}(5\cdot 10^{-4})} = 2000\ \mathrm{m/s}$$

Seismic refraction

Ex. 2. 12 geophones (G1-G12) spaced 1 m apart with seismic source on G1



$$h = \frac{v_1 t_I}{2\cos i_c}$$

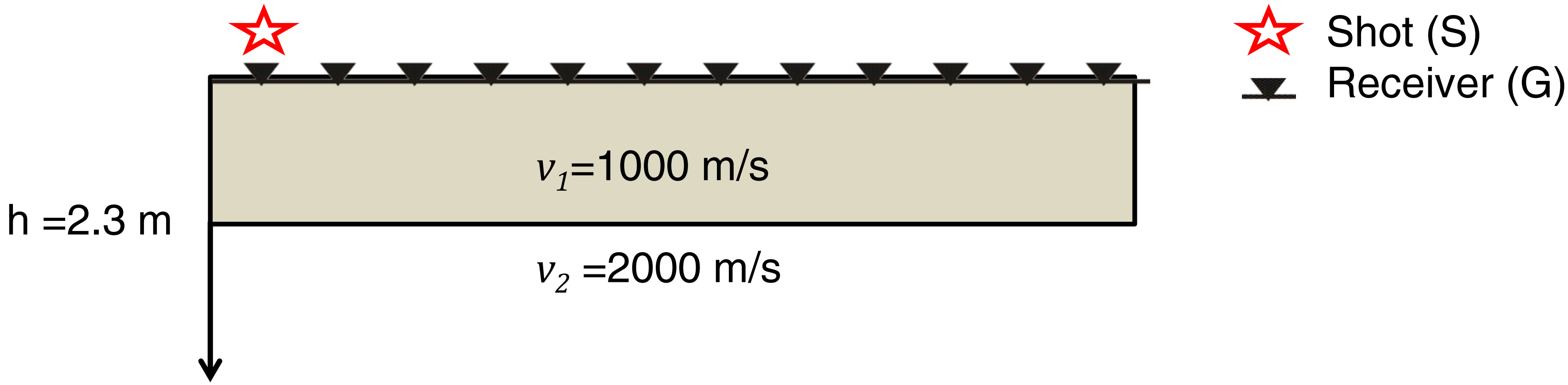
$$i_c = \sin^{-1}\left(\frac{v_1}{v_2}\right) = \sin^{-1} {}^{1}\!/_{2} = 0.5236 \text{ rad}$$

$$h = \frac{1000 \cdot 4 \cdot 10^{-3}}{2 \cdot \cos(0.5236)} = \frac{4}{2 \cdot 0.866} = 2.3 \text{ m}$$

Seismic refraction – Final model

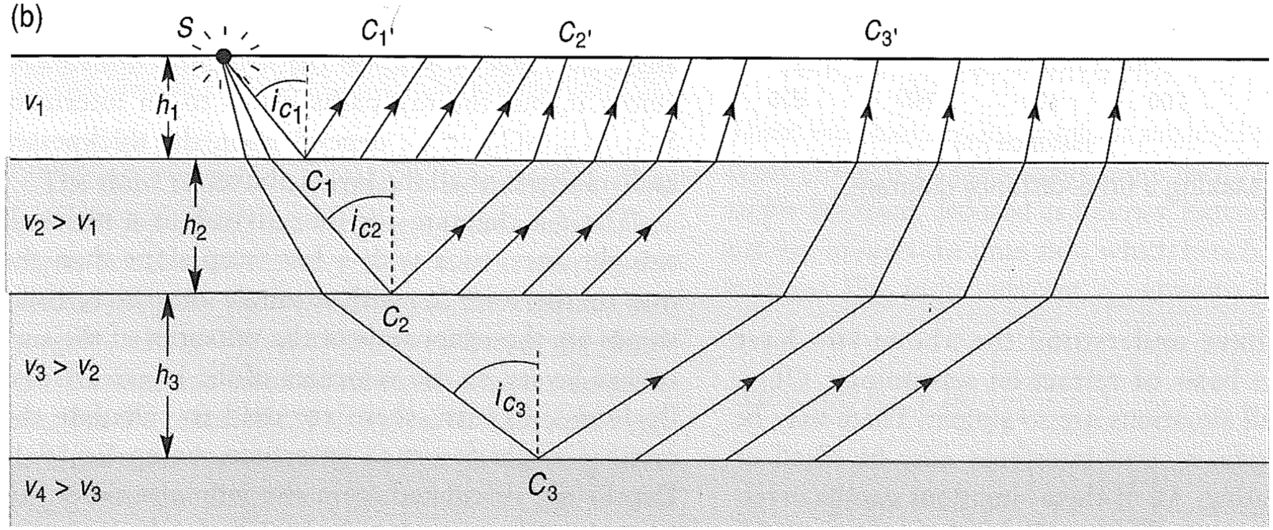
Ex. 2. 12 geophones (G1-G12) spaced 1 m apart with seismic source on G1

Using the refraction method, we reconstruct a 1-D two-layer model



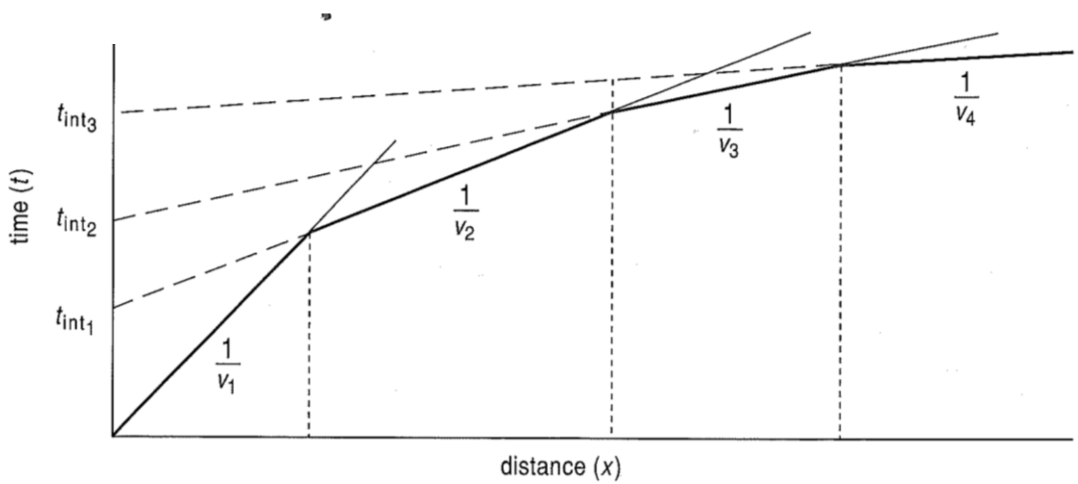
Special cases - Multiple layers

Seismic refraction can also detect multiple layers (we need large offsets)



The velocities are easily found from the slopes on the *x-t plot*

The thicknesses by the sequential calculation of the intercept times



Special cases - Velocity inversion

Overall seismic velocities progressively increase with depth. But this is not always true!

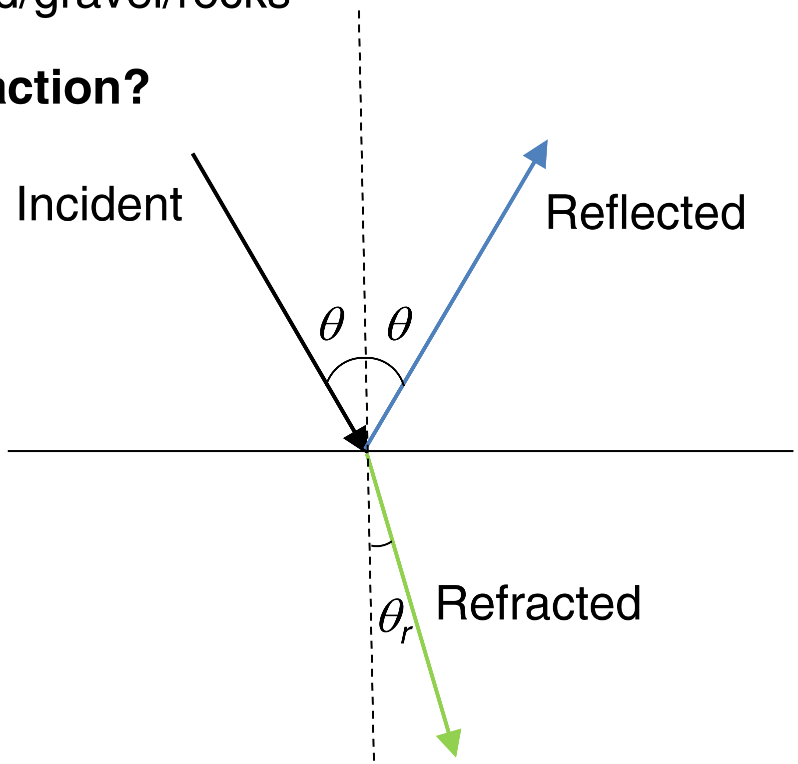
There are special cases, where we can have velocity inversion, that is a decrease of velocity with depth. i.e. silt/clay/peat below sand/gravel/rocks

Q. How we can deal with this with seismic refraction?

Snell's Law

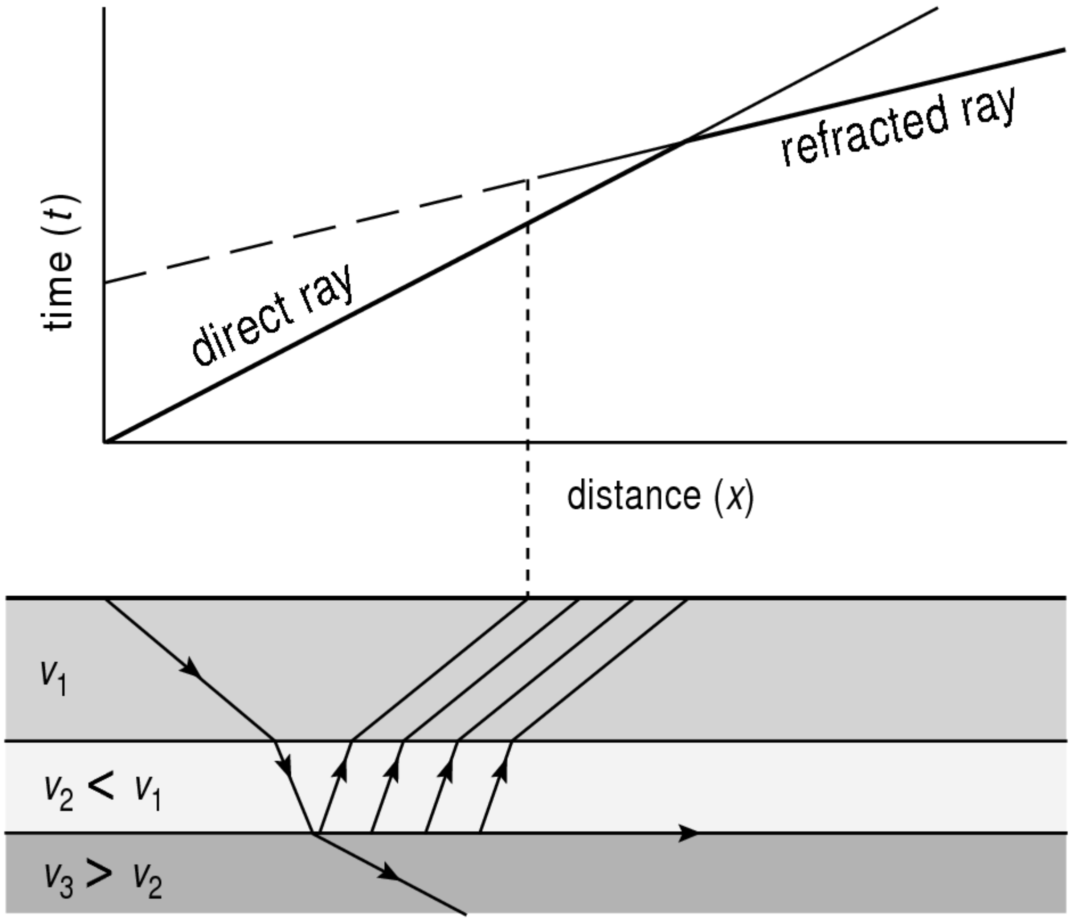
$$\frac{\sin \theta}{\sin \theta_r} = \frac{v_1}{v_2}$$

$$v_2 < v_1 \Rightarrow \theta_r < \theta$$



A. None of the refracted energy returns back to surface. We cannot apply the seismic refraction method to detect the velocity inversion!

Special cases - Velocity inversion: three-layer model example



I see only two layers in the *x-t* plot.

The effective thickness of the 1st and 3rd layers can be biased.

I need to know the presence of low-velocity layers by a priori information.

Special cases - Diffraction

For non-planar interfaces we can apply the same rules seen before only if the radius of curvature is much greater than wavelength.

If radius of curvature ≈ wavelength we have DIFFRACTION phenomena.

The edge becomes a point source of seismic waves following the Huygens' principle: diffraction can represent noise to the recorded signal if we aim to discern refractions or reflections.