SAPIENZA
UNIVERSITÀ DI ROMA

Environmental geophysics

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1. Seismic methods

Basic principles

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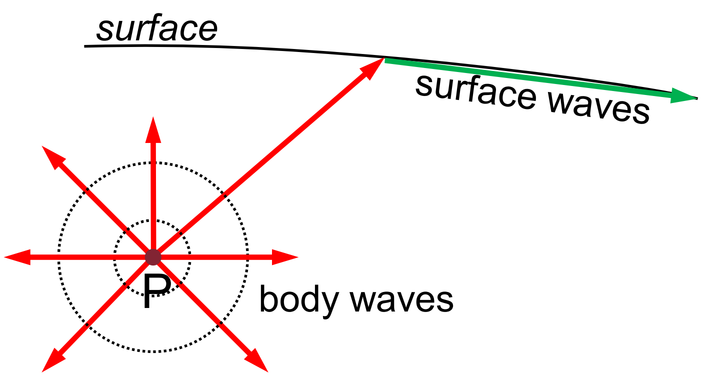
Seismic waves

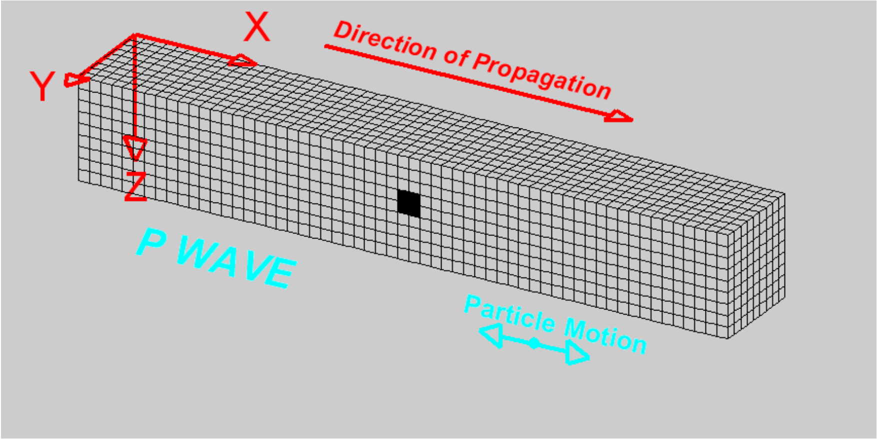


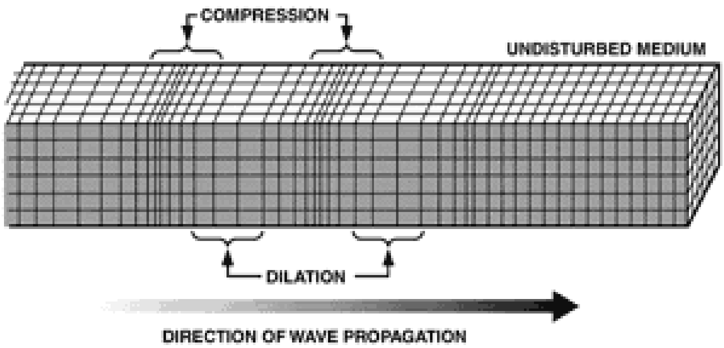
When a stress is applied (or released) on a point source P the corresponding strain propagates out of the source both within the body (**body waves**) and on surface (**surface waves**).

The body waves can be divided into **primary** and **secondary** waves, while the most studied surface waves are the **Rayleigh** and **Love** waves.

Body waves - P-wave

- The **wave** propagates **parallel** to the direction of **particle motion**.
- Particle motion consists of alternating **compression and dilation** (extension).
- Material returns to its original shape after the wave passes.

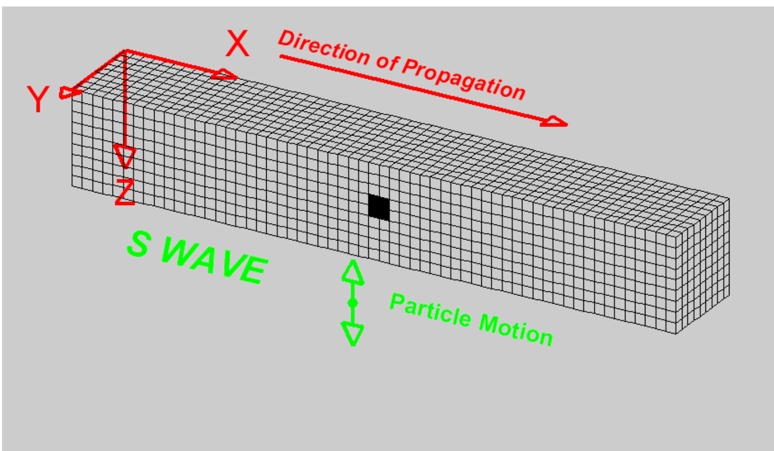




This wave is called compressional, longitudinal or **primary** (**P**), because it represents the first arrival

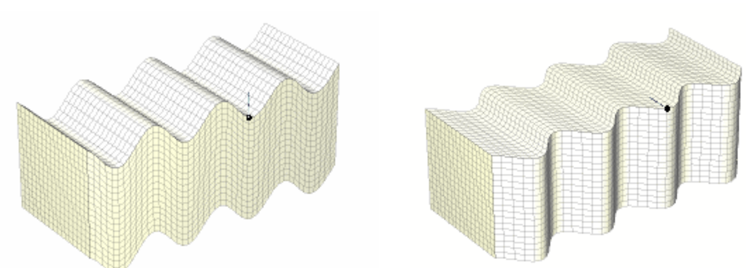
Body waves - S-wave

- The **wave** propagates **normal** to the direction of **particle motion**.
- Particle motion consists of alternating **transverse** motion.
- Material returns to its original shape after the wave passes.



Transverse motion can be in the vertical (z) direction (**SV wave**) or in the horizontal (y) direction (**SH wave**)

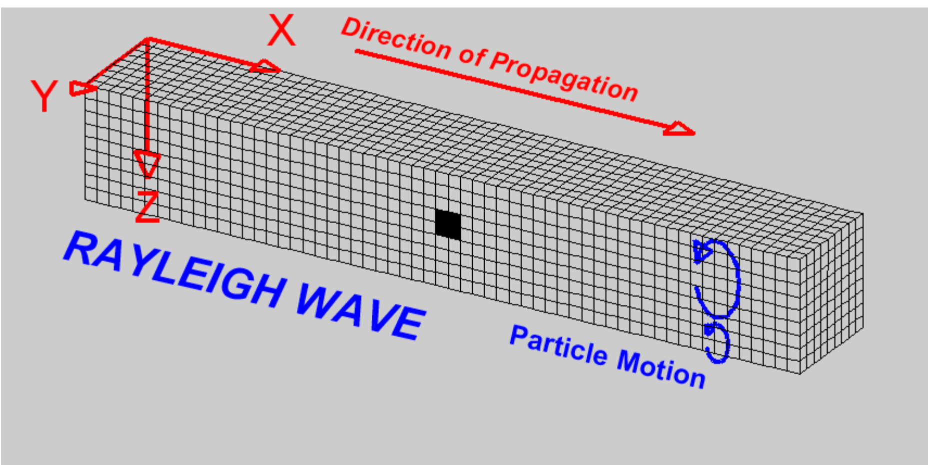
SV wave **SH wave**

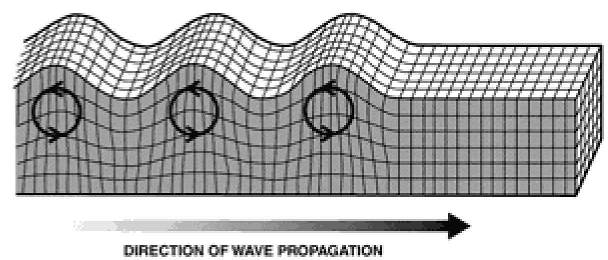


This wave is called **shear, transverse** or **secondary (S)**, because it represents the second arrival.

Surface waves – Rayleigh wave

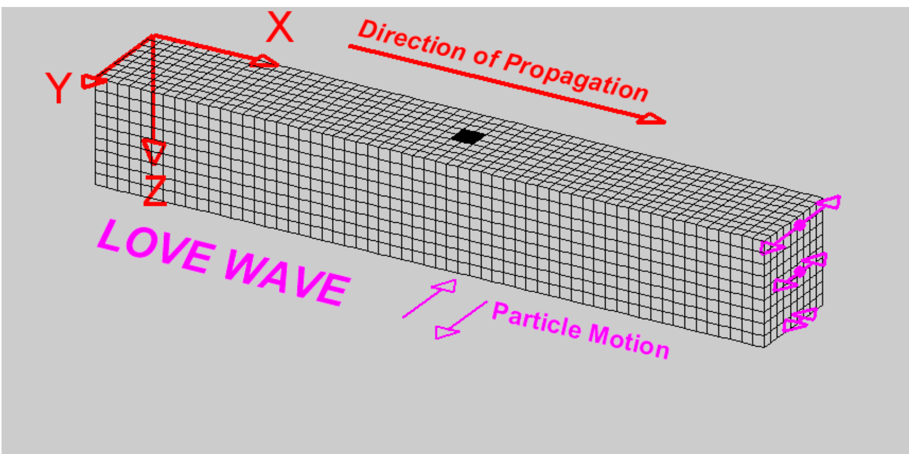
- Particle motion consists of **retrograde elliptical motions** in the **vertical plane** and **parallel to the direction of propagation**.
- Amplitude decreases with depth
- Material returns to its original shape after the wave passes.

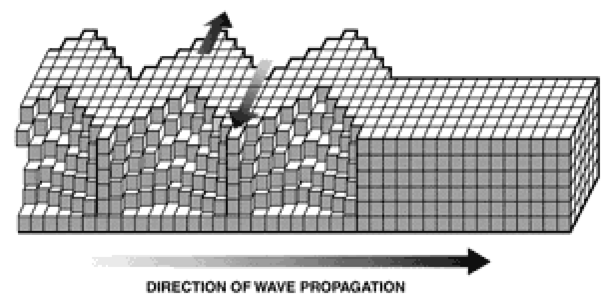




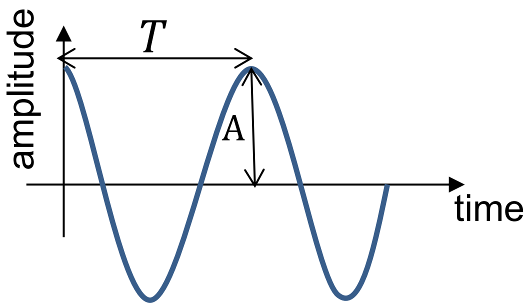
Surface waves - Love wave

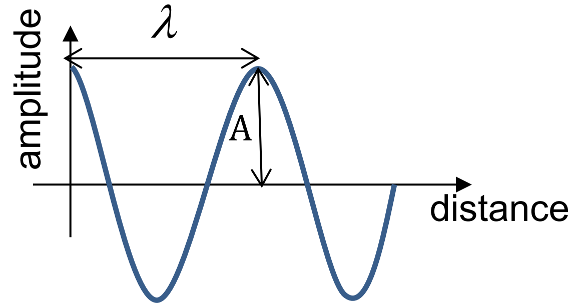
- Particle motion consists of **alternating transverse motions**, horizontal and **perpendicular to the direction of propagation** (transverse).
- Amplitude decreases with depth.
- Material returns to its original shape after the wave passes.





Wave parameters



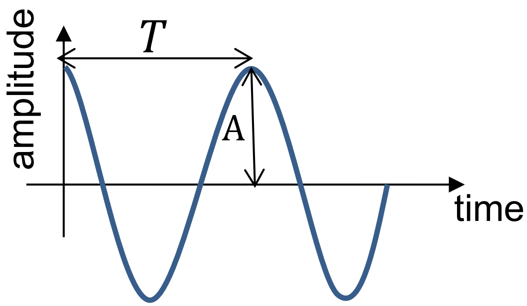


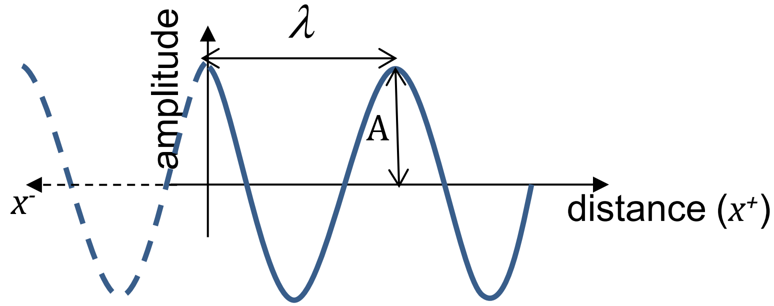
- **period** (*T*): duration of an oscillation cycle [s]
- **wavelength** (λ): distance of an oscillation cycle [m]
- **peak amplitude** (*A*): maximum absolute value of signal

From these, we can derive:

- **frequency** (*f*): $f = \frac{1}{T}$ [Hz]
- **angular frequency** (ω): $\omega = \frac{2\pi}{T} = 2\pi f$ [rad/s]
- **velocity** (*v*): $v = \frac{\lambda}{T} = \lambda f$ [m/s]
- **wavenumber** (*k*): $k = \frac{2\pi}{\lambda} = \frac{\omega}{v}$ [rad/m]

Wave parameters





Monochromatic signal = signal with only one frequency = sinusoidal signal

In this case the signal is:

$$u(t) = A\cos\left(\frac{2\pi}{T}t\right) = A\cos(\omega t)$$

$$u(x) = A\cos\left(\pm\frac{2\pi}{\lambda}x\right) = A\cos(\pm kx)$$

That is, using complex notation:

$$u(t) = \Re e\{A\mathrm{e}^{i\omega t}\}$$

$$u(x) = \Re e\{A\mathrm{e}^{\pm ikx}\}$$

Since the observed signal are real-valued (e.g. displacements), in the following we will use a complex notation neglecting the imaginary part.

For a wave propagating through space and time, the resulting signal is:

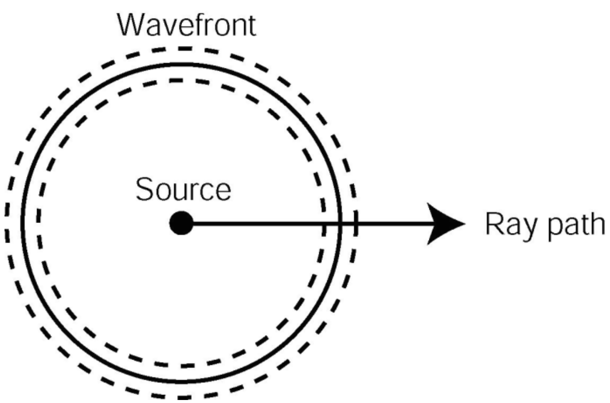
$$u(x,t) = A\mathrm{e}^{i(\omega t \pm kx)}$$

Propagation

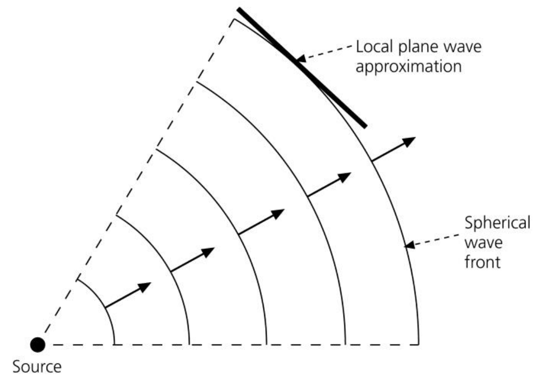
The **wavefront** is the surface where the elastic wave has a constant phase

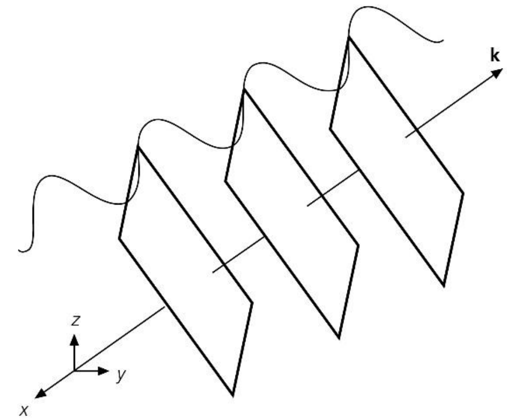
The **ray** is the path that one parcel of the wavefront travels along.

Spherical wavefront



Plane wavefront





These hypothesis hold for:

- Point source

- Point source, far enough from the source
- Plane source with stress distributed over a surface

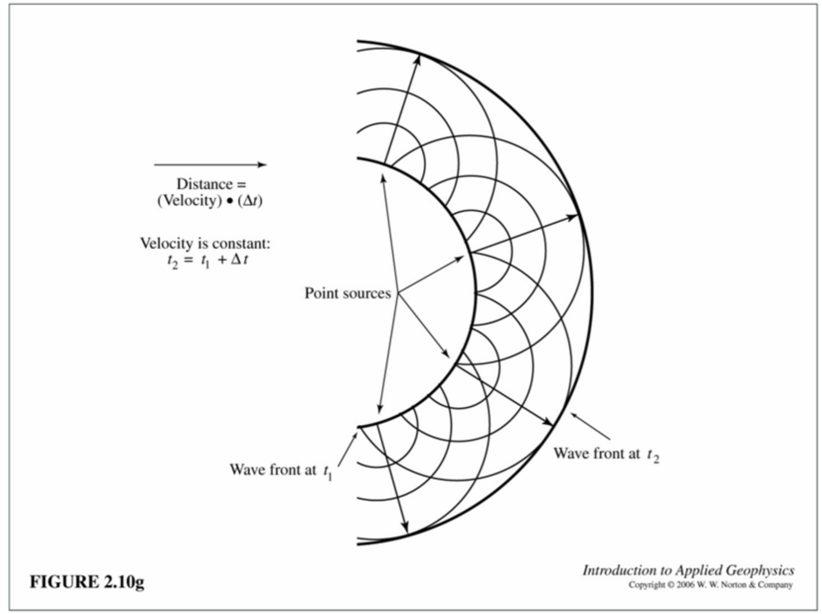
Propagation

Huygens principle:

• each point on wavefront acts as a point source

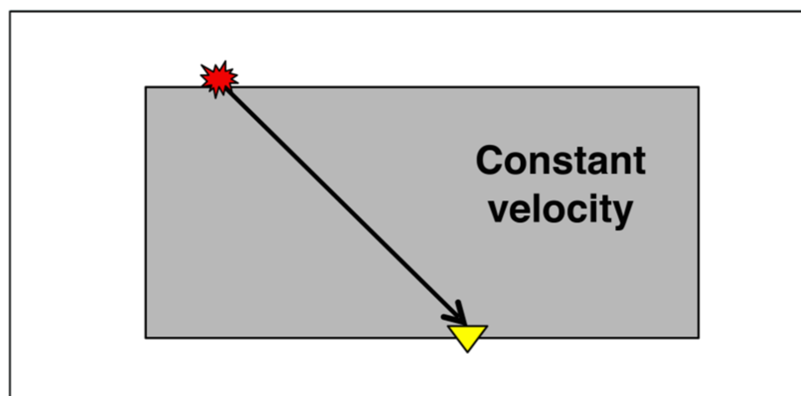
• spherical waves radiate outward from each point source

• envelope of waves is the overall wavefront



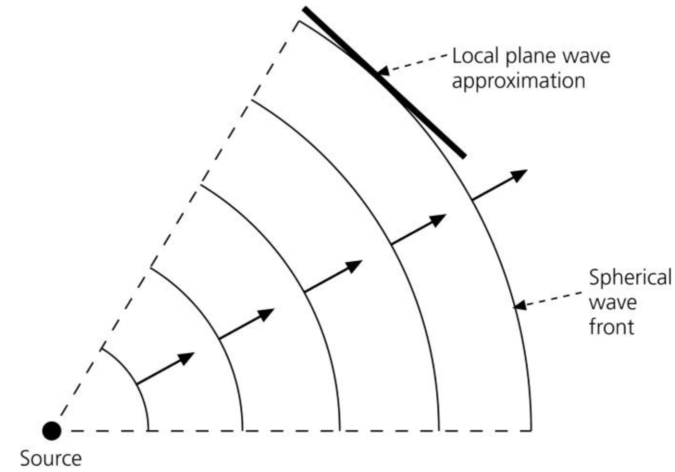
Fermat's Principle:

• rays propagate along the path which yields the smallest travel time (principle of least time)



Elastic wave fields loose energy through Earth due to:

- **geometrical spreading**: the wavefront must always contain a constant amount of energy E_0



The energy of a unit area of the growing wave front is equal to:

$$E(r) = \frac{E_0}{4\pi r^2}$$

surface of a sphere

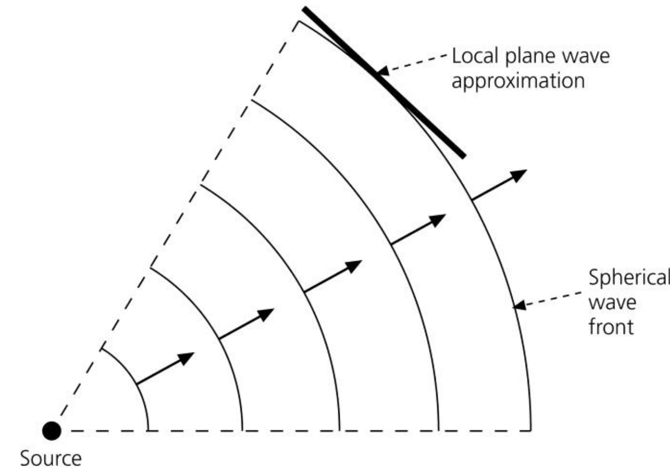
Therefore, the energy decreases proportionally to r^2

Q. What about the wave amplitude? It decrease proportionally to…

Attenuation

Elastic wave fields loose energy through Earth due to:

• **geometrical spreading**: the wavefront must always contain a constant amount of energy E_0



The energy of a unit area of the growing wave front is equal to:

$$E(r) = \frac{E_0}{4\pi r^2}$$

surface of a sphere

Therefore, the energy decreases proportionally to r^2

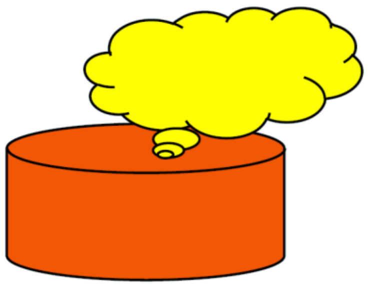
Q. What about the wave amplitude? It decrease proportionally to…

A. Proportionally to r! Remember that $E \propto a^2$

Attenuation

Elastic wave fields loose energy through Earth due to:

- **intrinsic attenuation:** energy loss due to shear heating at grain boundaries, mineral dislocations etc.



It can be evaluated though a “quality factor” (Q):

$$Q = \frac{2\pi E}{\Delta E}$$

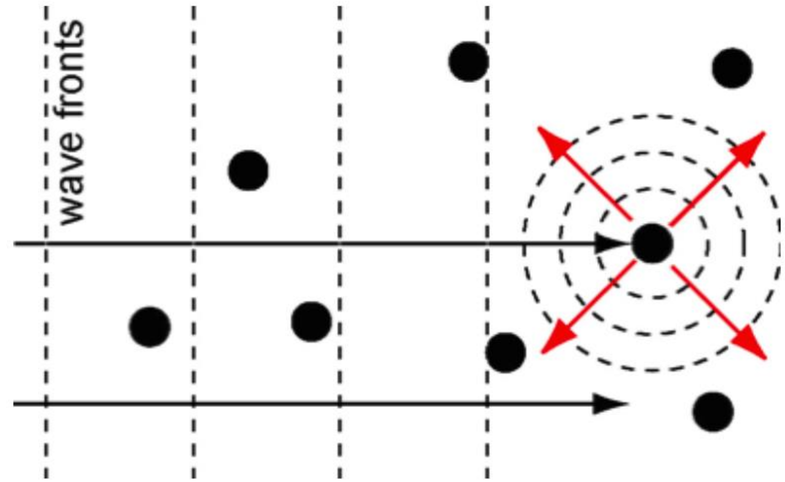
Q is a measure of the fractional loss of energy per cycle (oscillation) of the seismic wave

high Q = little energy loss

Attenuation

Elastic wave fields loose energy through Earth due to:

• **scattering:** most materials contain small heterogeneities (grains, mineral boundaries, pore edges, cracks, etc.)



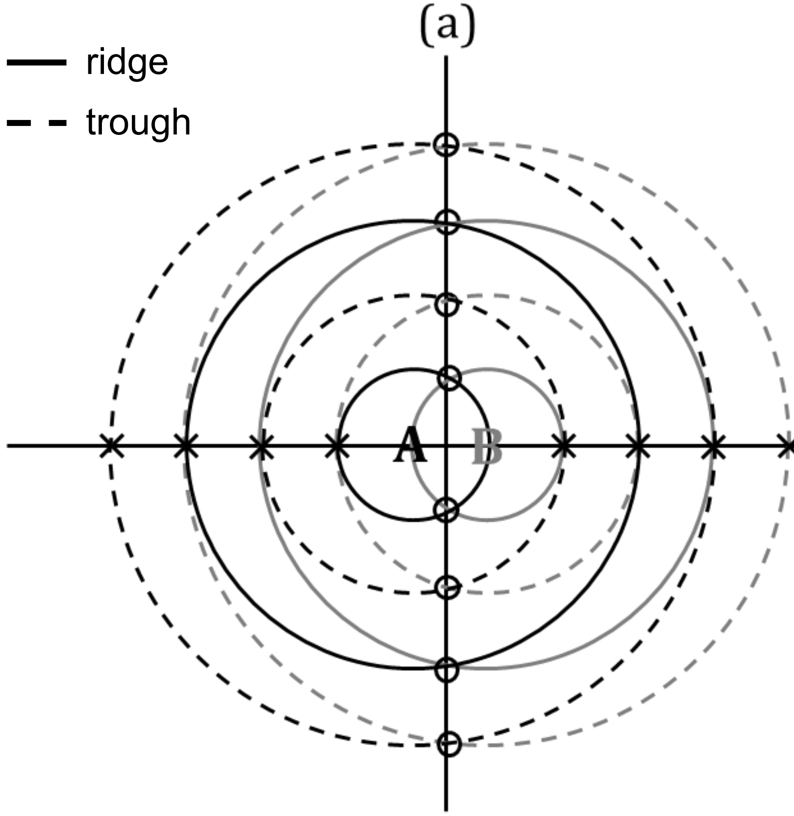
Some seismic energy is scattered when it encounters these features: acording to Huugens' principles the small heterogeneities are new sources of waves.

This phenomenon depends on the ratio of the heterogeneity size to the wavelength (it must be comparable)

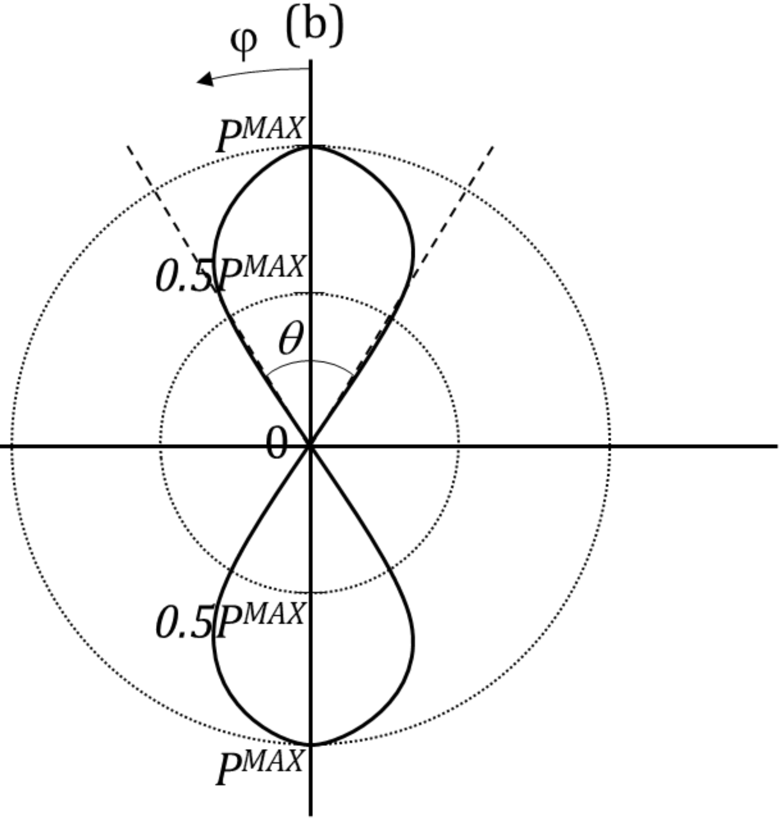
Interference

Waves can interact and the resulting wave is the effect of the superposition of two or more wave fronts

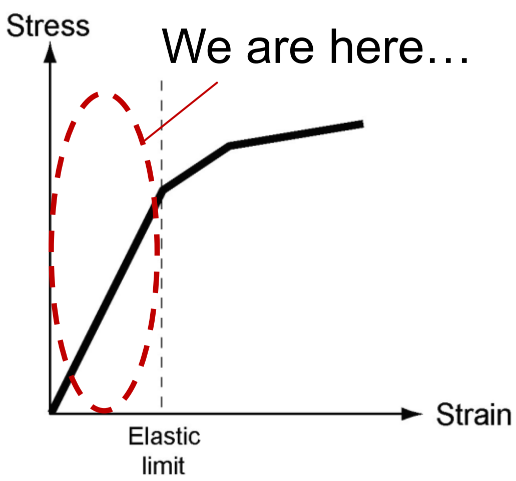
Interference
2 sources at $d=\lambda/2$



Radiation pattern



Linear elasticity

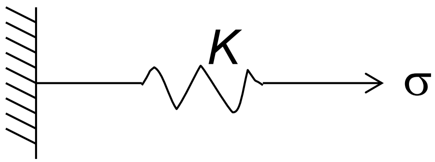


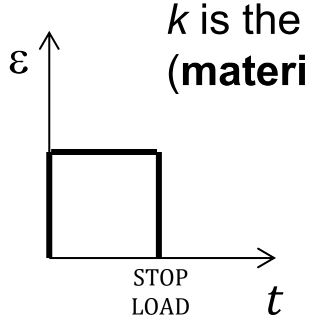
Seismic sources release energy as waves that causes a certain level of **stress** and a consequent **strain**, though not permanent for our low-energy applications (**elastic behaviour**)

Stress σ ⟷ Strain ε

$$\varepsilon = f\,(\sigma)$$

$$\sigma = f^{-1}\,(\varepsilon)$$





k is the spring constant (**material property**)

Hooke's Law: Strain is directly proportional to the stress (**linear elasticity**)

stress

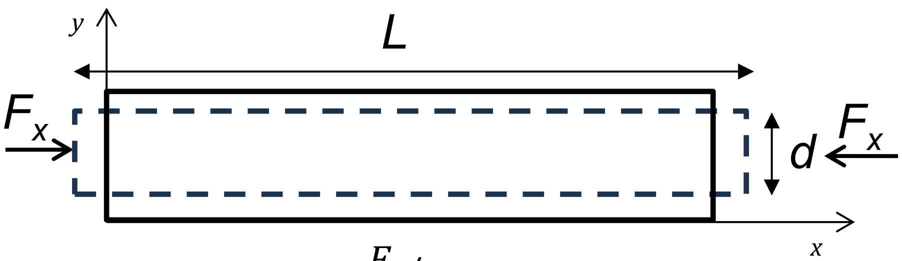
$$F = k\Delta x$$

strain

Elastic parameters

Displacement $\mathbf{u} \equiv (u, v, w)$

1. Longitudinal (compression/extension) stress



$$E = \frac{long.stress}{long.strain} = \frac{F_x/_A}{\Delta L/_L} = \frac{\sigma_{xx}}{\varepsilon_{xx}} = \frac{\sigma_{xx}}{\frac{\partial u}{\partial x}}$$

$$\varepsilon_{xx} = \frac{1}{E}\,\sigma_{xx}$$

E: Young's modulus [Pa]

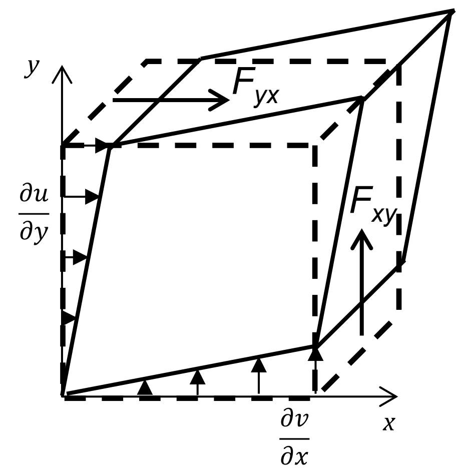
$$\nu = -\frac{lat.strain}{long.strain} = -\frac{\Delta d/_d}{\Delta L/_L} = -\frac{\varepsilon_{yy}}{\varepsilon_{xx}} = -\frac{\frac{\partial v}{\partial y}}{\frac{\partial u}{\partial x}} = -\frac{\varepsilon_{zz}}{\varepsilon_{xx}} = -\frac{\frac{\partial w}{\partial y}}{\frac{\partial u}{\partial x}}$$

ν: Poisson's ratio [dim less]

$$\varepsilon_{yy} = -\frac{\nu}{E}\,\sigma_{xx}$$

$$\varepsilon_{zz} = -\frac{\nu}{E}\,\sigma_{xx}$$

2. Shear stress



$$\mu = G = \frac{sh.stress}{sh.strain} = \frac{F_{xy}/_A}{\tan(\theta)} = \frac{\sigma_{xy}}{\gamma_{xy}} = \frac{\sigma_{xx}}{\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}}$$

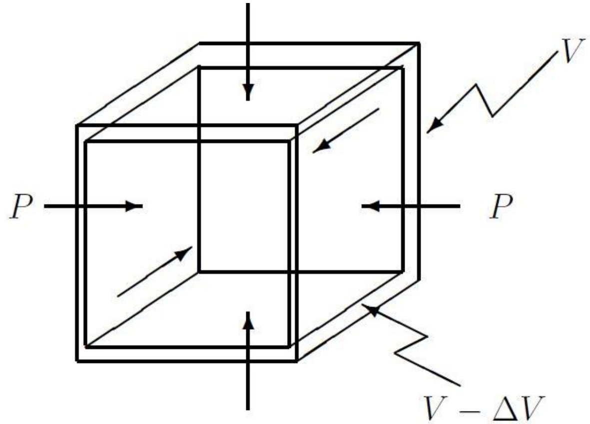
$$\gamma_{xy} = \frac{1}{\mu}\,\sigma_{xy}$$

$$\sigma_{xy} = \mu\gamma_{xy}$$

μ: Shear modulus [Pa]

Elastic parameters

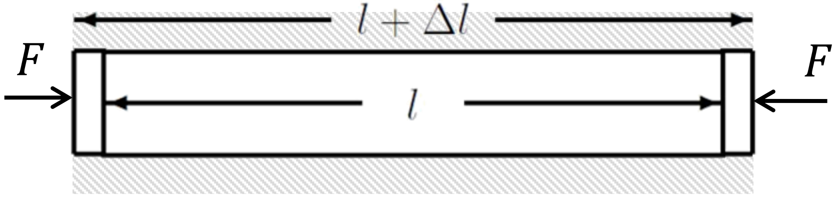
3. Volumetric (bulk stress)



$$K = \frac{vol.stress}{vol.strain} = \frac{P}{\Delta V/V}$$

Bulk modulus [Pa]

4. Longitudinal stress with no lateral strain

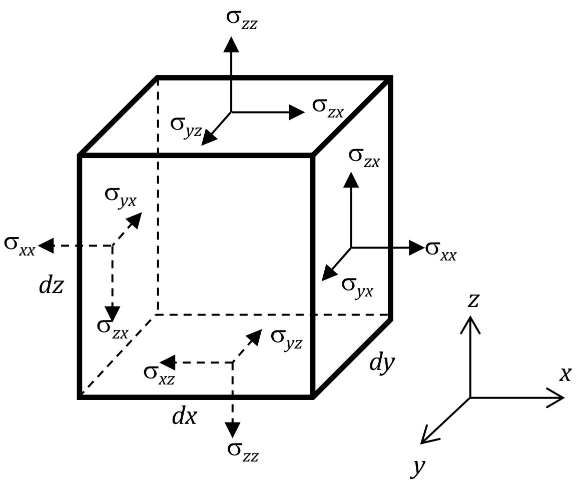


$$M = \frac{long.stress}{long.strain} = \frac{F/A}{\Delta l/l}$$

Longitudinal modulus [Pa]

$$M = K + \frac{4}{3}\mu$$

Linear elastic homogeneous isotropic stress-strain relationship



$$\begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix} = \begin{bmatrix} 1/E & -\nu/E & -\nu/E & & & \\ -\nu/E & 1/E & -\nu/E & & 0 & \\ -\nu/E & -\nu/E & 1/E & & & \\ & & & 1/\mu & 0 & 0 \\ & 0 & & 0 & 1/\mu & 0 \\ & & & 0 & 0 & 1/\mu \end{bmatrix} \begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix}$$

inverting

$$\begin{bmatrix}\sigma_{xx}\\ \sigma_{yy}\\ \sigma_{zz}\\ \sigma_{xy}\\ \sigma_{xz}\\ \sigma_{yz}\end{bmatrix} = \begin{bmatrix} \lambda+2\mu & \lambda & \lambda & & & \\ \lambda & \lambda+2\mu & \lambda & & 0 & \\ \lambda & \lambda & \lambda+2\mu & & & \\ & & & \mu & 0 & 0 \\ & 0 & & 0 & \mu & 0 \\ & & & 0 & 0 & \mu \end{bmatrix} \begin{bmatrix}\varepsilon_{xx}\\ \varepsilon_{yy}\\ \varepsilon_{zz}\\ \gamma_{xy}\\ \gamma_{xz}\\ \gamma_{yz}\end{bmatrix}$$

x-components:

$$\sigma_{xx} = (\lambda + 2\mu)\frac{\partial u}{\partial x} + \lambda\frac{\partial v}{\partial y} + \lambda\frac{\partial w}{\partial z}$$

$$\sigma_{xy} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

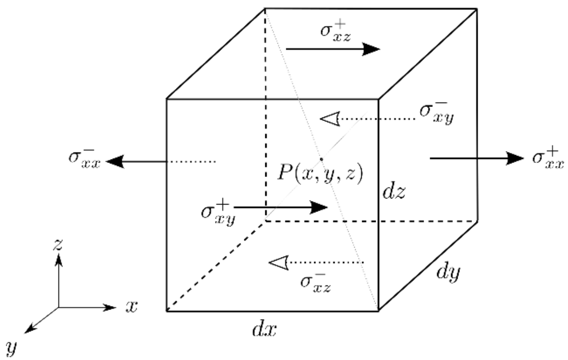
$$\sigma_{xz} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

λ, μ: Lamé parameters

The **linear elastic homogeneous isotropic** stress-strain relationship is defined by only **two constants** (**Lamé parameters**)

Seismic waves

Wave motion



$$\sum_i F_{x,i} = ma \quad \begin{cases} m = \delta V = \delta dxdydz \\ a = \dfrac{\partial^2 u}{\partial t^2} = \ddot{u} \end{cases}$$

δ: density
V: volume

$$\frac{\partial \sigma^{xx}}{\partial x}\mathrm{dx}(\mathrm{dydz}) + \frac{\partial \sigma^{xy}}{\partial y}\mathrm{dy}(\mathrm{dxdz}) + \frac{\partial \sigma^{xz}}{\partial z}\mathrm{dz}(\mathrm{dxdy}) = \delta\ddot{u}\ dxdydz$$

$$\frac{\partial \sigma^{xx}}{\partial x} + \frac{\partial \sigma^{xy}}{\partial y} + \frac{\partial \sigma^{xz}}{\partial z} = \delta\ddot{u}$$

Insert linear elasticity

$$\lambda\frac{\partial}{\partial x}\frac{\partial u}{\partial x} + \lambda\frac{\partial}{\partial x}\frac{\partial v}{\partial y} + \lambda\frac{\partial}{\partial x}\frac{\partial w}{\partial z} + \mu\frac{\partial}{\partial x}\frac{\partial u}{\partial x} + \mu\frac{\partial}{\partial x}\frac{\partial v}{\partial y} + \mu\frac{\partial}{\partial x}\frac{\partial w}{\partial z} + \mu\frac{\partial^2 u}{\partial x^2} + \mu\frac{\partial^2 u}{\partial y^2} + \mu\frac{\partial^2 u}{\partial z^2} = \delta\ddot{u}$$

$$(\lambda+\mu)\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) + \mu\left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2}\right) = \delta\ddot{u}$$

1-D (scalar) equation of motion

$$(\lambda+\mu)\frac{\partial}{\partial x}(\nabla\cdot\mathbf{u}) + \mu\nabla^2 u = \delta\ddot{u}$$

extension in 3-D

3-D (vector) equation of motion

$$(\lambda+\mu)\boldsymbol{\nabla}(\nabla\cdot\mathbf{u}) + \mu\nabla^2\mathbf{u} = \delta\ddot{\mathbf{u}}$$

Stress increment (rate)

$$\Delta\sigma^{xx} = \sigma^{xx+} - \sigma^{xx-} = \frac{\partial \sigma^{xx}}{\partial x}\mathrm{dx}$$

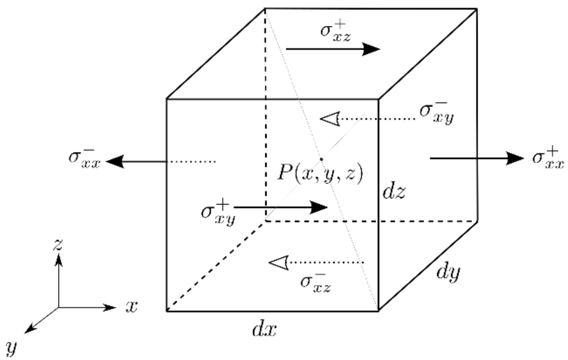
$$\Delta\sigma^{xx} = \left[(\lambda+2\mu)\frac{\partial^2 u}{\partial x^2} + \lambda\frac{\partial}{\partial x}\frac{\partial v}{\partial y} + \lambda\frac{\partial}{\partial x}\frac{\partial w}{\partial z}\right]\mathrm{dx}$$

$$\Delta\sigma^{xy} = \left[\mu\frac{\partial}{\partial x}\frac{\partial v}{\partial y} + \frac{\partial^2 u}{\partial y^2}\right]\mathrm{dy}$$

$$\Delta\sigma^{xz} = \left[\mu\frac{\partial}{\partial x}\frac{\partial w}{\partial z} + \frac{\partial^2 u}{\partial z^2}\right]\mathrm{dz}$$

Seismic waves

Wave motion



$$(\lambda + \mu)\boldsymbol{\nabla}(\nabla \cdot \mathbf{u}) + \mu\nabla^2\mathbf{u} = \delta\ddot{\mathbf{u}}$$

3-D (vector) equation of motion (coupled)

1. Apply divergence (scalar) operator $(\nabla \cdot)$

$$(\lambda + \mu)\nabla^2(\nabla \cdot \mathbf{u}) + \mu\nabla^2(\nabla \cdot \mathbf{u}) = \delta\frac{\partial^2}{\partial t^2}(\nabla \cdot \mathbf{u})$$

$$(\lambda + 2\mu)\nabla^2(\nabla \cdot \mathbf{u}) = \delta\frac{\partial^2}{\partial t^2}(\nabla \cdot \mathbf{u})$$

Pure volumetric strain (P)

2. Apply curl (**vector**) *operator* ($\boldsymbol{\nabla}$ x)

$$\mu\nabla^2(\boldsymbol{\nabla}\,\mathrm{x}\,\mathbf{u}) = \delta\frac{\partial^2}{\partial t^2}(\boldsymbol{\nabla}\,\mathrm{x}\,\mathbf{u})$$

Pure shear strain (S)

Sharing the same form WAVE EQUATION

$$\frac{\partial^2 f}{\partial t^2} = v^2\nabla^2 f$$

$$v_P = \alpha = \sqrt{\frac{\lambda+2\mu}{\delta}}$$

P-wave velocity

$$v_S = \beta = \sqrt{\frac{\mu}{\delta}}$$

S-wave velocity

$$u(x,t) = Ae^{i(\omega t \pm kx)}$$

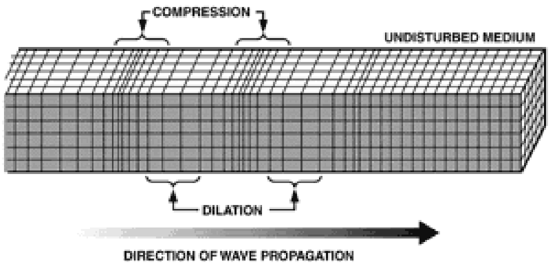
Solution for a plane P-wave in the x-dir

P-wave velocity

$$v_P = \sqrt{\frac{\lambda + 2\mu}{\delta}} = \sqrt{\frac{E(1-\nu)}{\delta(1+\nu)(1-2\nu)}}$$

density [kg/m³]

P-wave velocity [m/s]



Dimensional analysis

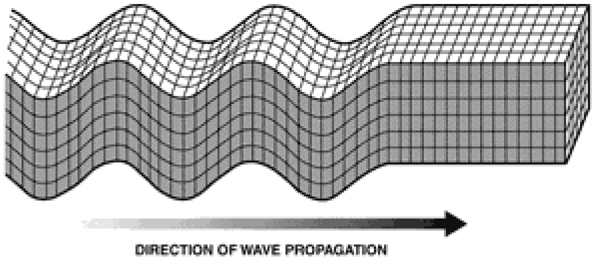
$$v_P = \left(\frac{[\mathrm{Pa}]}{[\mathrm{kg/m^3}]}\right)^{1/2} = \left(\frac{[\mathrm{N/m^2}]}{[\mathrm{kg/m^3}]}\right)^{1/2} = \left(\frac{[\frac{\mathrm{kg}}{\mathrm{m}^{\cancel{2}}}\frac{\cancel{\mathrm{m}}}{\mathrm{s}^2}]}{[\mathrm{kg/m^3}]}\right)^{1/2} = \left(\frac{[\mathrm{kg/ms^2}]}{[\mathrm{kg/m^3}]}\right)^{1/2}$$

$$v_P = \left(\left[\frac{\cancel{\mathrm{kg}}}{\cancel{\mathrm{m}}\mathrm{s}^2}\right]\left[\frac{\mathrm{m}^{\cancel{3}\,2}}{\cancel{\mathrm{kg}}}\right]\right)^{1/2} = \left(\left[\frac{\mathrm{m}^2}{\mathrm{s}^2}\right]\right)^{1/2} = [\mathrm{m/s}]$$

S-wave velocity

$$v_S = \sqrt{\frac{\mu}{\delta}} = \sqrt{\frac{E}{\delta}\frac{1}{2(1+\nu)}}$$

S-wave velocity [m/s]

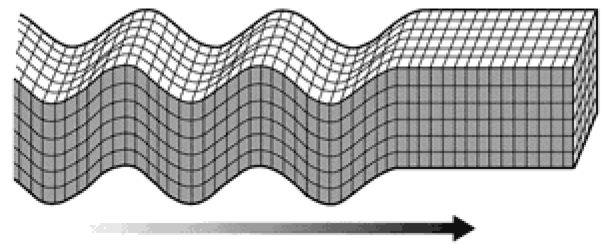


Q. Which is the S-wave velocity in water?

S-wave velocity

$$v_S = \sqrt{\frac{\mu}{\delta}} = \sqrt{\frac{E}{\delta}\frac{1}{2(1+\nu)}}$$

S-wave velocity [m/s]



Q. Which is the S-wave velocity in water?

A. The S-wave velocity in water is null!

v_S^{water}=0 water does not carry shear waves

Q. Why?

A. Because its shear modulus is null!

$$\mu = \frac{sh.stress}{sh.strain} = \frac{arbitrary}{\infty} = 0$$

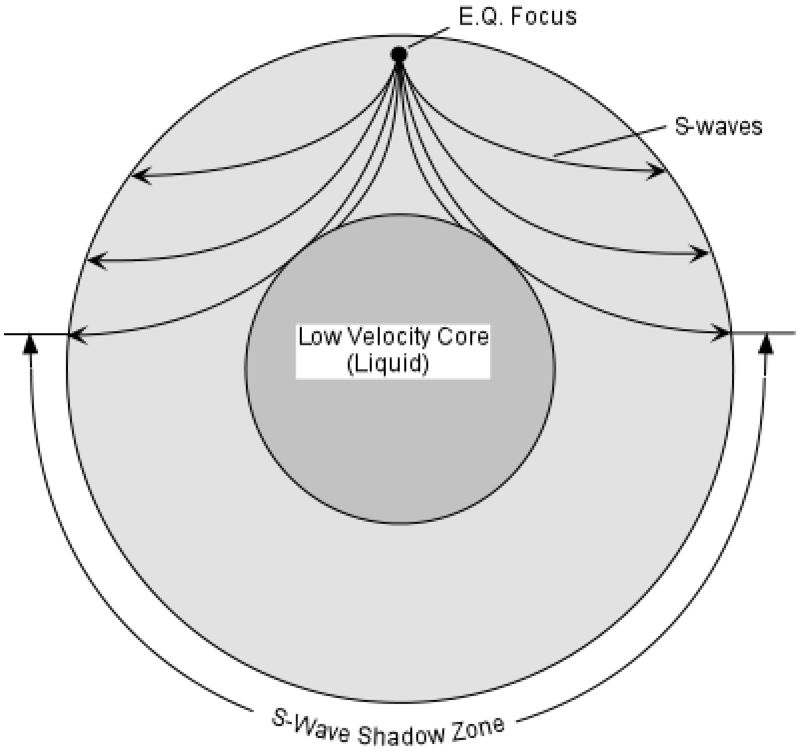
In water, the shear modulus is zero because if you apply an arbitrary shear force *F*, the liquid simply flows, and the shear strain becomes arbitrarily large:

shear strain → ∞

$$v_S = \sqrt{\frac{\mu}{\delta}} = \sqrt{\frac{0}{1000\mathrm{kg/m^3}}} = 0$$

Elastic moduli from seismic velocities

Fluids do not carry shear waves. This knowledge, combined with earthquake observations, is what lead to the discovery that the earth's outer core is a liquid rather than a solid – "shear wave shadow".



Elastic moduli from seismic velocities

Given the seismic velocities v_P and v_S, i.e. from geophysical measurements, we can derive the elastic moduli:

$$M = \delta {v_P}^2$$

$$\nu = \frac{{v_P}^2 - 2{v_S}^2}{2({v_P}^2 - {v_S}^2)}$$

$$K = \delta\left({v_P}^2 - \frac{4}{3}{v_S}^2\right)$$

$$\mu = \delta {v_s}^2$$

In a first approximation:

$$v_S \approx 0.6 v_P$$

$${v_P}^{freshwater} \approx 1450 \text{ m/s}$$

$${v_P}^{saltwater} \approx 1530 \text{ m/s}$$

$${v_P}^{air} \approx 330 \text{ m/s (speed of sound)}$$

Velocity of rocks and soils

Type of formation	P wave velocity (m/s)	S wave velocity (m/s)	Density (g/cm^3)	Density of constituent crystal (g/cm^3)
Scree, vegetal soil	330-700	100-300	1.7-2.4	-
Dry sands	400-1200	100-500	1.5-1.7	2.65 quartz
Wet sands	1500-2000	100-500	1.9-2.1	2.65 quartz
Saturated shales and clays	1100-2500	200-800	2.0-2.4	-
Marls	2000-3000	750-1500	2.1-2.6	-
Saturated shale and sand sections	1500-2200	500-750	2.1-2.4	-
Porous and saturated sandstones	2000-3500	800-1800	2.1-2.4	2.65 quartz
Limestones	3500-6000	2000-3300	2.4-2.7	2.71 calcite
Chalk	2300-2600	1100-1300	1.8-3.1	2.71 calcite
Salt	4500-5500	2500-3100	2.1-2.3	2.1 halite
Anhydrite	4000-5500	2200-3100	2.9-3.0	-
Dolomite	3500-6500	1900-3600	2.5-2.9	(Ca, Mg) CO_3 2.8-2.9
Granite	4500-6000	2500-3300	2.5-2.7	-
Basalt	5000-6000	2800-3400	2.7-3.1	-
Gneiss	4400-5200	2700-3200	2.5-2.7	-
Coal	2200-2700	1000-1400	1.3-1.8	-
Water	1450-1500	-	1.0	-
Ice	3400-3800	1700-1900	0.9	-
Oil	1200-1250	-	0.6-0.9	-

Velocity vs. porosity

Soils and rocks are multiphase media

The effective velocity is the combination of the velocity of the single component

Solid	**Liquid**	**Gas**
Minerals	Mainly water	Mainly air
(**often high velocity**)	(**P-wave velocity=1450-1530 m/s**)	(**P-wave velocity=330-340 m/s**)

Velocity of the multiphase medium
Combination of the three contributions

Velocity vs. porosity

The overall rock properties are the average of the matrix (V_M) and pore fluid (V_F) properties, weighted by the porosity ϕ:

Time-average (Wyllie) equation (empirical)

$$\frac{1}{v_P} = \frac{\phi}{v_F} + \frac{1-\phi}{v_M}$$

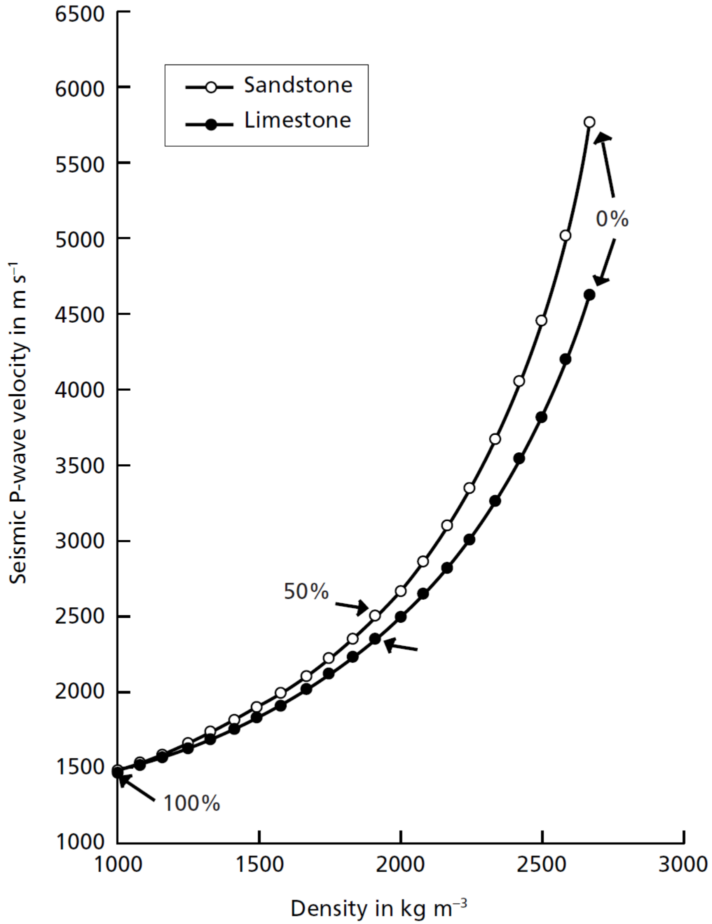
Ex. 1: travertine having a matrix velocity of 3 km/s, a porosity of 30% (0.3) filled by fresh water (V_F = 1.45 km/s)

$$\frac{1}{v_P} = \frac{0.3}{1.45} + \frac{0.7}{3} = 0.207 + 0.233 = 0.44$$

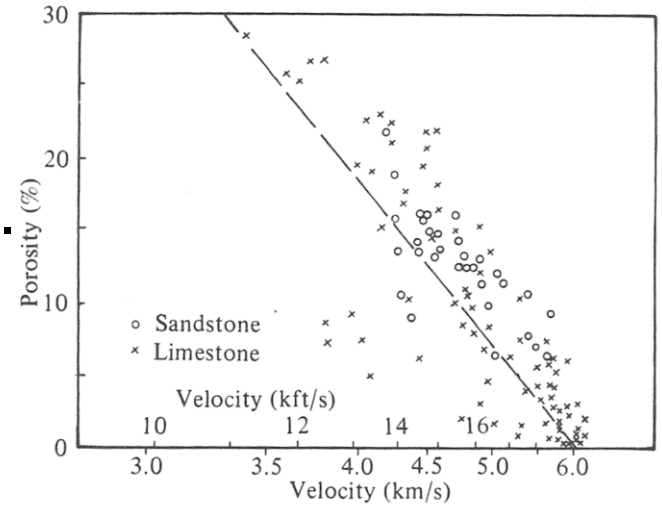
$$v_P = \frac{1}{0.44} = 2.27\ \mathrm{km/s}$$

Velocity vs. porosity, density, pressure and saturation

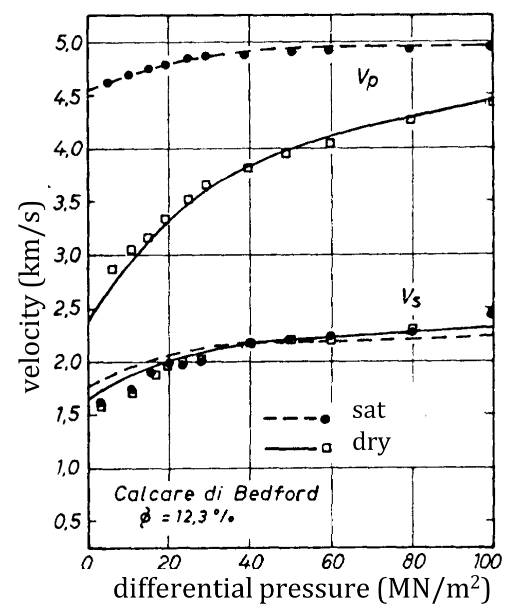
Velocity vs. density



Porosity vs. velocity



Velocity vs. pressure and saturation



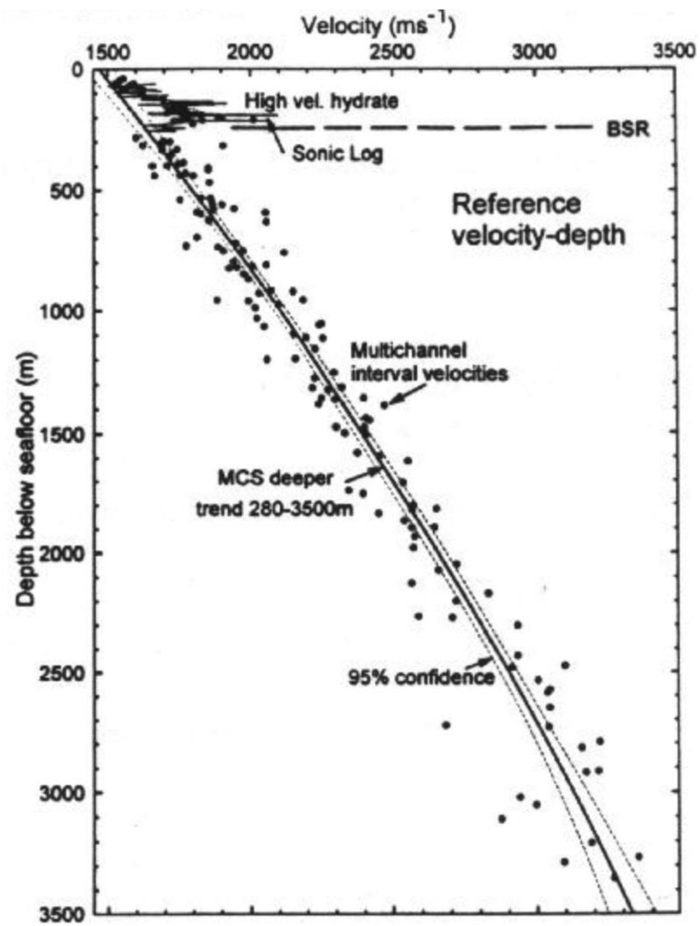
Velocity vs. depth

Seismic velocities generally increase with depth due to:

$$v = C(zT)^{1/6}$$

z – depth
T – age of the rock
C – experimental constant

- increased compaction and consequent reduced pore space
- elastic moduli increase with pressure



However, near surface or within anthropogenic elements, we can have some velocity decrease **(velocity inversion)**