ES. 1

$M =$ "n° medici nella commissione"

i) $M \in \{0,1,2,3\}$ q.c.

$$P(M=0) = \frac{\binom{6}{3}}{\binom{9}{3}} = \frac{20}{84} \qquad P(M=1) = \frac{\binom{6}{2}\binom{3}{1}}{\binom{9}{3}} = \frac{45}{84}$$

$$P(M=2) = \frac{\binom{6}{1}\binom{3}{2}}{\binom{9}{3}} = \frac{18}{84} \qquad P(M=3) = \frac{\binom{3}{3}}{\binom{9}{3}} = \frac{1}{84}$$

$$\mathbb{E}M = \frac{45}{84} + \frac{36}{84} + \frac{3}{84} = 1$$

ii) $I =$ "n° infermieri nella commissione"

$$P(M=1, I=1) = \frac{\binom{3}{1}^3}{\binom{9}{3}} = \frac{27}{84} = 0{,}32$$

iii) $A =$ "presidente è medico"

$$P(A) = P\left(A, \bigcup_{i=0}^{3} (M=i)\right)$$

$$= \sum_{i=1}^{3} P(A, M=i) = \sum_{i=1}^{3} P(A \mid M=i)\, P(M=i)$$

$$= \frac{45}{84} \cdot \frac{1}{3} + \frac{18}{84} \cdot \frac{2}{3} + \frac{1}{84} \cdot 1 = \frac{1}{3}$$

iv) $P(M=1 \mid A) = \dfrac{P(M=1, A)}{P(A)}$

$$= \frac{\frac{45}{84} \cdot \frac{1}{3}}{\frac{1}{3}} = \frac{45}{84} = 0{,}53$$

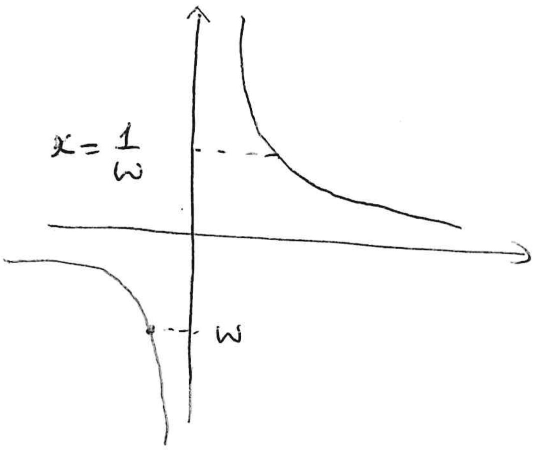
ES. 2

i) $F(w) = P\left(\frac{1}{X} < w\right)$

per $w<0$

$$= \int_{\{x > \frac{1}{w}\}} f(x)\,dx = \int_{\frac{1}{w}}^{0} \frac{1}{\pi} \frac{dx}{1+x^2} = -\frac{1}{\pi} \operatorname{arctg} \frac{1}{w}$$

$$= \frac{1}{\pi} \operatorname{arctg} \frac{1}{|w|}$$



per $w>0$

$$= \frac{1}{2} + \frac{1}{\pi} \int_{\frac{1}{w}}^{+\infty} \frac{dx}{1+x^2} =$$

$$= 1 - \frac{1}{\pi} \operatorname{arctg} \frac{1}{w}$$

poiché $\operatorname{arctg} w + \operatorname{arctg} \frac{1}{w} = \frac{\pi}{2}$

$$F_{\frac{1}{X}}(w) = \begin{cases} \frac{1}{2} - \frac{1}{\pi} \operatorname{arctg} |w| & w<0 \\ \frac{1}{2} + \frac{1}{\pi} \operatorname{arctg} w & w>0 \end{cases}$$

$f_{\frac{1}{X}}(w) = \dfrac{1}{\pi(1+w^2)} \quad w \in \mathbb{R}$ $\qquad \frac{1}{X} \sim$ Cauchy standard

ii) $f_{1+\frac{1}{X}}(z) = f_{\frac{1}{X}}(z-1) = \dfrac{1}{\pi\left(1+(z-1)^2\right)}$ $\qquad z \in \mathbb{R}$

poiché $\quad Z = 1+\frac{1}{X} = 1+W \qquad W = Z-1$

$$\left[\begin{array}{l} \text{oppure con la f.c.} \quad H_Z(\vartheta) = e^{-|\vartheta| + i\mu} \\ \qquad\qquad \Rightarrow \quad f_Z(z) = \dfrac{1}{\pi}\,\dfrac{1}{(z-1)^2+1} \end{array}\right]$$

iii) $P\left(\dfrac{1}{1+\frac{1}{X}} < w\right) = P\left(\dfrac{1}{1+Y} < w\right)$ $\qquad$ dove $Y = \frac{1}{X}$ (vedi punto i)

$$= P(1 < w + Yw)$$

$$= P\left(Y > \frac{1-w}{w}\right)$$

$$= \int_{\frac{1-w}{w}}^{+\infty} \frac{dy}{\pi(1+y^2)}$$

$$f_{\frac{1}{1+\frac{1}{X}}}(w) = \frac{d}{dw}\int_{\frac{1-w}{w}}^{+\infty} \frac{dy}{\pi(1+y^2)} = -\frac{1}{\pi\left(1+\left(\frac{1-w}{w}\right)^2\right)} \cdot \frac{-w-(1-w)}{w^2}$$

$$= \frac{1}{\pi}\,\frac{1}{w^2+(1-w)^2} = \frac{1}{\pi}\,\frac{1}{2w^2-2w+1}$$

$$= \frac{1}{\pi}\,\frac{\frac{1}{2}}{\left(w-\frac{1}{2}\right)^2+\frac{1}{2^2}} \quad \Rightarrow \quad \frac{1}{1+\frac{1}{X}} \sim \text{Cauchy}\left(\tfrac{1}{2},\tfrac{1}{2}\right)$$

iv) $f_Z(z) = f_W(z-1) = \frac{1}{\pi} \frac{1/2}{(z-1-\frac{1}{2})^2 + \frac{1}{2^2}} = \frac{1/2}{\pi (z-\frac{3}{2})^2 + \frac{1}{2^2}}$ $\quad z \in \mathbb{R}$

dove $Z = 1 + W$ $\qquad$ e $W = \frac{1}{1+\frac{1}{X}}$

$W = Z - 1$

$1 + \frac{1}{1+\frac{1}{X}} \sim \text{Cauchy}\left(\frac{1}{2}, \frac{3}{2}\right)$