# CHAPTER 7
# DEMAND FOR INSURANCE

## Why buy insurance?

- Demand for insurance driven by the **fear of the unknown** - **uncertainty**
  - Hedge against risk -- the possibility of bad outcomes
- Risk of ***income loss***
  - income loss because sick
  - + income need for healthcare
- Purchasing insurance means forfeiting income in good times to get money in bad times
- Insurance contract:
  - Pay a premium
  - Receive a pay-out if sick
    - If bad times avoided, then money lost
    - Ex: The individual who buys health insurance but never visits the hospital might have been better off spending that income elsewhere.

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# Risk aversion

- Hence, risk aversion drives demand for insurance
  - The individual is risk averse if s/he prefers the certainty equivalent to a "lottery" with the same expected value
- We can model risk aversion through utility from income *U(I)*
  - Utility increases with income: $U'(I) > 0$
  - Marginal utility for income is declining: $U''(I) < 0$
  - A measure of (absolute) risk aversion: $-\frac{U''(I)}{U'(I)}$

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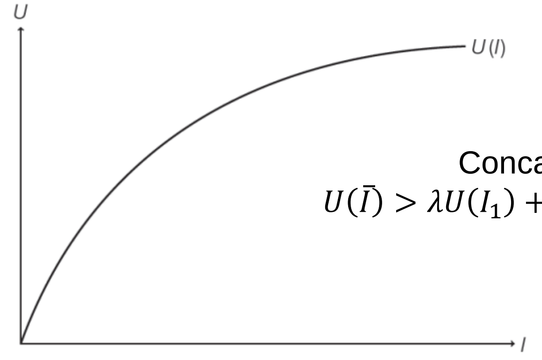
# Income and utility

- Graphically,
  - Utility increasing with income $U'(I) > 0$
  - Marginal utility decreasing $U''(I) < 0$




Concavity

$$U(\bar{I}) > \lambda U(I_1) + (1-\lambda)\, U(I_2)$$

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# Adding uncertainty

- An individual does not know whether she will become sick, but she knows the probability of sickness is ***p*** between 0 and 1
  - Probability of sickness is *p*
  - Probability of staying healthy is *1 - p*

- If she gets sick, medical bills and missed work will reduce her income
  - $I_S$ = income if she does get sick
  - $I_H > I_S$ = income if she remains healthy



# Expected value

- The **expected value** of a random variable X, E[X], is the sum of all the possible outcomes of X weighted by each outcome's probability
  - If the outcomes are $x_1, x_2, \ldots, x_n$, and the probabilities for each outcome are p1, $p_2, \ldots, p_n$ respectively, then:

$$E[X] = p_1 x_1 + p_2 x_2 + \cdots + p_n x_n$$

- In our individual's case, the formula for expected value of income *E[I]*:

$$\boldsymbol{E[I] = p\, I_S + (1- p)\, I_H}$$

## Example: expected value

- Suppose we offer a starving graduate student a choice between two possible options, a lottery and a certain payout:

  **A:** a lottery that awards $500 with probability 0.5 and $0 with probability 0.5.

  **B:** a check for $250 with probability 1.

- The expected value of both the lottery and the certain payout is $250:

  $E[I] = p\, I_S + (1-p)\, I_H$

  $E[A] = .5(500) + .5(0) = \$250$

  $E[B] = 1(250) = \$250$



## People prefer certain outcomes

- Studies find that **most people prefer certain payouts over uncertain scenarios**

- What does that imply about the utility function?

- To answer this question, we need to define expected **utility** for a lottery or uncertain outcome.

# Expected Utility

- ***The expected utility*** from a random payout X **E[U(X)]** is the sum of the utility from each of the possible outcomes, weighted by each outcome's probability.

- If the outcomes are $x_1, x_2, \ldots, x_n$, and the probabilities for each outcome are $p_1, p_2, \ldots, p_n$ respectively, then:
  - $\mathbf{E[U(X)]} = p_1 U(x_1) + p_2 U(x_2) + \cdots + p_n U(x_n)$

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# Example

- The preference for option B over option A implies that expected utility from B, is greater than expected utility from A:

$$E[U(B)] \geq E[U(A)]$$
$$U(\$250) \geq 0.5\ U(\$500) + 0.5\ U(\$0)$$

- In this case, even though the expected values of both options are equal, the individual **prefers the certain payout over the uncertain one**.
  - **This is risk aversion**
  - as opposed to risk neutrality and risk loving

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# Expected utility without insurance

- Lottery scenario similar to case of insurance customer
  - She gains a high income $I_H$ if healthy, and low income $I_S$ if sick.
- Uncertainty about which outcome will happen, though she knows the probability of becoming sick is *p*
  - ***Expected utility*** E[U(I)] is:

$$E[U(I)] = p\ U(I_S) + (1-\ p)\ U(I_H)$$

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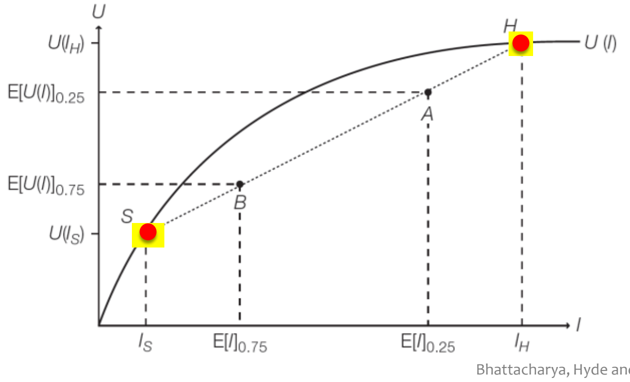
# E[U(I)] and probability of sickness

- Consider a case where the person is sick with certainty (p = 1):
  - $E[U] = U(I_S)$ equals the utility from certain income $I_S$ (Point S)
- Consider case where person has no chance of becoming sick (p = 0):
  - $E[U] = U(I_H)$ equals utility from certain income $I_H$ (Point H)

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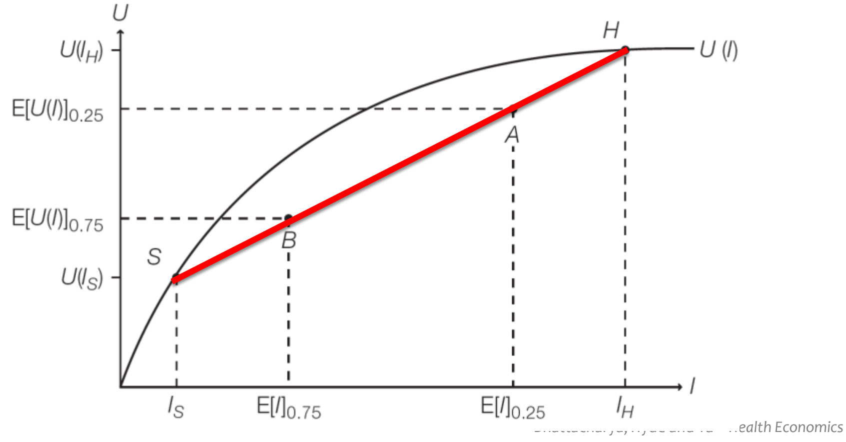
# What if *p* lies between 0 and 1?

- For p between 0 and 1, ***expected utility*** ***falls on a line segment between S and H***

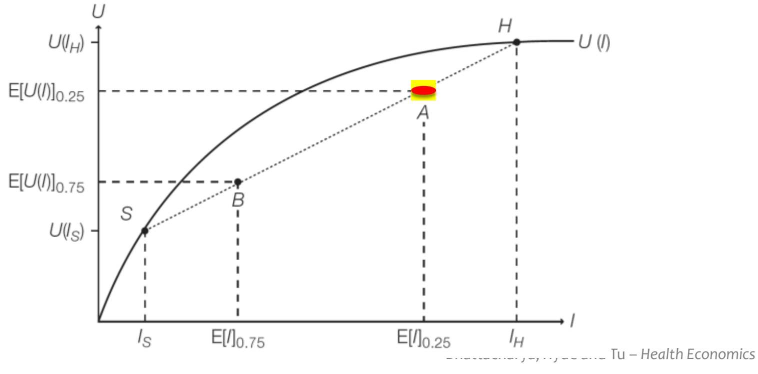



# *Ex:* p = 0.25

- For p = 0.25, person's expected **income** is:

$$E[I] = 0.25 \cdot I_S + (1 - .25) \cdot I_H$$

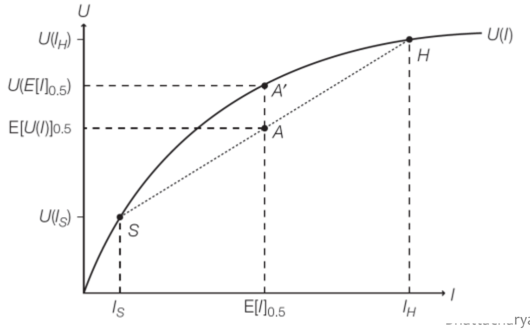
- **Utility** at that expected income is E[U(I)] (Point A)



# Expected utility and expected income

- Crucial distinction between
  - Expected utility E[U(I)]
  - Utility from expected income U(E[I])

***For risk-averse people, U(E[I]) > E[U(I)]***

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# An example

- $I_1$=100 $I_2$=400
- $p_1$=0.25 $p_2$=0.75
- U(I)=log(I)
- E(I)=?
- U(E(I))=?
- E(U(I))=?

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## Risk-averse individuals

Synonymous definitions of risk-aversion:

- ***Prefer certain outcomes to uncertain ones with the same expected income.***
- ***Prefer the utility from expected income to the expected utility from uncertain income***
  - $U(E[I]) > E[U(I)]$
- ***Concave utility function***
  - $U'(I) > 0$
  - $U''(I) < 0$



## A basic health insurance contract

- Customer pays an upfront fee
  - Payment *r* is known as the *insurance premium*
- If ill, customer receives *q* -- the *insurance payout*
- If healthy, customer receives nothing

- Either way, customer loses the upfront fee
- Customer's final income is:
  - Sick: $I_S + q - r$
  - Healthy: $I_H + 0 - r$

## Income with insurance

- Let $I_H'$ and $I_S'$ be income with insurance
  - Sick: $I_S' = I_S + q - r$
  - Healthy: $I_H' = I_H + 0 - r$

- Remember that risk-averse individuals want to **avoid uncertainty** →

$$I_H' = I_S'$$



## Full insurance

- **Full insurance means no income uncertainty**

$$I_H' = I_S'$$

- Final income is **state-independent**
  - Regardless of healthy or sick, final income is the same

- Risk-averse individuals prefer **full** insurance to **partial** insurance (given the same price)

# Full insurance payout

- State independence implies

$$I_H' = I_S'$$

- So

$$I_H + 0 - r = I_S + q - r$$

$$I_H = I_S + q$$

$$\boldsymbol{q = I_H - I_S}$$

- The payout from a full insurance contract is difference between incomes without insurance



# Actuarially fair insurance

- Actuarially fair means that insurance is a **fair** bet
  - i.e. the premium equals the expected payout

$$\boldsymbol{r = p\, q}$$

- Insurer makes zero profit/loss from actuarially fair insurance *in expectation*

# Actuarially fair, full insurance

- **Healthy State**

$$\begin{aligned} I'_H &= I_H - r \\ &= I_H - pq \\ &= I_H - p(I_H - I_S) \\ &= pI_S + (1-p)I_H \\ I'_H &= \mathrm{E}[I]_p \end{aligned}$$

- **Sick State**

$$\begin{aligned} I'_S &= I_S - r + q \\ &= I_S - pq + q \\ &= I_S - p(I_H - I_S) + (I_H - I_S) \\ &= pI_S + (1-p)I_H \\ I'_S &= \mathrm{E}[I]_p \end{aligned}$$

Notice consumers with actuarially fair, full insurance achieve their *expected income* with certainty!



# Insurance and risk aversion

- As we have seen, by reducing uncertainty, insurance can make risk-averse individuals better off.

- Relative to the state of no insurance, with insurance she loses income in the healthy state ($I_H > I'_H$) and gains income in the sick state ($I_S < I'_S$).
  - In other words, the risk-averse individual willingly sacrifices some good times in the healthy state to ease the bad times in the sick state.

# Insurer profits

- Now consider the same insurance contract from the point of view of the insurer
  - Premium *r*
  - Payout *q*
  - Probability of sickness *p*
  - E[Π] = Expected profits

$$E[\Pi(p,q,r)] = (1-p)r + p(r-q)$$
$$= r - pq$$

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# Fair and unfair insurance

- In a perfectly competitive insurance market, profits will equal zero

$$E[\Pi(p,q,r)] = 0 \qquad \Longrightarrow \qquad r = pq$$

  - Same definition as actuarially fair!
- An insurance contract which yields positive profits is called **unfair insurance:**

$$E[\Pi(p,q,r)] > 0 \qquad \Longrightarrow \qquad r > pq$$

- An insurer would never offer a contract with negative profits

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# Full vs. partial insurance

- Partial insurance does not achieve *state-independence*

**Full insurance**

$$I_S' = I_H'$$
$$I_S - r + q = I_H - r$$
$$I_S + q = I_H$$
$$q = I_H - I_S$$

**Partial insurance**

$$I_S' < I_H'$$
$$I_S - r + q < I_H - r$$
$$I_S + q < I_H$$
$$q < I_H - I_S$$

- Size of the payout q determines the fullness of the contract
  - Closer q is to $I_H - I_S$, the fuller the contract

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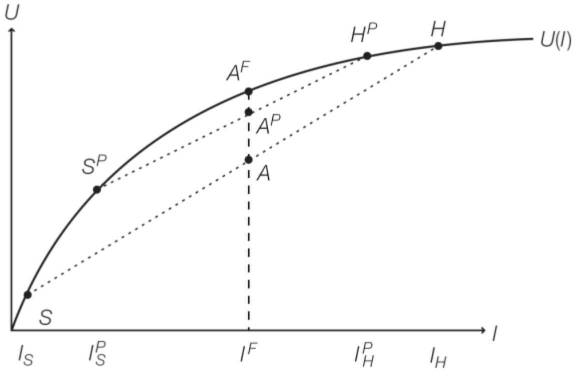
# Comparing insurance contracts

- $A^F$ -- Actuarially fair & full
- $A^P$ -- Actuarially fair & partial
- A -- Uninsurance
- $U(A^F) > U(A^P) > U(A)$

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# The ideal insurance contract

- For anyone risk-averse, actuarially fair & full insurance contract offers the most utility
  - Hence, it is called the **ideal insurance contract**

- Ideal and non-ideal insurance contracts:

| | Fair | Unfair |
|---|---|---|
| Full | $r = pq$<br>$q = I_H - I_S$ | $r > pq$<br>$q = I_H - I_S$ |
| Partial | $r = pq$<br>$q < I_H - I_S$ | $r > pq$<br>$q < I_H - I_S$ |

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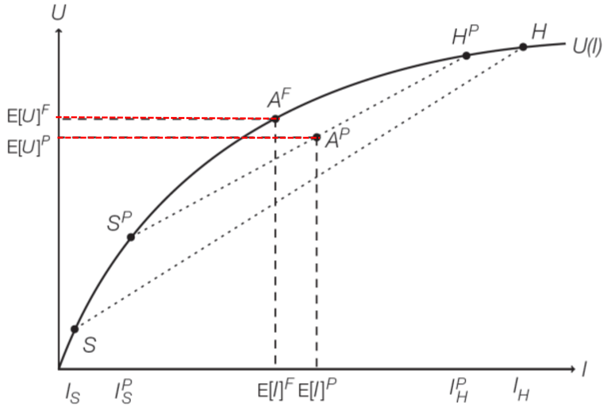
# Comparing non-ideal contracts

- $A^F$ – Full but actuarially unfair contract
- $A^P$ – Partial but actuarially fair contract

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# Comparing non-ideal contracts

- In this case, $U(A^F) > U(A^P)$
  - Even though $A^F$ is actuarially unfair, its relative fullness (i.e. higher payout) makes it more desirable
- But notice if contract $A^F$ became more unfair, then expected income $E[I]$ falls
  - If too unfair, $A^F$ may generate less utility than $A^P$
- Similarly, $A^P$ may become more full by increasing its payout
  - Uncertainty falls, so point $A^P$ moves
  - At some point, this consumer will be indifferent between the two contracts



# Conclusion

- Demand for insurance driven by risk aversion
  - Desire to reduce uncertainty
  - Diminishing marginal utility from income
  - $U(I)$ is concave, so $U''(I) < 0$
  - $U(E[I]) > E[U(I)]$
- Risk aversion can explain not only demand for insurance but can also help explain
  - Large family sizes
  - Portfolio diversification
  - Farmers scattering their crops and land holdings