RISPOSTE

1 1) $k = \frac{e-1}{e+1}$; $\mathbb{E}X = 2\,\frac{e-1}{e+1}$; $\mathbb{V}\mathrm{ar}(X) = 2\,\frac{(e-1)(3-e)}{(e+1)^2}$ ·

2) $f_Y(x) = \begin{cases} 0, & x \le 0 \\ (1+e^{-\sqrt{x}})/(4\sqrt{x}), & 0 < x \le 1 \\ e^{-\sqrt{x}}/(4\sqrt{x}), & x > 1 \end{cases}$ 3) $Y_n \xrightarrow{\text{q.c.}} 1$.

2 1) $H(u) = 2(u \sin u + \cos u - 1)/u^2$. 2) $k = 1/\pi^2$; $F_{(X,Y)}(x,y) =$
$= (\arctan x + \pi/2)(\arctan y + \pi/2)/\pi^2$; $P((X,Y) \in Q) = 1/16$. 3) $Y_n \xrightarrow{\text{p}} 0$.

3 1) $f_X(x) = \lambda\alpha x^{\alpha-1} e^{-\lambda x^\alpha}$, $x > 0$; $\mathbb{E}X^r = \lambda^{r/\alpha}\Gamma(1 + r/\alpha)$.

2) $F_{(Y,Z)}(y,z) = \begin{cases} 0, & y \text{ e/o } z \le 0 \\ z^n - (z-y)^n, & 0 < y \le z \le 1 \\ z^n, & y \le z,\ 0 \le z \le 1 \\ 1 - (1-y)^n, & 0 \le y \le 1,\ z \ge 1 \\ 1, & y \ge 1,\ z \ge 1 \end{cases}$ 3) $6/7$.

4 1) 0,65; 0,22; 8/35. 2) $Y \sim$ Cauchy. 3) $Z_n \xrightarrow{\text{d}} Z \equiv X + k$, con $F_Z(z) = F_X(z-k)$.

5 1) $\binom{9}{3} 2^6/3^9$; $\binom{9}{3}\binom{6}{3}/3^9$; $\binom{9}{2}\binom{7}{3} 3!/3^9$.

6 1) 11/32. 2) $p_1q_2q_3 + q_1p_2q_3 + q_1q_2p_3$; $1 - q_1q_2q_3$. 3) $P(\text{libero}) = \begin{cases} (p' + p'')^2/4, & \text{(a)} \\ (p'^2 + p''^2)/2, & \text{(b)} \end{cases}$

7 1) 5/9. 2) 1/2 e 1/3 ovvero 1/3 e 1/2. 3) $p^{n-1}q + pq^{n-1}$.

8 1) $a = 1/2$; $F(x) = (1 + \sin x)/2$, $-\pi/2 < x < \pi/2$; $\sqrt{2}/4$. 2) $f_T(t) = 1/t$, $t \in (1, e)$.

3) $f_Z(z) = \dfrac{\exp\{-(-1 + \sqrt{1+z})^2/2\} + \exp\{-(1 + \sqrt{1+z})^2/2}{2\sqrt{2\pi(1+z)}}$, $z > -1$.

9 1) 4/15; 1/4. 2) $f_Y(y) = \dfrac{1}{\pi\,|a|}\,\dfrac{1}{\sqrt{1 - y^2/a^2}}$, $|y| < |a|$. 3) $Z_n \xrightarrow{\text{q.c.}} 1$.

10 1) $f_Z(z) = 1/z^2$, $z > 1$. 2) Anche $Y_n \xrightarrow{\text{q.c.}} X$. 3) X assume i valori 0,1,2,3 con probabilità 0,032+2/30, 0,144+8/30, 0,216+5/30, 0,108.

11 1) $(2/3)^n$; $1/3^n$; $(2^n-1)/3^{n-1}$. 2) $Y_n \xrightarrow{\text{d}} \chi^2_1$. 3) $Z \sim \text{Espon}(\lambda)$.

12 1) $P(X=1)=1/3$, $P(X=r)=2/15$, $r=2,3,\dots,6$; $\mathbb{E}X=3$.
2) $f_Z(z)=4\log z/z^3$, $z>1$. 3) $Y_n \xrightarrow{\text{p}} 1/\lambda$.

13 1) $P(\text{prende il treno})=\begin{cases} 2/3, & \text{sotto casa} \\ 161/225, & \text{a 5 min.} \end{cases}$
2) $f(y)=1/(2\sqrt{1-y})$, $0<y<1$. 3) $Z_n \xrightarrow{\text{p}} 0$.

14 1) $P(X=1)=3/8$, $P(X=r)=1/8$, $r=2,3,\dots,6$; $3/8$.
2) $p_n(n+1)/(\mathbb{E}R+1)$, dove $R=$ “num. di palline rosse nell'urna e $p_n=P(R=n)$.

15 1) $F_Z(z)=z/(1+z)$, $0<z\le 1$. 2) $Y_n \xrightarrow{\text{d}} \mathcal{N}(0,1)$.

16 1) $Z \sim \text{Unif}(-1,1)$. 2) $Y_n \xrightarrow{\text{d}} Y$, con $-Y \sim \text{Espon}(1)$.

17 1) $Y_n \xrightarrow{\text{p}} 0$. 2) $1/(6-5q_1q_2)$; $5p_1/(6-5q_1q_2)$; $5q_1p_2/(6-5q_1q_2)$; equo se $p_1=$ $=1/5$ e $p_2=1/4$. 3) $2/(3\sqrt{h})$ se $h\ge 1$; $1-h/3$ se $0<h\le 1$.

18 1) $X_n \xrightarrow{\text{d}} \text{Unif}(0,\lambda)$. 2)

x	0	1	2	3	4
$P(X=x)$	p_0^2	$2p_0p_1$	$2p_0p_2+p_1^2$	$2p_1p_2$	p_2^2

con $p_0=\binom{b}{2}\Big/\binom{a+b}{2}$, $p_1=ab\Big/\binom{a+b}{2}$, $p_2=\binom{a}{2}\Big/\binom{a+b}{2}$.
3) $Z \sim \text{Unif}(0,1)$.

19 1) $[3^n-3(2^n-1)]/3^n$. 2) $Z \sim \text{Unif}(0,3)$. 3) Anche $Y_n \xrightarrow{\text{q.c.}} X$.

20 1) 0,15; 2/3. 2) $f_Z(z)=6(1-2\sqrt{z}+z)$, $0<z<1$. 3) $Y_n \xrightarrow{\text{d}} \text{Espon}(1)$.

21 1) 29/12. 2) $Y_n \xrightarrow{\text{d}} \text{Gamma}(\lambda=1,\nu=r)$. 3) $\text{Unif}(0,\sqrt{3}/4)$.

22 1) $P(X=x)=\begin{cases} [1-(1/6)^{x-1}]/5, & x=2,\dots,5 \\ [(1/6)^{x-6}-(1/6)^{x-1}]/5, & x=6,7,\dots \end{cases}$ 2) $Z \sim \text{Unif}(0,1)$.

23 1) $\mathbb{E}Z=1/a+1/b$; $f_Z(z)=\begin{cases} \dfrac{ab}{b-a}(\mathrm{e}^{-az}-\mathrm{e}^{-bz}), & z>0,\ a\ne b \\ \text{Gamma}(z;\lambda=b,\nu=2), & a=b \end{cases}$
2) $f_{W_n}(w)=-n^2w^{n-1}\log w$, $0<w<1$; $W_n \xrightarrow{\text{q.c.}} 1$.

24 1) $P(A \text{ vince})=\dfrac{a^2+ab}{a^2+ab+b^2}=1-P(B \text{ vince})$. 2) $f_U(u)=\frac{u}{2}$, $0<u<2$; $f_V(v)=$ $=\dfrac{1}{2}-\dfrac{|v|}{4}$, $-2<v<2$. 3) $F_{Y_n}(y)=\begin{cases} 0, & y\le -\log n \\ (1-\mathrm{e}^{-y}/n)^n, & y>-\mathit{logn} \end{cases} \longrightarrow \exp\{-\mathrm{e}^{-y}\}$.

25 1) 1/2; 7/13; $[1+(2/3)^n]^{-1}$; 1. 3) $X_n \xrightarrow{\text{p}} 0$; $r<2$.

2) $f_Y(y)=2/\sqrt{2\pi}\exp\{-(y-1)^2/2\},\ y>1;\ \mathbb{E}Y=1+\sqrt{2/\pi}$.

26 1) $1-\dfrac{(1+\lambda/2)}{e^{\lambda/2}}\cdot$ 2) $f_Y(y)=\begin{cases}0, & y\le 2\\ 2y^{-2}, & 2<y<4\\ (2y^{3/2})^{-1}, & y>4\end{cases}$ 3) $Y_n \xrightarrow{\text{d}} \text{Unif}(0,1)$.

27 1) 224/323; 96/323; 3/323. 2) $\text{Binom}(n=s, p=\lambda/(\lambda+\mu))$. 3) $Y_n \xrightarrow{\text{p}} 0$.

28 1) 20/36; 15/36; 1/36. 3) 1/2.

2) $F_{Z_n}(z)=\dfrac{\lambda_n/\mu_n}{\lambda_n/\mu_n+(1-z)/z},\ 0<z<1;\quad Z_n\begin{cases}\xrightarrow{\text{p}} 1, & \text{(a)}\\ \xrightarrow{\text{d}} \text{Unif}(0,1), & \text{(b)}\end{cases}$

29 1) $p_r=\begin{cases}[1-(2/3)^{n+1}]^{-1}/3, & r=0\\ p_0(2/3)^{|r|}/2, & r=\pm1,\dots,\pm n\end{cases}$ 2) $F_Z(z)=1-2e^{-\lambda z/2}+e^{-\lambda z},\ z>0$.

3) $V_n \xrightarrow{\text{d}} \text{Binom}(n=1, p=P(U\le 1/2))$. Anche $V_n \xrightarrow[\text{m.r.}]{\text{q.c.}} V\equiv\begin{cases}0, & U\le 1/2\\ 1, & U>1/2\end{cases}$

30 1)

x	0	1	2
$P(X=x)$	30/200	116/200	54/200

2) $P(A)=\dfrac{30}{61}=1-P(B)$.

3) $f_Y(y)=\dfrac{2}{3}\dfrac{y+1}{y^3},\ y>1$.

31 1) $f_Z(z)=2(1-z),\ 0<z<1$. 2) $Z_n \xrightarrow{\text{q.c.}} Z\equiv\begin{cases}0, & X<Y\\ 1/2, & X=Y\\ 1, & X>Y\end{cases}\sim \text{Unif}\{0,1\}$.

32 1) $1\Big/\dbinom{n}{3}$; $(n-i)\Big/\dbinom{n}{3}$; $3/n$. 2) $F_Z(z)=1-\dfrac{\mu}{\mu+\lambda(z-1)},\ z>1$.

3) $Z_n \xrightarrow{\text{q.c.}} 1/2$.

33 1) $\dfrac{n-1}{n+80}\cdot$ 2) 8/9.

34 1) p^2+2pqp_1; $2pq$, $2pqp_1$; $\dfrac{2pqp_1}{p^2+2pqp_1}\cdot$ 2) $F_Y(y)=\begin{cases}0, & y\le 1\\ 1-1/\sqrt{y}, & 1<y\le 4\\ 1/2, & 4<y\le 5\\ 1, & y>5\end{cases}$

3) $Y_n \xrightarrow{\text{q.c.}} 1$.

35 1) $p_v=\begin{cases}e^{-\lambda}\lambda^v/v!, & v=0,1,\dots,n-1\\ \displaystyle\sum_{k=n}^{+\infty}e^{-\lambda}\lambda^k/k!, & v=n\end{cases}$; $(1+b)\mathbb{E}V-bn$. 2) 1/3.

3) $f_{(Y_n,Z_n)}(y,z)=n(n-1)(z-y)^{n-2},\quad 0<y<z<1;$

$F_{T_n}(t)=\begin{cases}0, & t\le 0\\ n(1-t)t^{n-1}+t^n, & 0<t<1\\ 1, & t\ge 1\end{cases}=F_{\text{Beta}(n-1,2)}(t);\quad T_n \xrightarrow{\text{q.c.}} 1$.

36 1) $6/11$; $1/2$. 2) $1/[2(1+|z|)^2]$. 3) No; $\mathcal{N}(0,(1-1/4^{n+1})/3)$; $\mathcal{N}(0,1/3)$.

37 1)

z	20	30	40	50	60
p_z	0,2	0,2	0,15	0,3	0,15

2) Si : $F_{(U,V)}(u,v) = (1-\mathrm{e}^{-u}-u\mathrm{e}^{-u})/(1+1/v)$, u e $v>0$. 3) $Y_t \xrightarrow[t\to 1]{\mathrm{d}} \mathrm{Gamma}(\lambda=1,\nu=2)$.

38 1) $\sum_{k=0}^{3} p_k \dfrac{a+k}{a+b}$; $p_0 = a^3/n^3$; $p_1=(3a^2b+3ab+b)/n^3$; $p_2=(3ab^2+3b^2-3ab-3b)/n^3$; $p_3=(b^3-3b^2+2b)/n^3$. 2) $\dfrac{30}{61}$; $\dfrac{31}{61}\cdot$ 3) $\dfrac{57}{81}$; $\dfrac{18}{81}$; $\dfrac{6}{81}\cdot$

39 2) $P(X=x_k)=\dfrac{\alpha-1}{\alpha^k}$, $\alpha>1$. 3) $F_Z(z)=\begin{cases}0, & z\le 0\\ 1-\mathrm{e}^{-z}, & 0<z\le 1\\ 1, & z>1\end{cases}$

40 1) $Y_n \xrightarrow{\mathrm{d}} \mathrm{Unif}(0,1)$. 2) Non converge. 3) $Z_n \xrightarrow{\mathrm{q.c.}} 1$.

41 1) $\dfrac{n}{2^{n-2}(1+n)}$ 2) $F_Z(z)=\begin{cases}0, & z\le 1\\ 1/(1+1/z), & z>1\end{cases}$ 3) $F_{Y_n}(y) \longrightarrow \begin{cases}0, & y\le 1\\ 1-1/y, & y>1\end{cases}$

42 1) $\dfrac{10}{11}\cdot$ 2) $f_Y(y)=\dfrac{4}{\sqrt{2\pi}}\,y\,\mathrm{e}^{-y^4/2}$, $y>0$. 3) $Y_n \xrightarrow{\mathrm{p}} 1/2$.

43 1) $\dbinom{2n}{n}\dfrac{1}{4^n}\cdot$ 2) $\int_0^\lambda y^{k-1}\mathrm{e}^{-y}\,dy\,/(k-1)!$ 3) $H_{Z_n}(u)=\prod_{k=1}^{n}\dfrac{k}{k-iu}\cdot$

44 1) $Y\sim \mathrm{Poisson}(\lambda p)$. 2) $\dfrac{1}{2}\cdot$ 3) $H(u)=\dfrac{(\lambda/n)\,\mathrm{e}^{iu/n}}{1-(1-\lambda/n)\,\mathrm{e}^{iu/n}}\cdot$

45 1) $\dfrac{1}{N}$; $\sum_{j=k}^{n}\dfrac{1}{j}\dbinom{n}{j}p^j q^{n-j}$. 2) $\dfrac{5}{6}\cdot$ 3) $X_n \xrightarrow{\mathrm{q.c.}} 0$; $Y_n \xrightarrow{\mathrm{q.c.}} 1$.

46 1) $f(x)=\begin{cases}31/40, & 0<x<1\\ 9/40, & 1<x<2\\ 0, & \text{altrove}\end{cases}$ 2) $\dfrac{1}{4}+\dfrac{1}{2}\log 2$; $\dfrac{1}{2}-\dfrac{1}{2}\log 2$; $\dfrac{1}{4}$; 0. 3) $H_{Y_n}(u) = \left[\dfrac{1}{p}\,\mathrm{e}^{iu\sqrt{q/n}}-\dfrac{q}{p}\,\mathrm{e}^{iu/\sqrt{nq}}\right]^{-n}$.

47 1) $Z\sim\mathrm{Poisson}(\mu q)$. 2) $X\sim\mathrm{Espon}(1)$; $Y\sim\mathrm{Gamma}(\lambda=1,\nu=2)$. 3) $Y_n \xrightarrow{\mathrm{d}} \mathrm{Cauchy}(1/2)$.

48 1) $\begin{cases}1, & x\le 0\\ (1-x)^n+nx(1-x)^{n-1}, & 0<x\le 1\\ 0, & x\ge 1\end{cases}$; $(1-y)^n+n(y-x)(1-y)^{n-1}$,

$0<x<y<1$. 2) $f_Y(y)=\frac{1}{\pi}(y-y^2)^{-1/2},\ y\in(0,1)$. 3) $Y_n \begin{cases} \text{non converge}, & \alpha<3/2 \\ \xrightarrow{\text{d}} \mathcal{N}(0,1/3), & \alpha=3/2 \\ \xrightarrow{\text{p}} 0, & \alpha>3/2 \end{cases}$

49 1) Geom(p=1/2). 2) $Y \sim$ Cauchy$(1/t)$. 3) $Y_n \xrightarrow{\text{d}}$ Cauchy(t).

50 1)

x_r	4	5	6	7
p_r	$\frac{6\cdot7\cdot8}{10\cdot11\cdot12}$	$\frac{3\cdot4\cdot6\cdot7}{10\cdot11\cdot12}$	$\frac{3\cdot4\cdot5\cdot6}{10\cdot11\cdot12}$	$\frac{4\cdot5\cdot6}{10\cdot11\cdot12}$

2) $P(A) = \frac{6}{16} = 1 - P(B)$.

51 1) $X+Y \sim$ Gamma$(\lambda,2)$;

$$F_G(g)=\begin{cases} 0, & g\le -bM \\ 1-\left(1+\lambda\frac{g+bM}{a+b}\right)\cdot\exp\left\{-\lambda\frac{g+bM}{a+b}\right\}, & -bM<g\le aM \\ 1, & g>aM \end{cases}$$

2) $k=2$; $F_Z(z)=\frac{3z^2+2z}{3(1+z)^2},\ z>0$.

52 1)

n	0	1	2
p_n	$e^{-\lambda t}$	$\lambda t\, e^{-\lambda t}$	$1-(\lambda t+1)e^{-\lambda t}$

2) $k=2$; $F_Z(z)=\frac{3z^2+4z}{3(1+z)^2},\ z>0$.

53 1) $Z_n \xrightarrow{\text{q.c.}} X/Y$. 2) $Y_n \xrightarrow{\text{m.q.}} 1/2$. 3) $Y_n \xrightarrow{\text{q.c.}} X$.

54 1) $1/3^6$; $(2^7-2)/3^6$; $(3^6-2^7+1)/3^6$. 2) U e V i.i.d. $\sim\mathcal{N}(0,2)$; $ac+bd=0$. 3) $Y_n \xrightarrow{\text{d}}$ Cauchy$(0,1)$.

55 1) $\frac{1}{3}, \frac{1}{3}, \frac{1}{2}$. 2) $F_Y(y)=1-\frac{p\,e^{-y}}{1-q\,e^{-y}},\ y>0$. 3) $Y_r \xrightarrow{\text{d}} Y$, con $f_Y(y)=(1+y)^{-2},\ y>0$.

56 1) $P(N_1=r)=\frac{1}{2}\binom{n}{r}\vartheta_1^r(1-\vartheta_1)^{n-r}\left\{1+\left(\frac{1-\vartheta_2}{\vartheta_1}\right)^r\left(\frac{\vartheta_2}{1-\vartheta_1}\right)^{n-r}\right\},\ r=0,\ldots,n$. $P(H_1 \mid N_1=k)=\left\{1+\left(\frac{1-\vartheta_2}{\vartheta_1}\right)^k\left(\frac{\vartheta_2}{1-\vartheta_1}\right)^{n-k}\right\}^{-1}$. $\vartheta_1=1-\vartheta_2$; si.

2) No; $k=1$; $f_{X+Y}(s)=\begin{cases} s^2, & 0<s<1 \\ 2s-s^2, & 1<s<2 \\ 0, & \text{altrove} \end{cases}$ 3) $\mathbb{E}(\eta_n)^r \longrightarrow \exp\left\{\frac{1}{2}r^2\log^2 z\right\}$.

57 1) $p_1=0,\ p_r=\frac{1-[-1/(n-1)]^{r-1}}{n} \longrightarrow \frac{1}{n}$. 2) $f_X(t)=f_Y(t)=e^{-(t+1)},\ t>-1$; indip.; $f_Z(z)=(z+2)\,e^{-(z+2)},\ z>-2$. 3) $H_{Y_n}(u)=(1-iu/n)^{-n}$; $Y_n \xrightarrow{\text{p}} 1$.

58 1) $\frac{2}{n+1}$. 2) $\left\{r, \left(\frac{n-2}{n-1}\right)^{r-2}\frac{1}{n-1};\ r=2,3,\ldots\right\}$. 3) $Z_\lambda \xrightarrow[\lambda\to0]{\text{p}} 0$;

$$F_{Z_\lambda}(z) \xrightarrow[\lambda\to+\infty]{} \begin{cases} 0, & z\le 0 \\ e^{-1/z}, & z>0 \end{cases}$$

59 1) $P(X=i \mid Y=0)=\begin{cases}1/19, & i=0\\ 2/19, & i=1,2,\dots,9\end{cases}$ 2) $\binom{r-i-1}{k-3}p^k q^{j-k}$.
3) 0; 1; $\alpha_n \xrightarrow{\text{d}} \mathcal{N}(0,1)$.

60 1) $P(T)=p_O+p_{AB}+p_A^2+p_B^2-p_O p_{AB}$. $p_{AB}/P(T)$.
2) $\mathbb{E}Y=\begin{cases}-\infty, & \alpha\le -1\\ 1-(\alpha+1)^{-2}, & \alpha>-1\end{cases}$
3) $Y_n \xrightarrow{\text{p}} Y \overset{\text{q.c.}}{=} 1$. $\mathbb{E}Y_n^k \overset{\text{(def.nte)}}{=} \dfrac{n}{n-k\log n} \longrightarrow 1=\mathbb{E}Y^k$.

61 1) $P(E)=\begin{cases}(n-1)/2^n, & p=q=1/2\\ pq(p^{n-2}+qp^{n-3}+\dots+q^{n-2}), & p\ne q\end{cases}$ 2) $Y\sim \text{Espon}(\alpha)$.
3) $X_n+Y_n \xrightarrow{\text{m.r.}} 1+Y$.

62 1) $P(kT)=\frac{1}{2}\left\{\binom{n}{k}p^k q^{n-k}+I_{\{n\}}(k)\right\}\cdot$ 2) $f_Z(z)=(1+|z|)^{-2}$, $|z|<1$.
3) $Z_n \xrightarrow{\text{m.r.}} X$.

63 1) 11/30. 2) $Z\sim \text{Unif}(0,a)$. Si; $f_Z(z)=2z/a^2$, $0<z<a$.
3) $F_{Y_t|(X_t>t)}(y)=F_{\text{Espon}(1)}(y)\longrightarrow F_{\text{Espon}(1)}(y)$. $F_{Y_t|(X_t>t)}(y)\longrightarrow F_{\text{Espon}(1)}(y)$.

64 1) $3/3^n$; $3(2^n-2)/3^n$; $1-3/3^n-3(2^n-2)/3^n$. 2) $f_Y(y)=\exp\{1/y-\mathrm{e}^{1/y}\}/y^2$.

65 1) $p_r=\binom{2n-r-1}{n-1}\Big/2^{2n-r-1}$. 2) $(1-\mathrm{e}^{-\mu})/\mu$.
3) $f_Y(y)=(2y\log^2 y)^{-1}$, $y\in(0,1/\mathrm{e})\cup(\mathrm{e},+\infty)$.

66 1) $F_Z(z)=\begin{cases}0, & z\le 1\\ (z-1)/(z+1), & z>1\end{cases}$ 2) $Y_n \xrightarrow{\text{p}} 1$. 3) $F_{Y_n}(y)\longrightarrow \begin{cases}\mathrm{e}^y, & y\le 0\\ 1, & y>0\end{cases}$

67 1) $\dfrac{(3n-1)(n+1)}{n(2n+1)^2}=P(A)$; $1-P(A)$; 0; $\dfrac{1}{2(2n+1)}$; $\dfrac{n+1}{2n(2n+1)}\cdot$
2) $F_Y(y)=\begin{cases}0, & y\le 0\\ y/[2(y+1)], & 0<y\le 2\\ 1-y/(y^2-1), & y>2\end{cases}$ 3) $Y_n \xrightarrow{\text{p}} 0$.

68 1) $\dfrac{2\alpha p_1}{2\alpha p_1+(1-\alpha)(1-p_1)}\cdot$ 2) $2/\pi$.
3) $F_{Z_n}(z)=\begin{cases}0, & z\le 0\\ z/[2n(1-z)], & 0<z\le n/(n+1)\\ 1-n(1-z)/(2z), & n/(n+1)<z<1\\ 1, & z\ge 1\end{cases}$; $Z_n \xrightarrow[\text{m.r.}]{\text{q.c.}} 1$.

69 1) $\frac{6(n-3)}{n(n-1)}\cdot$ 2) $F_Z(z)=\begin{cases}0, & z\le 0\\ \frac{13}{18}+\frac{2}{3}z+\frac{1}{2}z^2, & 0<z\le 1/3\\ 1, & z>1/3\end{cases}$ 3) $Z_n \xrightarrow{\text{p}} 0.$

70 1) $\frac{(1-\gamma)\alpha}{(1-\gamma)\alpha+\gamma(1-\beta)}\cdot$ 2) y. 3) $\forall n\in\mathbb{N},\ Y_n\sim\text{Beta}(a/n,1).$ $Y_n \xrightarrow[\text{m.r.}]{\text{q.c.}} 0.$

71 1) $\frac{p^3}{1-q^3};\ \frac{q}{(1+q)^2}\cdot$ 2) $Z\sim\text{Espon}(1).$ 3) $Y_n \xrightarrow{\text{q.c.}} X.$

72 1) $x/2;\ x/4.$ $3/2;\ 5/4.$ 2) $n!$ 3) $S_n\sim\text{Gamma}(\lambda=1,\nu=n),\ Z_n \xrightarrow{\text{q.c.}} 1.$

73 1) $p_{y\cdot}=(1-q^2)q^{2y-2},\ y=1,2,\dots;\ p_{\cdot z}=(1-q)(2q^{z-1}-q^{2z-2}-q^{2z-1}),\ z=1,2\dots;$
$p_{y,z}=\begin{cases}(1-q)^2q^{2y-2}, & y=z=1,2,\dots\\ 2(1-q)^2q^{y+z-2}, & y=1,2,\dots \text{ e } z=y+1,y+2,\dots\end{cases}$
2) $f_Z(z)=\frac{1}{z}(1-\log z)^{-2},\ 0<z<1.$

74 1) $(2pq)^{n-1}p^2;\ p^2/(1-2pq);\ (2pq)^{n-1}q^2;\ q^2/(1-2pq).$

75 1) $Z_n \xrightarrow{\text{p}} 1.$ 2) $F_{Y_n}(y)\longrightarrow\begin{cases}\mathrm{e}^y, & y\le 0\\ 1, & y>0\end{cases}$ 3) $Z_n \xrightarrow{\text{p}} 1.$

76 1) $\frac{2n(n^2-n)!(n+k)!}{k!(n^2)!}\cdot$ 2) $F_Z(z)=\begin{cases}\frac{1}{2(1-z)}, & z\le 0\\ 1-\frac{2^{1-z}-1}{2(1-z)}, & z>0\end{cases}$

3) $F_{Z_n}(y)=\begin{cases}0, & z\le 0\\ 2^{n-1}z^n, & 0<z\le 1/2\\ 1-2^{n-1}(1-z)^n, & 1/2\le z<1\\ 1, & z\ge 1\end{cases}$; $Z_n \xrightarrow{\text{p}} \frac{1}{2}.$

77 1) $M_i=$(la persona sulla sedia n. i muore), $E=$(nessuno muore).
a) $P(M_i)=1/6=$cost.; $E=\emptyset$. b) $P(M_i)=(5/6)^{i-1}/6;\ P(E)=(5/6)^6.$
2) $Z\sim\text{Unif}(0,2).$ 3) $Y_n \xrightarrow{\text{q.c.}} 1/\mathrm{e}.$

78 1) $N_2\sim\text{Binom}(n,p^2).$ 2) $f_Z(z)=\begin{cases}\mathrm{e}^{-1}, & -1<z<0\\ 1-\mathrm{e}^{-1}, & 0<z<1\\ 0, & \text{altrove}\end{cases}$ 3) $Y_n \xrightarrow{\text{d}}$ Cauchy.

79 1)

t	0	3	4
p_t	2/5	2/5	1/5

; no. 2) 1/4. 3) $Y_n \xrightarrow{\text{q.c.}} \mathrm{e}^{-1/2}.$

80 1) $\frac{n-1}{2(n+1)};\ \frac{1}{2};\ \frac{1}{n-1}\cdot$ 2) $X\sim\text{Unif}(0,1);$ $f_Y(y)=y^{-2},\ y>1;$ indip.;

$$f_Z(z)=\begin{cases}\dfrac{1}{z-1}-\dfrac{1}{z}, & z<-1\\ \dfrac{1}{z-1}+1, & -1<z<0\\ 0, & \text{altrove}\end{cases}$$

3) $F_{Y_N}(y)=1-\exp\{y+\mu(1-\mathrm{e}^{-y})\}$; $Y_N \xrightarrow[\mu\to 0]{\mathrm{d}} Y_0 \sim \mathrm{Espon}(1)$.

81 1) $(L-2d)^3/L^3$. 2) $X+Y\sim \mathrm{Binom}(2n,p)$; $X \mid (X+Y=m) \sim \mathrm{Ipergeom}(n,n,m)$.
3) $T_n \sim t$ di Student con n g.d.l.; $T_n \xrightarrow{\text{q.c.}} X_0$.

82 1) $\frac{4}{9}\cdot$ 2) $\frac{2}{5}\cdot$ 3) $H_{U_n}(u)=[1-c\,|u|^\alpha/n+o(u^\alpha/n)]^n \longrightarrow \exp\{-c\,|u|^\alpha\}$;

$$U_n \xrightarrow{\mathrm{d}} \begin{cases}\text{Gaussiana inversa}, & \alpha=1/2\\ \mathrm{Cauchy}(0,c), & \alpha=1\\ \mathcal{N}(0,2c), & \alpha=2\end{cases}$$

83 1) $\dfrac{1}{n^2}$; $3\dfrac{n-1}{n^2}$; $\dfrac{(n-1)(n-2)}{n^2}\cdot$ 2) $Z \stackrel{\mathrm{d}}{=} W \sim \mathrm{Espon}(\log 2)$; $\dfrac{11}{16}\cdot$ 3) $F_Z(z)=\exp\{\mu(\mathrm{e}^{-1/(\mu z)}-1)\}$, $z>0$; $Z \xrightarrow[\mu\to 0]{\mathrm{p}} 0$; $F_Z(z) \xrightarrow[\mu\to+\infty]{} F(z)=\mathrm{e}^{-1/z}$, $z>0$.

84 1) $p^2/(1-pq)$; $q/(1-pq)$.
2) $X\sim\{r,\ pq^r;\ r=0,1\ldots\}$; $Y\sim\{s,\ (s+1)p^2q^s;\ s=0,1,\ldots\}$.
3) $F_k(z)=\dfrac{1-\mathrm{e}^{-k(1/z-1)}}{k(1/z-1)}$, $0<z<1$; $Z_k \xrightarrow[k\to 0]{\text{q.c.}} 0$; $Z_k \xrightarrow[k\to+\infty]{\text{q.c.}} 1$.

85 1) $\dfrac{n_1n_3+n_2n_3}{n_1n_2+n_1n_3+n_2n_3}\cdot$ 2) $f_Y(y)=\dfrac{4d^2}{\pi(d^4+4y^2)}$, $y>0$; $\mathbb{E}H=+\infty$.
3) $Y_n \xrightarrow{\mathrm{d}} \mathrm{Unif}(0,1)$.

86 1) $0,61$; $0,8479$; $12/61$. 2) $\frac{1}{2}(p'+p'')$; $\frac{1}{2}(p'^2+p''^2)$; $\dfrac{p'^2+p'^2}{p'+p''}$.

87 1) $P(Z=z)=\begin{cases}\dfrac{1}{1+q}, & z=1\\ p^2\dfrac{q^{m+n-2}}{1-q^{m+n}}, & 1<z=m/n\end{cases}$ 2) $Y\sim\mathrm{EsponSimm}(1)$.

88 1) $P(Z_n=z)=\begin{cases}\dfrac{n-1}{n}\,\mathrm{e}^{-\lambda}, & z=0\\ \mathrm{e}^{-\lambda}\dfrac{\lambda^z}{z!}\,\dfrac{n-1+z/\lambda}{n}, & z=1,2,\ldots\end{cases}$; $Z_n \xrightarrow{\mathrm{d}} \mathrm{Poisson}(\lambda)$.

2) $F_\alpha(z) \xrightarrow[\alpha\to 1]{} F_{X+Y_1}(z)=\begin{cases}0, & z\le 0\\ z-\log(z+1), & 0<z\le 1\\ 1-\log(1+1/z), & z>1\end{cases}$; $Z_\alpha \xrightarrow[\alpha\to+\infty]{\mathrm{p}} X$.

89 1) $X\sim\mathrm{Poisson}(\lambda p)$; $Y\sim\mathrm{Poisson}(\lambda q)$. 3) $Z_n \xrightarrow{\mathrm{d}} \mathcal{N}(0,1/\lambda^2)$.

2) $F_{X_t}(x)=\begin{cases}0, & x\leq -t\\ 1-\mathrm{e}^{-\lambda(t+x)/2}, & -t<x\leq t\\ 1, & x>t\end{cases}$

90 1) $\frac{20}{29}\cdot$ 2) $f_X(x)=3x^2,\ 0<x<1;\ f_Y(y)=\frac{3}{2}(1-\sqrt{|y|}),\ |y|<1;$ X ed Y dipendenti e incorrelate. 3) $H_{Y_n}(u)=\left[\frac{\sin(u/\sqrt{n})}{u/\sqrt{n}}\right]^{2n}$; $Y_n \xrightarrow{\mathrm{d}} \mathcal{N}(0,2/3)$.

91 1) $\frac{a}{a+b};\ \frac{a+c}{a+b+c};\ \frac{a}{a+b}\cdot$ 2) $F(q)=\frac{\lambda(1-\mathrm{e}^{-\mu q})-\mu(1-\mathrm{e}^{-\lambda q})}{\lambda-\mu},\ 0<q\leq k.$
3) $Z_\mu \xrightarrow[\mu\to 0]{\mathrm{p}} Y.$

92 1) $\frac{91}{420};\ \frac{29}{420};\ \frac{300}{420}\cdot$ 2) $f_{W_1}(w)=1-|w|,\ |w|<1;\ f_{W_2}(w)=1-|w-1|,$ $0<w<2.$ 3) $H_{U_n}(u)=\mathrm{e}^{iu}\left(\frac{p}{1-q\mathrm{e}^{iu/n}}\right)^n$; $U_n \xrightarrow{\text{q.c.}} \frac{1}{p}\cdot$

93 1) $\frac{(1-\alpha)(1-\beta)}{1-\alpha\beta};\ \frac{\alpha(1-\alpha)(1-\beta)}{1-\alpha^2\beta}\cdot$ 2) $\alpha<0,\ Y\sim$ Beta$(-\lambda/\alpha,1);\ \alpha=0,\ Y=1;$ $\alpha>0,\ F_Y(y)=1-y^{-\lambda/\alpha},\ y>1.$ 3) $T_n \xrightarrow{\mathrm{d}}$ Poisson$(\frac{q}{1-q})\cdot$

94 1) $i<j,\ 2/(2j-1);\ i=j,\ 1/(2j-1);\ i>j,\ 0.$ 2) $Y\sim$ Geom$(1-1/\mathrm{e})$; Espon(1).
3) $Z_n \xrightarrow{\text{q.c.}} 1.$

95 1) $P(Z=1)=a+b-2ab=1-P(Z=-1)$; X e Z indipendenti per $b=1/2$; Y e Z indipendenti per $a=1/2$. 2) $Z\sim$ Unif$(0,1)$. 3) $f_{Z_k}(z)=\frac{1}{k^2}-\mathrm{e}^{-k^2/z}\left(\frac{1}{k^2}+\frac{1}{z}\right),\ y>0;$ $Z_k \xrightarrow[k\to 0]{\mathrm{p}} 0;\ Z_k \not\xrightarrow[k\to+\infty]{} .$

96 1) $\frac{3}{280};\ \frac{3}{70};\ \frac{15}{56}\cdot$ 2) $f_U(u)=\frac{\mathrm{e}^{-u}}{1-\mathrm{e}^{-1}},\ 0<u<1;\ V\sim$ Espon(1).
3) $F_{Z_n}(z)\longrightarrow \mathrm{e}^{-1/z},\ z>0.$

97 1) $\frac{27}{27+25/\mathrm{e}^2}\cdot$ 2) $Z\sim$ Espon(2). 3) $Y_n \xrightarrow{\mathrm{p}} 1;\ Z_n \xrightarrow{\mathrm{d}}$ Espon(1).

98 1) $2p^2-2p+1;\ 1/2;\ 4p^3-6p^2+3p.$ 2) Triangolare(0,2).
3) $F_{Z_n}(z)=\begin{cases}0, & z\leq 0\\ z^n/2, & 0<z\leq 1\\ 1-z^{-n}/2, & z\geq 1\end{cases}$; $Z_n \xrightarrow[\text{m.r.}]{\text{q.c.}} 1.$

99 1) $3/5;\ 0.$ 2) $2\leq k\leq n-1;\ P(X_i=0,X_j=0)=\frac{(n-k)(n-k-1)}{n(n-1)};$
$P(X_i=0,X_j=j)=\frac{k(n-k)}{n(n-1)}=P(X_i=i,X_j=0);\ P(X_i=1=X_j)=\frac{k(k-1)}{n(n-1)};$

$\mathbb{C}\mathrm{ov}(X_i, X_j) = \dfrac{-ijk(n-k)}{n^2(n-1)}\cdot$ 3) $Z_n \xrightarrow{\mathrm{d}} \text{Espon}(1)$.

100 1) $\frac{1}{2}\left[\left(\frac{1}{2}\right)^r + \frac{1}{4}\left(\frac{3}{4}\right)^{r-1}\right]$. 2) $43/2^6$.

101 1) $P(Z=0) = \dfrac{p}{1+q}$; $P(Z=z) = \dfrac{2pq^z}{1+q}$, $z \in \mathbb{N}$.

2) $F_{X_t}(x) = \begin{cases} 0, & x \le t \\ \mathrm{e}^{-\lambda(2t-x)}, & t < x \le 2t \\ 1, & x > 2t \end{cases}$

102 1) $p_k(h) = \begin{cases} 0, & k < h \\ 1/3, & k = h \\ \frac{1}{9}\left(\frac{5}{6}\right)^{k-h-1}, & k \ge h+1 \end{cases}$ 3) 0; $\dfrac{1}{n}$; $Y_n \xrightarrow[\text{m.q.}]{\text{q.c.}} 0$.

2) $F_Z(z) = \begin{cases} 0, & z \le 0 \\ z^2/4, & 0 \le z \le 2 \\ 1, & z \ge 2 \end{cases}$; $F_Q(q) = \begin{cases} 0, & q \le -\frac{1}{8} \\ \frac{1}{17}(1+8q)^2, & -\frac{1}{8} \le q \le 0 \\ 1 - \frac{4}{17}(2-q)^2, & 0 \le q \le 2 \\ 1, & q \ge 2 \end{cases}$

103 1) Per $p_1 = p$, $Z_{p_1} \sim \text{BinomNeg}(n=2, p)$; per $p_1 \ne p$, $P(Z_{p_1} = z) = pp_1q_1^{z-2}\dfrac{1-(q/q_1)^{z-1}}{1-q/q_1}$, $z \ge 2$; $Z_{p_1} \xrightarrow[p_1 \to p]{\mathrm{d}} \text{BinomNeg}(n=2, p)$. 2) $Z \begin{cases} \xrightarrow{\mathrm{p}} -1, & \text{per } \mu/\lambda \longrightarrow 0 \\ \xrightarrow{\mathrm{d}} \text{Unif}(-1,1), & \text{per } \mu/\lambda \longrightarrow 1 \\ \xrightarrow{\mathrm{p}} 1, & \text{per } \mu/\lambda \longrightarrow +\infty \end{cases}$

104 1)

x	1	2	3	4
p_x	$\frac{4}{32}$	$\frac{17}{32}$	$\frac{10}{32}$	$\frac{1}{32}$

2) $Y \sim \text{Geom}(p = 1 - \mathrm{e}^{-\lambda})$. 3) $Y_n \xrightarrow{\mathrm{d}} \text{Poisson}(1)$.

105 1) 3/7. 2) $\dfrac{\lambda}{\lambda+\mu}$; $F_{(W,U)}(w,u) = \left(1 - \mathrm{e}^{-(\lambda+\mu)u}\right)\left[\dfrac{\lambda}{\lambda+\mu}(1-\mathrm{e}^{-\mu w}) + \dfrac{\mu}{\lambda+\mu}(1-\mathrm{e}^{-\lambda w})\right]$, w e $u > 0$. 3) $F_{Y_n}(y) = \left(1 - \dfrac{1}{\pi}\arctan\dfrac{\pi}{ny}\right)^n$, $y > 0$.

106 1) $\dfrac{432}{2197}$; $\dfrac{433}{32\cdot 6^3}$; $\dfrac{144}{169}\cdot$ 3) $k = (1-\mathrm{e}^{-n^2\lambda})^{-1}$; $Z_n \xrightarrow{\mathrm{d}} \text{EsponSimm}(\lambda)$.

2) $f_T(t) = \begin{cases} \frac{1}{10}(1-\mathrm{e}^{1-t/10}), & 10 < t < 20 \\ \frac{\mathrm{e}-1}{10}\mathrm{e}^{1-t/10}, & t > 20 \end{cases}$; $\mathbb{E}T = 25$; $\mathbb{V}\mathrm{ar}(T) = 108{,}\overline{3}$.

107 1) $\dfrac{2}{3}$; $\dfrac{2}{9}$; $\dfrac{3^k}{3^k+1}$; $k > 4$. 2) $Z \sim \text{Unif}(0,1)$. 3) $Z_n \xrightarrow[\text{m.r.}]{\text{q.c.}} 0$.

108 1) $p[2(1-p)^2-(1-p)^4]+1-p$. 2) $\frac{1}{6}n(n+1)(2n+1)$; $\frac{\mathrm{O}(n^{4/3})}{\mathrm{O}(n^{3/2})} \longrightarrow 0$; Liapunov. 3) $X+Y \sim \text{Gamma}(\lambda=1, \nu=4)$.

109 1) $\left\{h,\ \frac{2(n-h)}{n(n-1)};\ h=1,2,\ldots,n-1\right\}$. 2) $f_Y(y)=2y,\ 0<y<1$. 3) $Z_n \xrightarrow{\text{q.c.}} 1$.

110 1) 0,9; 0,1. 2) $X \sim \text{Poisson}(\mu p)$; $Y \sim \text{Poisson}(\mu q)$; X e Y indipendenti. 3) $\exp\{2n[\cos(u/\sqrt{n})-1]\}$; $\exp\{2n(\mathrm{e}^{iu/n}-1)\}$; $U_n \xrightarrow{\text{d}} \mathcal{N}(0,2)$; $V_n \xrightarrow{\text{p}} 2$; $Z_n \xrightarrow{\text{d}} \mathcal{N}(0,1)$.

111 1) $\frac{r}{r+n}$; $\frac{r(r-1)}{(r+n)(r+n-1)}$; $\frac{r-2}{n+r-2}\cdot$ 2) $f_Z(z)=(1+z)^{-2},\ z>0$. 3) $Z_n \xrightarrow{\text{q.c.}} Z \equiv \frac{X}{X+1}$, con $F_Z(z)=2-1/z,\ 1/2\le z\le 1$.

112 1) $\frac{2^{n-k}}{2^n-1}\cdot$ 2) $f(x,y)=\frac{1}{\pi ab},\ (x,y)\in\mathcal{E}$; $f_X(x)=\frac{2}{\pi a}\sqrt{1-\frac{x^2}{a^2}},\ |x|<a$; $f_Y(y)=\frac{2}{\pi b}\sqrt{1-\frac{y^2}{b^2}},\ |y|<b$; dipend.; $\frac{a^2}{4}\cdot$ 3) $\sum_{n=1}^{+\infty}\frac{\mathbb{V}\text{ar}(X_n)}{n^2}\le\sum_{n=1}^{+\infty}\frac{1}{n^2}$; $p_n=1$.

113 1) $\frac{256}{525}\cdot$ 2) $p_1=\frac{b}{b+n}$; $p_2=\frac{n}{b+n}\ \frac{b}{b+n-1}$; $p_3=\frac{n}{b+n}\ \frac{n-1}{b+n-1}$; $P(\text{vince A})=\frac{p_1}{1-p_3}$; $P(\text{vince B})=\frac{p_2}{1-p_3}$; equo per $b=1$.

114 1) $P(C_k)=p_r q_r^{k-1},\ k=1,2,\ldots$ (cfr. Geom(p_r));
$P(T_n)=p_c p_r(1-p_c p_r)^{n-1},\ n=1,2,\ldots$ (cfr. Geom$(p_c p_r)$);
$P(T_n|C_k)=\binom{n-1}{k-1}p_c^k q_c^{n-k},\ k=1,2,\ldots;\ n=k,k+1,\ldots$ (cfr. BinomNeg(k,p_c)).
2) $Y \sim \text{Espon}(p\lambda)$. 3) $Y_n \xrightarrow[\text{m.r.}]{\text{q.c.}} 1$; $Z_n \xrightarrow{\text{d}} \mathcal{N}(0,1)$.

115 1) $Z_\lambda \xrightarrow[\lambda\to 0]{\text{p}} 1$; $Z_\lambda \xrightarrow[\lambda\to 1]{\text{d}} \text{Unif}(0,1)$; $Z_\lambda \xrightarrow[\lambda\to+\infty]{\text{p}} 0$.
2) $Z_n \xrightarrow{\text{q.c.}} Z \equiv \frac{X}{X+Y}$, con $F_Z(z)=\begin{cases}\frac{1}{2}\ \frac{z}{1-z}, & 0<z\le 1/2\\ 1-\frac{1}{2}\ \frac{1-z}{z}, & 1/2<z\le 1\end{cases}$

116 1) q^k; $1+n-kq^k$; $n/2$. 2) $\forall y>0,\ X|y \sim \text{Espon}(y)$; $\forall x>0,\ Y|x \sim \text{Gamma}(\lambda=1+x, \nu=2)$; $f_X(x)=(1+x)^{-2},\ x>0$; $Y \sim \text{Espon}(1)$; $\mathbb{E}(X|y)=1/y$; $\mathbb{E}(Y|x)=\frac{2}{1+x}$; $\mathbb{E}X=+\infty$; $\mathbb{E}Y=1$; la $\mathbb{C}\text{ov}(X,Y)$ non è definita; X e Y dipendenti.
3) $Y_n \xrightarrow{\text{d}} \mathcal{N}(0,1)$.

117 1) La procedura a) (se $p_1=p_2$, le due procedure sono equivalenti). 2) $F_Y(y)=\frac{1-\mathrm{e}^{-y}}{1-q\,\mathrm{e}^{-y}},\ y>0$. 3) $Z_\lambda \xrightarrow[\lambda\to 0]{\text{p}} 1$; $Z_\lambda \xrightarrow[\lambda\to 1]{\text{d}} Z$, con $F_Z(z)=1+\frac{1-z}{z}(\mathrm{e}^{-\frac{z}{1-z}}-1),\ 0<z\le 1$; $Z_\lambda \xrightarrow[\lambda\to+\infty]{\text{p}} 0$.

118 1) $\frac{1}{9}$; $\frac{5}{18}$; $\frac{7}{12}$; $\frac{2}{3}$. 2) $\frac{\nu-1}{\lambda}$. 3) $F_{Y_p}(y)=\frac{py}{1-qy}$, $0<y\le 1$; $Y_p \xrightarrow[p\to 0^+]{\mathrm{p}} 1$; $Y_p \xrightarrow[p\to 1^-]{\mathrm{p}} X_1 \sim \mathrm{Unif}(0,1)$.

119 1) Poisson($\lambda p_1 p_2$). 2) $F_Z(z)=\begin{cases}\frac{(2+z)^2}{6}, & -2<z\le 0\\ 1-\frac{(1-z)^2}{3}, & 0<z\le 1\end{cases}$ 3) $Y_n \xrightarrow{\mathrm{d}} \mathcal{N}(0,1/3)$.

120 1) $\frac{6}{n(n+1)(2n+1)}$; $\frac{1}{2}-\frac{3}{(n+1)(2n+1)}$. 2) $\frac{3-2\sqrt{2}}{3}\left[-3+6\sqrt{2}-8\sqrt{3-2\sqrt{2}}\right]\simeq$ 0,12. 3) 0; 1; $V_n \xrightarrow{\mathrm{q.c.}} 0$; $Z_n \xrightarrow{\mathrm{d}} \mathcal{N}(0,1)$.

121 1) $\binom{6}{2}9^4\Big/10^6 \simeq 0{,}098$; $\binom{6}{2}\binom{4}{2}8^2\Big/10^6 \simeq 0{,}006$; $\binom{6}{2}\binom{4}{2}\Big/10^6 = 9\cdot 10^{-5}$; $10\binom{6}{2}9^4\Big/10^6 - \binom{10}{2}\binom{6}{2}\binom{4}{2}8^2\Big/10^6 + \binom{10}{3}\binom{6}{2}\binom{4}{2}\Big/10^6 \simeq 0{,}736$. 2) $f_Z(z)=\frac{1}{\sqrt{z}}-1$, $z\in(0,1)$. 3) $F_Y(y)=\frac{1-\mathrm{e}^{-y}}{1-q\,\mathrm{e}^{-y}}$, $y>0$; $Y \xrightarrow[p\to 0^+]{\mathrm{p}} 0$; $Y \xrightarrow[p\to 1^-]{\mathrm{d}}$ Espon(1).

122 1) $\frac{27}{41}$; $\frac{3}{59}$. 2) 4. 3) $Y_n \sim \mathcal{N}(0, 1+(1+1/n)^2)$; $Y_n \xrightarrow{\mathrm{d}} \mathcal{N}(0,2)$.

123 1) Il più forte. 2) $\frac{1}{\mathrm{e}-1}$; $F_Z(z)=\frac{1-\mathrm{e}^{-z}}{1-\mathrm{e}^{-1}}$, $0<z\le 1$. 3) $Z_n \xrightarrow{\mathrm{p}} 1$.

124 1) $P(Y=n)=\frac{1}{6}\sum_{k=1}^{6}\left(\frac{1}{2}\right)^k\left[1-\left(\frac{1}{2}\right)^k\right]^{n-1}$, $n=1,2,\ldots$ 2) $F_{X^R}(y)=\begin{cases}(y+1)/4, & -1<y\le 0\\ (y+2\sqrt{y}+1)/4, & 0<y\le 1\end{cases}$ 3) $H_{X_k}(t)=\left(\frac{1}{2}+\frac{\mathrm{e}^{it}}{2}\right)^2=H_{\mathrm{Binom}(2,1/2)}(t)$, $k=1,2,\ldots,n$; $Y_n\sim \mathrm{Binom}(2n,1/2)$; $a_n=n$; $b_n=\sqrt{n/2}$.

125 1)

i	0	1	2
p_i	$(1-p)^2$	$2p(1-p)^2$	$p^2(3-2p)$

; $\mathbb{E}I=2p(1+p-p^2)$. 2) $F_{Z_n}(z)= 1-(1-z^2)^n$, $0<z\le 1$; $\mathbb{E}Z_n=\frac{2^{2n}(n!)^2}{(2n+1)!}=\frac{1}{2}\frac{\Gamma(1/2)\Gamma(n+1)}{\Gamma(n+1+1/2)}$. 3*a*) $Z_n\sim \mathrm{Espon}(n\lambda_n)$; 3*bi*) $Z_n \xrightarrow{\mathrm{d}}$ Espon(1); 3*bii*) $Z_n \not\rightarrow$; 3*biii*) $Z_n \xrightarrow{\mathrm{p}} 0$.

126 a1) $\binom{6}{2}9^4\Big/10^6 \simeq 0{,}098$; $\binom{6}{2}\binom{4}{2}8^2\Big/10^6 \simeq 0{,}006$; $\binom{6}{2}\binom{4}{2}\Big/10^6 = 9\cdot 10^{-5}$; $10\binom{6}{2}9^4\Big/10^6 - \binom{10}{2}\binom{6}{2}\binom{4}{2}8^2\Big/10^6 + \binom{10}{3}\binom{6}{2}\binom{4}{2}\Big/10^6 \simeq 0{,}736$.

a2)

x	0	1	2
p_x	$(1-p)^2$	$2p(1-p)(1-p_1)$	$p^2+2pp_1(1-p)$

b1) $\binom{6}{2}5^4\Big/6^6$; $\binom{6}{2}\binom{4}{2}4^2\Big/6^6$; $\binom{6}{2}\binom{4}{2}\Big/6^6$;

$6\binom{6}{2}5^4\Big/6^6 - \binom{6}{2}^2\binom{4}{2}4^2\Big/6^6 + \binom{6}{3}\binom{6}{2}\binom{4}{2}\Big/6^6.$

b2) $P(Y=0)=(1-\alpha)^2+2\alpha(1-\alpha)(1-\alpha_1)$; $P(Y=1)=2\alpha\alpha_1(1-\alpha)$; $P(Y=2)=\alpha^2$.

c1) $(p+q/2)^2$; $2(p+q/2)(r+q/2)$; $(r+q/2)^2$.

c2) $\binom{Np}{2}\Big/\binom{N}{2} + \frac{1}{2}NpNq\Big/\binom{N}{2} + \frac{1}{4}\binom{Nq}{2}\Big/\binom{N}{2}$;

$\frac{1}{2}NpNq\Big/\binom{N}{2} + \frac{1}{2}NpNr\Big/\binom{N}{2} + 2\frac{1}{4}\binom{Nq}{2}\Big/\binom{N}{2}$;

$\frac{1}{4}\binom{Nq}{2}\Big/\binom{N}{2} + \frac{1}{2}NqNr\Big/\binom{N}{2} + \binom{Nr}{2}\Big/\binom{N}{2}$.

127 1) Binom$(N,1/2^n)$; $N/2^n$. 2) $F_{(Z,U)}(z,u)=\dfrac{u}{u+1}[1-(1+\vartheta z)\mathrm{e}^{-\vartheta z}]$, z e $u>0$.
3) Poisson(n); n; $1-13\,\mathrm{e}^{-3}$; $1/2$.

128 1) $Y_n \xrightarrow{\text{d}}$ Poisson(1). 2) $Y_n \xrightarrow{\text{q.c.}} X$.

129 1) $X_1 \sim$ Binom$(n,1/s)$; $X_2|X_1=x_1 \sim$ Binom$(n-x_1,1/(s-1))$; $(X_1,X_2)\sim$ Trinom$(n,1/s,1/s)$. 2) $\frac{1}{2}$. 3) $\mathbb{C}\mathrm{ov}(X_n,Y_n)=a_n-3a_n^2-3a_nb_n \longrightarrow \begin{cases}0, & \text{caso 1}\\ -1/8, & \text{caso 2}\end{cases}$;

$F_{(X_n,Y_n)}(x,y)=\begin{cases}0, & x \text{ e/o } y\le 0\\ a_n, & x \text{ e } y\in(0,1]\\ 3a_n, & x>1 \text{ e } y\in(0,1]\\ a_n+b_n, & x\in(0,1] \text{ ee } y>1\\ 1, & x \text{ e } y>1\end{cases}$; $(X_n,Y_n)\begin{cases}\xrightarrow{\text{p}}(1,1), & \text{caso 1}\\ \xrightarrow{\text{d}}(X,Y)^*, & \text{caso 2}\end{cases}$;
$*(X,Y)$assume i valori (0,0), (0,1), (1,0) con prob. 1/4, 1/4, 2/4.

$P(Z_n=0)=a_n$; $P(Z_n=1)=2a_n+b_n$; $P(Z_n=2)=1-3a_n-b_n$;

$Z_n\begin{cases}\xrightarrow{\text{p}} 2, & \text{caso 1}\\ \xrightarrow{\text{d}} \text{Binom}(1,3/4), & \text{caso 2}\end{cases}$

130 1) $P_n=\frac{1}{2}+\frac{1}{2}(p-q)^{n-1}$. 2) $Y_n \sim$ Binom$(1,2pq)$; (Y_n,Y_{n+1}) assume i valori $(0,0),(0,1),(1,0),(1,1)$ con probabilità $1-3pq$, pq, pq, pq; Y_ne Y_{n+1} indipendenti se e solo se $p=\frac{1}{2}$; $\mathbb{E}Z_n=2npq$; $\mathbb{V}\mathrm{ar}(Z_n)=2npq(1-2pq)+2(n-1)pq(1-4pq)$.
3) $Y_n \xrightarrow{\text{q.c.}} -1$; $Z_n \not\longrightarrow$ (in realtà Z_n diverge q.c. a $-\infty$).

131 1) $\frac{5}{12}$; $\frac{5}{12}$; $\frac{55}{216}$. 2) $\frac{1}{2}$; $-\frac{1}{6}+\frac{5}{2}$. 3) $F_{Z_n}(z)=\begin{cases}\frac{1}{2}z^{1/n}, & 0<z\le 1\\ \frac{1}{2}z^n, & 1<z\le 2^{1/n}\end{cases}$; $Z_n \xrightarrow[\text{m.r.}]{\text{q.c.}}$ $[X]\sim$ Unif$\{0,1\}$.

132 1) 0,36; 0,06; 0,7; 18/41. 2) q^n; pq^{n-1}; Geom(p). 3) $F_{Y_n}(y)=1+\mathrm{e}^{-\lambda n}-\mathrm{e}^{-\lambda y}$, $0<y\le n$; $Y_n \xrightarrow{\text{q.c.}} X$.

133 1) Geom$(2pq)$; $\frac{1}{2}$; $\frac{1}{2pq}$; $\frac{p^2}{p^2+q^2}$. 2) $F_Z(z)=0$, $z\le 0$; $F_Z(z)=\sum_{r=0}^{k-1}\mathrm{e}^{-\lambda}\frac{\lambda^r}{r!}+$

$(z-k)\mathrm{e}^{-\lambda}\dfrac{\lambda^k}{k!},\ z\in(k,k+1],\ k=0,1,2,\ldots;\quad F_W(w)=(w+k)\mathrm{e}^{-\lambda}\dfrac{\lambda^k}{k!}+\sum_{r=k+1}^{+\infty}\mathrm{e}^{-\lambda}\dfrac{\lambda^r}{r!},$
$w\in(-k,-k+1],\ k=0,1,2,\ldots;\ F_W(w)=1,\ w>1.$ 3) $U_n\xrightarrow{\text{q.c.}}X_0/3\sim\mathcal{N}(0,1/3);$
$V_n\xrightarrow{\text{q.c.}}X_0/\sqrt{\mu_{2k}}\sim\mathcal{N}(0,1/\mu_{2k})$ con $\mu_{2k}=\mathbb{E}X_j^{2k}=\dfrac{(2k)!}{2^k k!};$
$W_n\xrightarrow{\text{q.c.}}X_0^2/3\sim\text{Gamma}(\lambda=3/2,\nu=1/2).$

134 1) $k+X\sim\text{BinomNeg}(k,p)$, e $P(X=x)=\binom{k+x-1}{k-1}p^kq^x,\ x=0,1,\ldots;\ \mathbb{E}X=kq/p,\ \mathbb{V}\text{ar}(X)=kq/p^2;\ X\xrightarrow[k\to+\infty]{\text{d}}\text{Poisson}(\lambda).$
2) $U\sim\text{Espon}(2\lambda);\ V\sim\text{Espon}(\lambda);\ \forall(u,v)\mathbb{R}^2,\ F_{(U,V)}(u,v)=F_U(u)F_V(v)\iff U$ e V indipendenti.
3) $\forall\alpha>0,\ F_{Z_n}(z)\longrightarrow F(z)=\begin{cases}0, & z\le 0\\ \exp\{-z^{-\alpha}\}, & z>0\end{cases}$

135 1) $\dfrac{n!}{h!j!k!}p_1^hp_2^j(1-p_1-p_2)^k$, con$p_1=1-\mathrm{e}^{-\lambda a}$ e $p_2=\mathrm{e}^{-\lambda a}-\mathrm{e}^{-\lambda b}$. Ovvero, posto $R=$"numero di lampadine di durata $<$ a ed $S=$"numero di lampadine di durata $\in(a,b)$: $(R,S)\sim\text{Trinom}(n,p_1,p_2)$.
2) $F_Y(y)=\begin{cases}py+\frac12qy^2, & 0\le y\le 1\\ 1-\frac12q(2-y)^2, & 1\le y\le 2\end{cases}$ 3) $U_n\begin{cases}\text{non converge}, & \text{per }\delta<1\\ \xrightarrow{\text{d}}\text{Cauchy}, & \text{per }\delta=1\\ \xrightarrow{\text{p}}0, & \text{per }\delta>1\end{cases}$

136 1)$\frac12(\lambda+\mu),\ \frac12(\lambda^2+\mu^2),\ \dfrac{\lambda^2+\mu^2}{\lambda+\mu},\ \dfrac{\lambda^n}{\lambda^n+\mu^n}\cdot$ 2) X e Y i.i.d. $\sim\text{Geom}(1/2);\ F_Z=F_X^2=F_Y^2$, ovv. $P(Z=n)=\dfrac{1}{2^n}\left(2-\dfrac{3}{2^n}\right),\ n\in\mathbb{N};\ \dfrac23;\ \dfrac12;\ \dfrac34\cdot$ 3) $\mathbb{E}X_n=\dfrac{\lambda}{\lambda-1},\ \mathbb{V}\text{ar}(X_n)=\dfrac{\lambda}{\lambda-2}-\left(\dfrac{\lambda}{\lambda-1}\right)^2$ per $\lambda>2$, e $\mathbb{V}\text{ar}(X_n)=+\infty$ per $1<\lambda\le 2$; $Y_n\sim\text{Espon}(\lambda);\ Z_n\xrightarrow{\text{q.c.}}\mathrm{e}^{1/\lambda}$.

137 1) $\dfrac{7}{17},\ \dfrac37;\ \dfrac{5}{18}\cdot$ 2) $f_R(r)=r\,\mathrm{e}^{-r^2/2},\ r>0;\ f_{(X,R)}(x,r)=\dfrac{r}{\pi\sqrt{r^2-x^2}}\mathrm{e}^{-r^2/2},\ |x|<r;\ f(x|r)=\dfrac{1}{\pi\sqrt{r^2-x^2}},\ |x|<r.$ 3) $Y_n\xrightarrow{\text{d}}\mathcal{N}(0,1);\ P(Y_n=\dfrac{2k-n}{\sqrt n})=\binom{n}{k}\dfrac{1}{2^n},\ k=0,1,\ldots,n.$

138 1) $\dfrac{27}{143};\ \dfrac59\cdot$ 2) $\dfrac{2}{\log 4-1};\ f_X(t)=\dfrac{\log(1+t)}{\log 4-1},\ 0<t>1;\ f_Y(t)=\dfrac{2}{\log 4+1}t\dfrac{1-t^2}{1+t^2},\ 0<t<1$; dipendenti. 3) $X_n\xrightarrow{\text{p}}0;\ Y_n\xrightarrow{\text{d}}\text{Unif}(0,2).$

139 1) 1/18. 2) 2; 21/25.

140 1) $Y_n\xrightarrow{\text{q.c.}}1$. 2) Z_α non converge (diverge a $-\infty$) per $\alpha\to 0$; $Z_\alpha\xrightarrow{\text{p}}X$ per $\alpha\to+\infty$.

141 1) a) 6 oppure 10; b) 8. 2) $\dfrac{X}{X+Y}|X+Y<1\sim\text{Unif}(0,1)$. 3) a) $H_{U_n}(u)=\left\{x\,\mathrm{e}^{iu(1-x)/\sqrt n}+(1-x)\,\mathrm{e}^{iux/\sqrt n}\right\}^n$; b) $\mathcal{N}(0,x(1-x))$.

142 1) a) $\frac{4}{15}$; b) $\frac{9}{11}$; c) $\frac{9}{13}$; d) $P(V=2)=\left(\frac{4}{15}\right)^3$; $P(V=1)=\frac{11}{5}\left(\frac{4}{15}\right)^3$; $P(V=0)=1-\frac{16}{5}\left(\frac{4}{15}\right)^3$. 2) U vale 0 o 1 con prob. 3/4, 1/4; V vale 0 o 1 con prob. 1/2, 1/2; $U+V$ vale 0, 1 o 2 con prob. 5/12, 5/12, 1/6. 3) $Z_n \xrightarrow[\text{m.r.}]{\text{q.c.}} 0$.

143 1) 7/96. 2) $X \sim \text{Gamma}(\nu=3, \lambda=1)$; $f_Y(y)=3\alpha\,\mathrm{e}^{-\alpha y}-6\alpha\,\mathrm{e}^{-2\alpha y}+3\alpha\,\mathrm{e}^{-3\alpha y}$, $y>0$; $\alpha=11/18$; $\mathbb{V}\text{ar}(X)=3<441/121=\mathbb{V}\text{ar}(Y)$. 3) $Z_n \xrightarrow{\text{q.c.}} I_{(X>Y)} \sim \text{Unif}\{0,1\}$.

144 1) a) $\frac{(n-3)(n-4)}{(n-1)(n-2)}$; b) $\frac{2}{n-3}$. 2) *i*) $X \sim \text{Espon}(1)$; $f_Y(y)=1-(1+1/y)\,\mathrm{e}^{-1/y}$, $y>0$; *ii*) $XY \sim \text{Unif}(0,1)$. 3) a) $Z_n \xrightarrow[\text{m.r.}]{\text{q.c.}} 1$; b) $f_{Z_n}(z)=\frac{(2n-1)!}{[(n-1)!]^2}\,\frac{z^{n-1}}{(z+1)^{2n}}$, $z>0$.

145 1) M_2 (le prob. sono proporz. a 3˙087, 3˙125, 1˙323). 2) $f_{X+Y}(z)=1/3$, $-1<z\leq 1$ e $f_{X+Y}(z)=1/6$, $1<z\leq 3$. 3) a) $2+n$ e 4; b) $\mathcal{N}(0,4)$.

146 1) 1/4. 2) $F_Z(z)=(3z-1)^2$, $1/3<z\leq 1/2$ e $F_Z(z)=1-3(1-z)^2$, $1/2<z\leq 1$. 3) $Z_\lambda \sim \text{EsponSimm}(\lambda/\sin\lambda)$ e $Z_\lambda \xrightarrow[\lambda\to 0]{\text{d}} \text{EsponSimm}(1)$.

147 1) a) 0,65; b) $\sigma[1+\cdots+(\delta-\sigma)^{n-2}]+(\delta-\sigma)^{n-1}/2$, dove $\delta=P(D_n|D_{n-1})$ e $\sigma=P(D_n|S_{n-1})$; c) 7/11. 2) $\text{Unif}(0,1)$; $\text{Gamma}(2,\lambda)$; $f_W(w)=2(1-w)$, $w\in(0,1)$. 3) $Z_n=\frac{X_1+\cdots+X_n}{\sigma\sqrt{n}}\Big/\sqrt{\frac{(Y_1/\sigma)^2+\cdots+(Y_n/\sigma)^2}{n}}=U_n/\sqrt{V_n}$; U_n, V_n indip.; $U_n \xrightarrow{\text{d}} U\sim\mathcal{N}(0,1)$, $V_n \xrightarrow{\text{q.c.}} 1$; $Z_n=U_n/\sqrt{V_n} \xrightarrow{\text{d}} U/\sqrt{1}\sim\mathcal{N}(0,1)$.

148 1) 3/8; 19/40; 9/19. 2) a) λ^2; b) $\text{Espon}(\lambda)$, $\text{Gamma}(2,\lambda)$; c) $\lambda\mathrm{e}^{-\lambda(y-x)}$, $0<x<y$; d) $X+1/\lambda$. 3) Espon(1).

149 1) a) $\frac{1}{6}\left(\frac{5}{6}\right)^{x-2}$, $x=2,3,\ldots$; b) $\frac{6!(x-1)}{(7-x)!6^x}$, $x=2,3,\ldots,7$. 2) $f_Z(z)=|4z-2|$, $0<z<1$. 3) $Y_n \xrightarrow[\text{m.r.}]{\text{p}} 0$.

150 1) $\binom{n}{2}\Big/2^n$; $(n-1)/2^n$; $(\frac{1}{2}n-1)(n-1)/2^n$. 2) $\text{Unif}(0,\pi)$. 3) $F_{Z_n}(z)\to 1-\frac{1-\mathrm{e}^{-z}}{z}$, $z>0$.

151 1) a) 1/2; b) 0, .1, .2, .3, .4; c) 1/4. 2) $F_Z(z)=\begin{cases}0 & z\leq 0\\ 1-\exp\{-\mu\left[1-\mathrm{e}^{-\frac{\lambda z}{1-z}}\right]\} & 0<z<1\\ 1-\mathrm{e}^{-\mu} & z=1\\ 1 & z>1\end{cases}$

3) $\mathcal{N}\left(0,\frac{p(1-p)}{2}\right)$.

152 1) 2/3; 4/5. 2) $F_U(u)=u(1-\log u)$, $0<u\le 1$; $F_V(v)=\sqrt{v}$, $0\le v\le 1$; $f_{(U,V)}(u,v)=-\frac{1}{2}\frac{\log u}{\sqrt{v}}$, u e $v\in(0,1)$. 3) $\mathcal{N}(0,1)$.

153 1) 5/6; 1/5. 2) $f_{(X,Y)}(x,y) = \frac{1}{2}$, $(x,y)\in S$; $Y \sim$ Triang$(-1,1)$; $f(x|y) = \frac{1}{2(1-|y|)}$, $y\in(-1,1)$, $x\in(|y|-1,1-|y|)$. 3) $X_n \xrightarrow[\not\mathrm{p}]{\mathrm{d}} \mathcal{N}(0,1)$.

154 1) $70/16^2$; 1/16. 2) Dipendenti; $X \sim$ Gamma$(\nu=\alpha,\lambda=1)$; $Y \sim$ Gamma$(\nu=\alpha+\beta,\lambda=1)$. 3) $X_n \xrightarrow[\text{q.c., m.r.}]{\text{d, p}} 0$.

155 1/90; p^5+5p^5q; $\frac{5q}{1+5q}$.

156 1) 3/8, 3/8, 2/8. 2) 1/24.

157 a) $F_Y(y) = \begin{cases} 0, & y\le 0 \\ \frac{2+y}{4}, & y\in(0,2] \\ 1, & y>2 \end{cases}$ b) No, è discontinua in 0. c) $\frac{1}{2}$.

158 b) $\mathcal{N}(0,2)$.

159 1) $Z \sim$ Triang$(0,2)$. 2) $Y_n \xrightarrow{\mathrm{d}}$ Espon(1).

160 1) $R \sim$ Binom$(n,p/2)$. 2) a) 60; b) no; c) $f_{X+Y}(z)=5z^5$, $z\in(0,1)$. 3) 1; χ^2_1; χ^2_1.

161 1) P(vince) è 7/12 se ripete e 8/12 se non ripete. 2) $f_{(U,V)}(u,v)=\lambda^2 e^{-\lambda v}$, $0<u<v$; $U \sim$ Espon(λ); $V \sim$ Gamma$(\nu=2,\lambda)$. 3) $Y_n \xrightarrow{\text{q.c.}} 0$.

162 1) No. 2) $F_Y(y)=\begin{cases} 0, & y\le 0 \\ 1/4, & 0<y\le 1 \\ y^2/4, & 1<y\le 2 \\ 1, & y>2 \end{cases}$; $\mathbb{E}Y=7/6$. 3) Sì; sì; no.

163 1) Poisson$(\mu/6)$. 2) $c=2$. 3) $U_n \xrightarrow[\text{q.c., m.r.}]{\text{d, p}} \vartheta$.

164 1) Per n pari, $p_d = \frac{2}{n-1}$, $d=1,2,\ldots,\frac{n}{2}-1$ e $p_{n/2}=\frac{1}{n-1}$; per n dispari, $p_d = \frac{2}{n-1}$, $d=1,2,\ldots,\frac{n-1}{2}$. 2) $f_Z(z)=\frac{1}{2(1+|z|)^2}$. 3) $Z_p \xrightarrow{\mathrm{p}} 0$, per $p\to 1$; Z_p non converge per $p\to 0$.

165 1) a) .05, .25, .90; b) .85; c) 4/85. 2) *a*) 2/3; b) $\forall x\in(-1,1)$, $Y|X=x \sim$ Unif$(0,1-|x|)$. 3) $Y_n \xrightarrow{\mathrm{d}} X_i$.

166 1) $P(X=4000)=5/18,\ P(X=5000)=10/18,\ P(X=6000)=3/18$. 2) $f_Z(z)=\dfrac{2}{(1+z)^2},\ z\in(0,1)$. 3) $Z_n \xrightarrow{\text{p}} 1$.

167 1) a) $P(H=0|B_2)=\dfrac{N(1-p)^2}{N+2},\ P(H=1|B_2)=\dfrac{2(1-p)(1+Np)}{N+2},\ P(H=2|B_2)=\dfrac{p(2+Np)}{N+2}$; b) $\mathbb{E}(H|B_2)=\dfrac{2[1+p(N+1)]}{N+2}$; c) $\lim_{N\to\infty}P(H=0|B_2)=(1-p)^2,\ \lim_{N\to\infty}P(H=1|B_2)=2p(1-p),\ \lim_{N\to\infty}P(H=2|B_2)=p^2$. 2) a) $k=1/(2\pi)$; c) no. 3) a) $H_{Y_n}(u)=\left[\dfrac{\mathrm{e}^{-iu/\sqrt{n}}}{1-iu/\sqrt{n}}\right]^n$; b) $\log H_{Y_n}(u)\to-\dfrac{1}{2}u^2$; c) $\mathcal{N}(0,1)$.

168 1) No; 111. 2) $Z_n \xrightarrow{\text{d}} \mathcal{N}(0,1)$; $V_n \xrightarrow{\text{q.c.}} 1$. 3) $X\sim \text{BinomNeg}\left(n=a, p=\dfrac{b}{b+1}\right)$.

169 1) $F_Z(z)=\begin{cases}0, & z\le 0\\ z(1-\mathrm{e}^{-\lambda z}), & 0<z\le 1\\ 1-\mathrm{e}^{-\lambda z}, & z>1\end{cases}$ 2) $P(Z_n=z)=\begin{cases}1/(2n), & z=0\\ 1/n, & 1\le z\le n-1\\ 1/(2n), & z=n\end{cases}$;

$Z_n/n \xrightarrow{\text{d}} \text{Unif}(0,1)$.

170 1) a) $p^2(2-p^2)$; b) $3p^2-3p^4+p^6$; c) $4pq^3+2p^2q^2$. 2) c) $F_{nX_n^r}(y)=1-\exp\{-\lambda_n(y/n)^{1/r}\}$, $y>0$;

$$nX_n^r\begin{cases}\stackrel{\text{d}}{\not\longrightarrow}, & \alpha<1/r\\ \xrightarrow{\text{d}} Y_1,\quad \text{con } F_{Y_1}(y)=1-\exp\{-y^{1/r}\},\ y>0, & \alpha=1/r\\ \xrightarrow{\text{p}} 0, & \alpha>1/r\end{cases}$$

3) $$F_Z(z)=\begin{cases}0, & z\le 0\\ z\mathrm{e}^{2-1/z}, & 0<z\le 1/2\\ 1-\mathrm{e}^{\frac{1-2z}{1-z}}(1-z), & 1/2<z<1\\ 1, & z\ge 1\end{cases}$$

171 1)

t	2	3	4	6
p_t	1/8	2/8	4/8	1/8

2) $F_{Y_n}(y)=\dfrac{ny}{n-y+1},\ 0\le y\le 1$; $Y_n \xrightarrow{\text{d}} \text{Unif}(0,1)$.

172 1) 5/9; 1/15; 16/45. 2) 7/8; 3/4; 1; 3/4. 3) $Y_n\sim\text{Espon}(n^{\beta-\alpha})$;

$$Y_n\begin{cases}\xrightarrow{\text{p}} 0, & \alpha<\beta\\ \xrightarrow{\text{d}} \text{Espon}(1), & \alpha=\beta\\ \stackrel{\text{d}}{\not\longrightarrow}, & \alpha>\beta\end{cases}$$

173 91/216; $2pq$; $4p^4q^2$; $\dfrac{p^2}{p^2+q^2}$

174 1) $\dfrac{n}{2^{n+1}}\to 0$; $\dfrac{k-1}{4(2k-1)}\to\dfrac{1}{8}$ 2)

t	0	10	20	30	40
p_t	4/16	4/16	4/16	2/16	1/16

175 $H_Z(u) = \dfrac{1}{1+u^2}$, $f_Z(z) = \dfrac{e^{-|z|}}{2}$; 0, 2; χ^2_1.

176 1) $F_Z(z) = \begin{cases} \dfrac{1}{2(1-z)}, & z \le 0 \\ 1/2, & 0 < z \le 1 \\ 1 - \dfrac{1}{2z}, & z > 1 \end{cases}$ 2) $Y_\lambda \begin{cases} \xrightarrow[\lambda\to 0]{\text{p}} 1 \\ \overset{\text{d}}{\not\longrightarrow}_{\lambda\to\infty} \end{cases}$

177 1) $3p^3q^4 + 3p^5q^2$; $3p^4q^3 + 3p^2q^5$; p. 2) $F_Z(z) = \begin{cases} 0, & z \le 0 \\ z/2, & z \in (0,1] \\ 1, & z > 1 \end{cases}$; no. 3) $V_n \sim \text{Poisson}(1-1/2^n) \xrightarrow{\text{d}} \text{Poisson}(1)$.

178 1) $\dfrac{8\cdot 28}{\binom{32}{5}}$; $\dfrac{8\cdot 28\cdot 6\cdot 23}{\binom{32}{5}\binom{27}{5}}$. 2) $F_A(a) = \begin{cases} 0, & a \le 0 \\ 1-\exp\{-2a\}, & 0 < a \le 1/2 \\ 1-\exp\{-\dfrac{1}{2(1-a)}\}, & 1/2 < a < 1 \\ 1, & a \ge 1 \end{cases}$ 3) $F_{Z_n}(z) = \begin{cases} 0, & z \le 0 \\ \dfrac{1}{1+n^2/z}, & z > 0 \end{cases}$, $Z_n \overset{\text{d}}{\not\to}$.

179 1) $\dfrac{k_1p_1 + k_2p_2}{k_1+k_2}$, $\dfrac{k_1p_1(1-p_1) + k_2p_2(1-p_2)}{(k_1+k_2)^2}$. 2) $f_X(x) = f_Y(x) = \dfrac{2}{\pi}\sqrt{1-x^2}$, $x \in (-1,1)$; no; $\dfrac{3}{8}$; sì. 3) $Y_n \xrightarrow{\text{q.c.}} e^{-1}$.

180 1)

v	-7	-2	0	1	2	3
p_v	1/8	1/8	1/8	2/8	2/8	1/8

; Unif$\{-3,2\}$. 2) $f_Z(z) = \dfrac{2}{(1+z)^2}$, $z > 1$. 3) $Y_n \xrightarrow[\text{m.r.}]{\text{q.c.}} 0$, $Z_n \xrightarrow[\text{m.r.}]{\text{q.c.}} 1$, $U_n \xrightarrow[\text{m.r.?}]{\text{q.c.}} 1$.

181 1) 4/7; 11/21. 2) a) 3/10; b) 1/2; 17/80; 23/80; c) $F_X(x) = \begin{cases} 0, & x \le -1 \\ \dfrac{1}{2} + \dfrac{3}{5}x - \dfrac{1}{10}x^3, & -1 < x \le 1 \\ 1, & x > 1 \end{cases}$

182 1) a) $\dfrac{1}{30}$; b) e c) $p(1+q)$ e $\dfrac{q}{1+q}$ se non ricorda l'eventuale errore, $p(1+30q/29)$ e $\dfrac{30q/29}{1+30q/29}$ se ricorda l'eventuale errore. 2) a) $X_R \sim \text{Binom}(10,p)$, $X_B \sim \text{Binom}(20,p)$; b) $p^{29}(20q+p)$; c) R.A.

183 1) $\dfrac{(1-p)(1-q_2)^2}{(1-p)(1-q_2)^2 + p(1-q_1)^2}$. 2) b) no; c) $\{0,1,2\}$, $[0,2]$; d) 0, 4/5, 4/5.

184 1) $f_{Y_n}(y) = \dfrac{(-\log y)^{n-1}}{(n-1)!}$, $0 < y < 1$. 2) a) $Y_n \xrightarrow{\text{d}} \mathcal{N}(0,1)$; b) $P(Y_n = y) =$

$\binom{n}{\sqrt{n}(y+\sqrt{n})/2}\frac{1}{2^n},\ y=\frac{-n}{\sqrt{n}},\frac{-n+2}{\sqrt{n}},\dots,\frac{n-2}{\sqrt{n}},\frac{n}{\sqrt{n}}.$

185 1) $f_{Y_1}(y)=\frac{y+1}{2},\ y\in(-1,1);\ f_{Y_2}(y)=-2y,\ y\in(-1,0);\ Y_3\sim \mathrm{Unif}(0,1)$. 2) $U_n \xrightarrow{\text{q.c.}} X_0/\sqrt{3}\sim\mathcal{N}(0,1/3)$.

186 1) $\dfrac{\binom{8}{1}\binom{4}{3}\binom{7}{1}\binom{4}{2}}{\binom{32}{5}};\ \dfrac{\binom{8}{1}\binom{4}{3}\binom{7}{1}\binom{4}{2}\cdot\binom{6}{1}\binom{4}{4}\binom{23}{1}}{\binom{32}{5}\cdot\binom{27}{5}}.$ 2) $X\sim \mathrm{Gamma}(\nu=2,\lambda=1);\ Y\sim \mathrm{Espon}(1);\ -Z\sim\mathrm{Espon}(1)$. 3) $Y_n\xrightarrow{\text{d}}\mathrm{Espon}(2)$.

187 1) $X_1\sim\mathrm{Bernoulli}(1/10)$; sì; no; $Y=\sum_{i=1}^{10}X_i$; 1. 2) a) $f_{(X,Y)}(x,y)=\frac{1}{2},\ (x,y)\in Q;\ f_X(x)=f_Y(x)=1-|x|,\ |x|<1$; b) no; c) $Z\sim\mathrm{Unif}(-1,1)$. 3) $F_{Z_n}(z)=\begin{cases}0, & z\le 1/\sqrt{n}\\ z^2, & 1/\sqrt{n}<z\le 1\\ 1, & z>1\end{cases}$; $Z_n\xrightarrow{\text{q.c.}}\sqrt{X}$, con $F_{\sqrt{X}}(z)=\begin{cases}0, & z\le 1\\ z^2, & 0<z\le 1\\ 1, & z>1\end{cases}$

188 1) $\{d,\ p_d=(n-1-d)/\binom{n}{2};\ d=0,1,\dots,n-2\}$.

2) $F_Y(y)=\begin{cases}e^{-\lambda(1+1/y)}-e^{-\lambda}, & y\le -1\\ 1-e^{-\lambda}, & -1<y\le 0\\ 1-e^{-\lambda}+e^{-\lambda(1+1/y)}, & y>0\end{cases}$ 3) $Y_n\xrightarrow{\text{q.c.}}Y\sim\mathrm{Binom}(1,e^{-\lambda})$.

189 1) 1/2. 2) 2; $F_Z(z)=\begin{cases}0, & z\le 0\\ z, & 0<z\le 1/2\\ 1-\frac{1}{4z}, & z>1/2\end{cases}$ 3) $F_{X_n}(x)=\begin{cases}0, & x\le 0\\ 1-1/n^2 & 0<x\le n^3\\ 1, & x>n^3\end{cases}$; $X_n \underset{\text{m/r.}}{\xrightarrow{\text{q.c.}}} 0$.

190 1) $P(B_1|N_2)=\frac{b}{b+n+c};\ F_X(x)=\begin{cases}0, & x\le b\\ \frac{n(n+c)}{(b+n)(b+n+c)}, & b<x\le b+c\\ \frac{n(n+c)+2bn}{(b+n)(b+n+c)}, & b+c<x\le b+2c\\ 1, & x>b+2c\end{cases}$ 2) $f_{(Y,Z)}(y,z)=f_Y(y)f_Z(z)=\begin{cases}\frac{1}{4\sqrt{z}}, & y \text{ e } z\in(0,1)\\ \frac{1}{4y^2\sqrt{z}}, & y>1 \text{ e } z\in(0,1)\end{cases}$ 3) $Z_n\xrightarrow{\text{d}}Z$ con $f_Z(z)=\frac{1}{(1+z)^2},\ z>0$.

191 1) $P(X=0)=q+pq^2,\ P(X=1)=P(X=2)=p^2q,\ P(X=3)=p^3$. 2) $X\sim\mathrm{Espon}(1);\ Y\sim\mathrm{Gamma}(\nu=2,\lambda=1);\ f_U(u)=\frac{1}{(1-u)^2},\ u\in(0,1/2)$. 3) $Y_n\xrightarrow{\text{d}}\mathrm{Espon}(1)$.