Es. 1

N = "n° mani per il giocatore A"

a) $N = \binom{52}{5}$

b) $P(4 \text{ assi ad } A) = \dfrac{48}{\binom{52}{5}}$

c) $P(\text{poker ad } A) = \dfrac{13 \cdot 48}{\binom{52}{5}}$

Es. 2 $\quad X, Y \sim Exp(\lambda)$

i) $V = \dfrac{Y}{X+Y} \in (0,1)$ p.c.

$$F_V(v) = \begin{cases} 0 & v \leq 0 \\ ? & 0 < v \leq 1 \\ 1 & v > 1 \end{cases}$$

$$F_V(v) = P\left(\frac{Y}{X+Y} < v\right) = P\left(Y < \frac{v}{1-v} X\right)$$

$$= \lambda^2 \int_0^{+\infty} e^{-\lambda x} \int_0^{\frac{v}{1-v}x} e^{-\lambda y}\, dy\, dx$$

$$= \lambda \int_0^{+\infty} e^{-\lambda x} \left(1 - e^{-\lambda x \frac{v}{1-v}}\right) dx$$

$$= 1 - \lambda \int_0^{+\infty} e^{-\frac{\lambda x}{1-v}} dx = 1 - (1-v) = v \qquad \text{per } 0 < v \leq 1$$

$$\Rightarrow \quad F_V(v) = \begin{cases} 0 & v \leq 0 \\ v & 0 < v \leq 1 \\ 1 & v > 1 \end{cases}$$

$$\Rightarrow \quad V \sim Unif(0,1)$$

ii) $F_{V^n}(v) = P(V^n < v) = P(V < v^{1/n}) = v^{1/n}$ per $0 < v \leq 1$

$$\Rightarrow \quad F_{V^n}(v) = \begin{cases} 0 & v \leq 0 \\ v^{1/n} & 0 < v \leq 1 \\ 1 & v > 1 \end{cases} \xrightarrow[n \to +\infty]{} \begin{cases} 0 & v \leq 0 \\ 1 & v > 1 \end{cases}$$

$$\Rightarrow \quad V^n \xrightarrow[P]{d} V = 0 \text{ p.c.}$$

Analogamente

$$F_{V^{1/n}}(v) = P(V < v^n) = \begin{cases} 0 & v \leq 0 \\ v^n & 0 < v \leq 1 \\ 1 & v > 1 \end{cases} \xrightarrow{n \to +\infty} \begin{cases} 0 & v \leq 1 \\ 1 & v > 1 \end{cases}$$

$$\Rightarrow V^{\frac{1}{n}} \xrightarrow[P]{d} V = 1 \text{ p.c.}$$